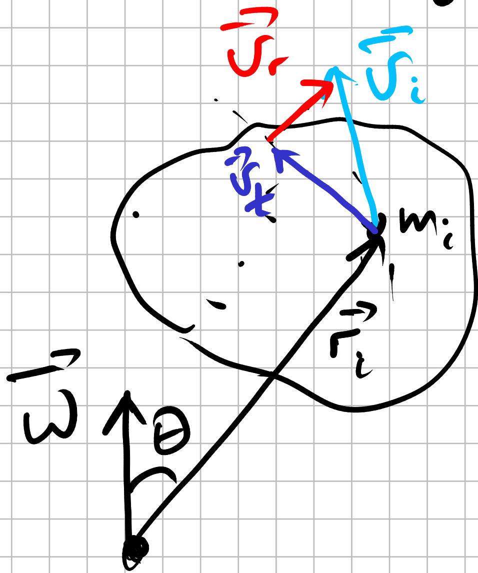
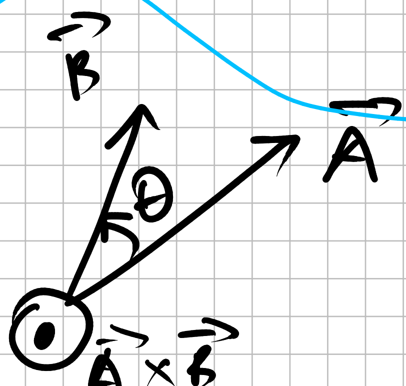
Kinetic energy for rotation around a fixed axis for rigid bodies



$$K = \sum_i \frac{m_i \vec{v}_i^{\,2}}{2} = \sum_i \frac{m_i\left(\cancelto{0}{\vec{v}_{r_i}} + \vec{v}_{t_i}\right)^2}{2}$$

$$= \sum_i \frac{m_i v_{t_i}^2}{2} =$$

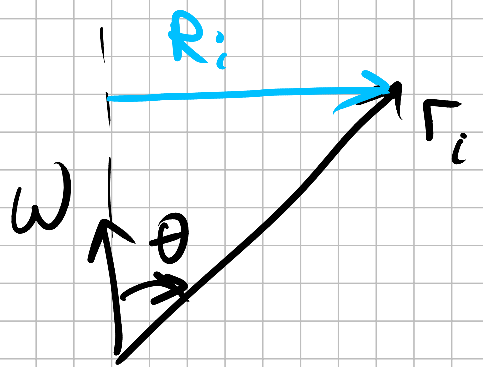
$$= \sum_j \frac{m_j}{2}\left(\underbrace{\vec{\omega}_j}_{=\vec{\omega}} \times \vec{r}_j\right)^2 = \sum_i \frac{m_i}{2}\left(\vec{\omega} \times \vec{r}_i\right)^2 =$$



$$\Longrightarrow \quad \vec{A} \times \vec{B} = A \cdot B \cdot \sin(\theta) \cdot \widehat{\text{direction}}$$

direction $\perp \vec{A}$ and $\perp \vec{B}$

$$K = \sum_i \frac{m_i}{2}\left(\vec{\omega}\times\vec{r_i}\right)^2 = \sum \frac{m_i}{2}\left(\omega\cdot \underbrace{r_i\cdot\sin\theta}_{R_i}\right)^2$$



distance to the axis

$$K = \underbrace{\left(\sum_i m_i R_i^2\right)}_{I_A \text{ - moment of inertia}} \cdot \frac{\omega^2}{2} \quad \Longleftrightarrow \quad m\frac{v^2}{2}$$

looks like

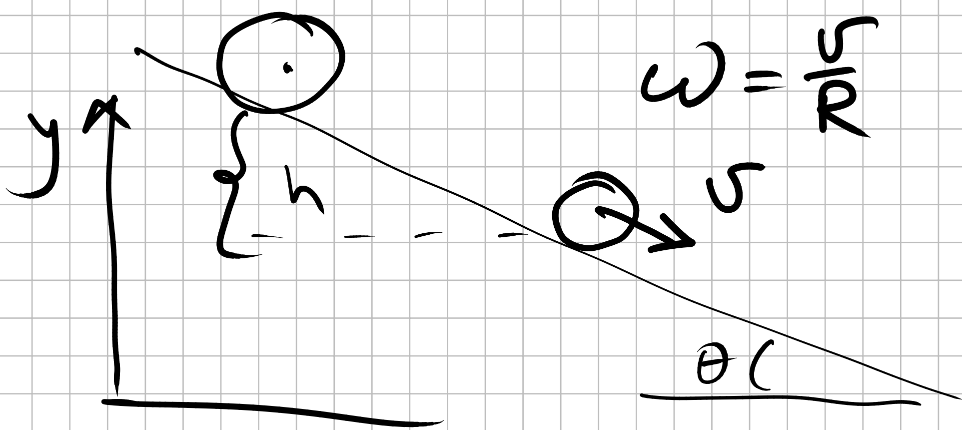
$$\boxed{K = I_A \frac{\omega^2}{2}}$$

relative to the axis

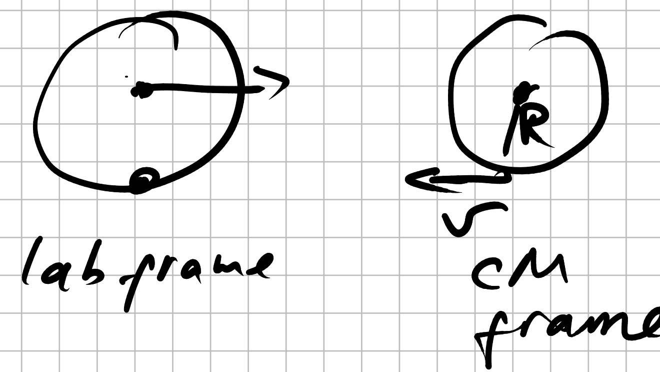
Kinetic energy of **moving** and **rotating** of a rigid body. (without derivation)

$$K = M_{total}\frac{v^2_{translation}}{2} + I_{CM}\frac{\omega^2}{2}$$

important
calculated with respect to CM



$$\omega = \frac{v}{R}$$



$$\omega = \frac{v}{R}$$

$$K + U = \frac{Mv^2}{2} + I_{cm} \cdot \frac{\omega^2}{2} + Mg\,y_f$$

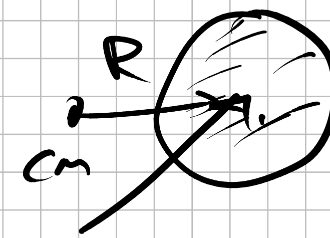
before $$= 0 + 0 + Mg\,y_i$$

$$\frac{Mv^2}{2} + I_{cm}\frac{\omega^2}{2} = Mg(y_i - y_f) = Mgh$$

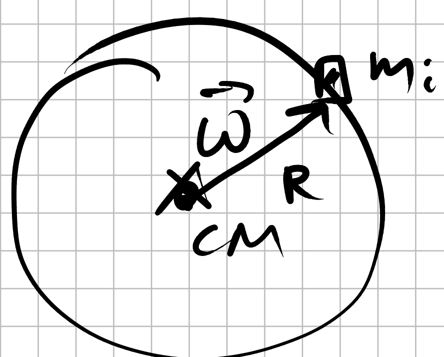
$$M\frac{v^2}{2} + \frac{I_{cm}\left(\frac{v}{R}\right)^2}{2} = Mgh$$

$$v^2 = \frac{2Mgh}{M + I_{cm}/R^2}$$

$$I_{CM} = \sum_i m_i R_i^2 = \int_V dm \cdot R^2$$



$$\vec{R}_{CM} = \frac{\sum m_i \vec{r}_i}{\sum_i m_i}$$



$$I_{\substack{loop \\ CM}} = \sum_i m_i R_i^2 = R^2 \sum_i m_i = M R^2$$

$$I_{\substack{disk \\ CM}} = \frac{M R^2}{2}$$