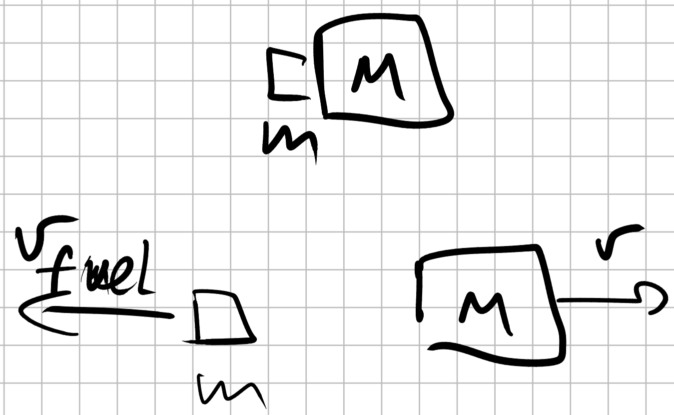
Rockets



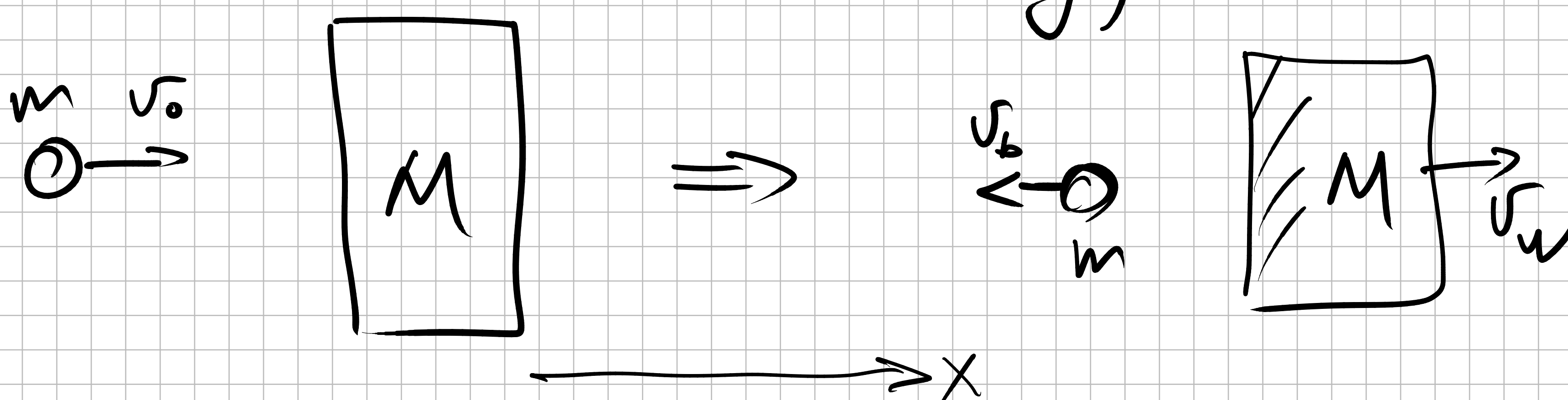
$$m v_{i_{fuel}}^{\;//0} + M v_{i_{Rocket}}^{\;//0}$$

$$\|$$

$$0 = m v_{f_{fuel}} + M v_{f_{Rocket}}$$

$$v_{f_{Rocket}} = -\frac{m v_{f_{fuel}}}{M}$$

Elastic bounce = Energy conserves



$$\vec{p_i} = \vec{p_f}$$

$$x: \quad m v_{0x} + M \cdot 0 = m v_{bx} + M v_{wx}$$

Energy conservation : $K_i = K_f$

$$\frac{m v_0^2}{2} + \frac{M \cdot 0^2}{2} = \frac{m v_b^2}{2} + \frac{M v_w^2}{2}$$

$$v_w = \frac{m v_0 - m v_b}{M} = \frac{m}{M}(v_0 - v_b) \quad (x)$$

$$\frac{m v_0^2}{2} = \frac{m v_b^2}{2} + \frac{M}{2} \cdot \left(\frac{m}{M}\right)^2 (v_0 - v_b)^2$$

$$v_0^2 = v_b^2 + \frac{m}{M}(v_0 - v_b)^2$$

$$|v_b| \le |v_0|$$

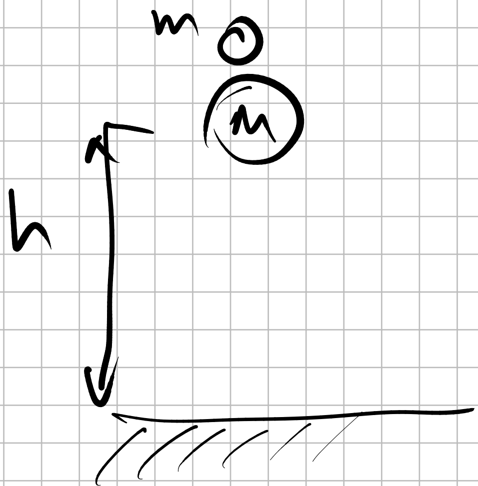
if $M \gg m$

$$v_0^2 \approx v_b^2$$

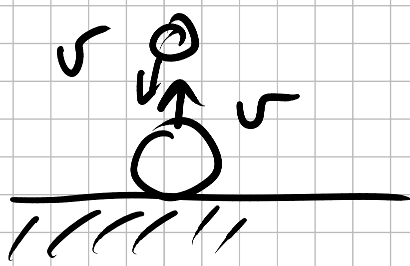
$$\boxed{\begin{aligned} v_{bx} &= -v_{0x} \\ v_w &\approx \frac{m}{M}\left(v_0 - (-v_0)\right) = \frac{2m}{M} v_0 \ll 1 \end{aligned}}$$

Light ball on top of the heavy ball

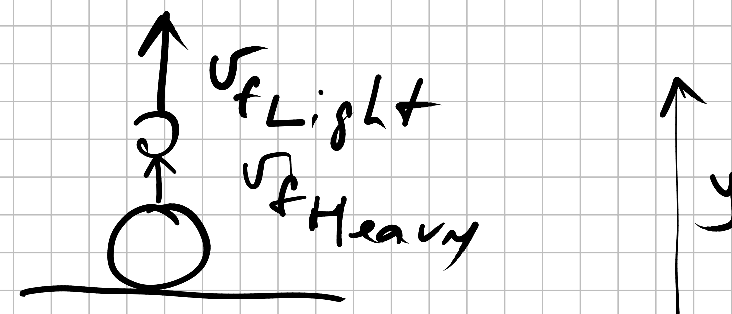
elastic case $\Rightarrow$ E = constant



$$mgh = \frac{mv^2}{2}$$



$$v = \sqrt{2gh}$$

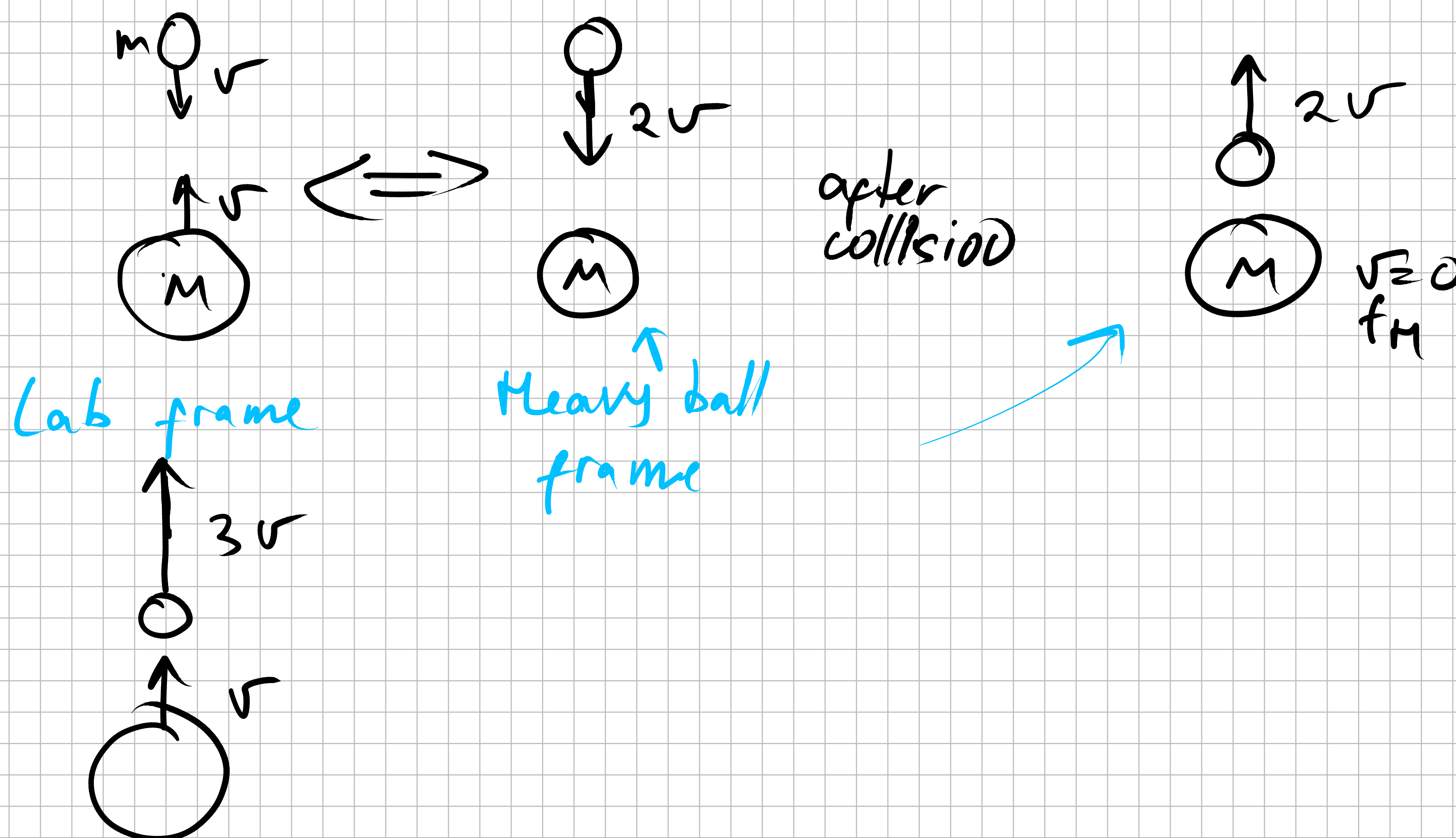


3 case

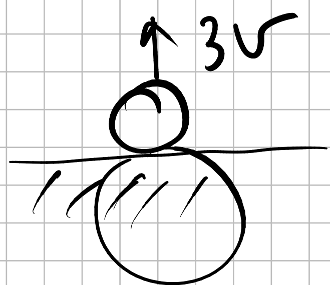
3 case: $\vec{P}_i = \vec{P}_f$ $\quad$ y: $-mv + Mv = mv_{fL} + Mv_{fH}$

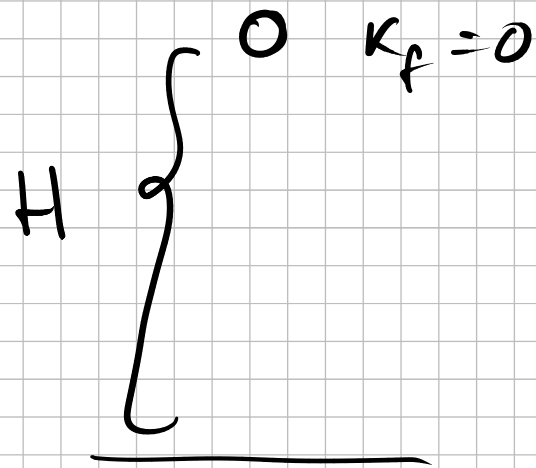
$K_i = K_f$ $\quad$ $\dfrac{mv^2}{2} + \dfrac{Mv^2}{2} = \dfrac{mv_{fL}^2}{2} + \dfrac{Mv_{fH}^2}{2}$

Trick: we move to the heavy ball reference frame



How high is the light ball





$$\frac{m(3v)^2}{2} = mgH$$

$$3^2 \cdot \frac{mv^2}{2} = 3^2 \frac{m}{2}\left(\sqrt{2gh}\right)^2$$

$$= 3^2 \frac{m}{\cancel{2}} \cancel{2}\cancel{g}h = m\cancel{g}H$$

$$\boxed{H = 3^2 \cdot h = 9 \cdot h}$$