Linear momentum

$$\vec{P} = m\vec{v}$$

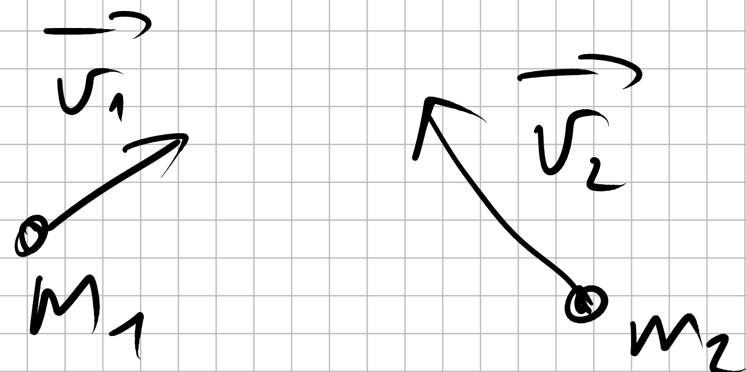
$$\frac{d\vec{P}}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a} = \vec{F}_{ext}$$

$$\approx$$

$$\frac{\Delta\vec{P}}{\Delta t} = \frac{\vec{P}_f - \vec{P}_i}{t_f - t_i}$$

Car collision $v_f = 0$, v_i:

$$\frac{\Delta\vec{P}}{\Delta t} = \frac{m \cdot 0 - m v_{ix}}{\Delta t}\,\hat{i}$$



$$\frac{d}{dt}\left(\vec{P} = m_1\vec{v}_1 + m_2\vec{v}_2\right)$$

$$\frac{d\vec{P}}{dt} = m_1\vec{a}_1 + m_2\vec{a}_2$$

$$\vec{F}_{1ext} + \vec{F}_{12} + \vec{F}_{2ext} + \vec{F}_{21}$$

on from

$$\vec{F}_{21} = -\vec{F}_{12}$$

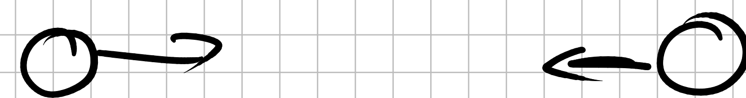
3rd Newton's law

$$\boxed{\frac{d\vec{P}}{dt} = \vec{F}_{1ext} + \vec{F}_{2ext} = \vec{F}_{Net\,ext}}$$

If there are no external forces
$\Longrightarrow$ total momentum conserves

Collisions $\nearrow$ elastic $\Longleftrightarrow$ Energy conserves
$\searrow$ inelastic $\Longleftrightarrow$ Energy does not conserve

1D



$m_1 \ v_{1x}$ $\qquad$ $m_2 \ v_{2x}$

$$\begin{cases} m_1 v_{1x_i} + m_2 v_{2x_i} = m_1 v_{1x_f} + m_2 v_{2x_f} \\ \text{elastic collision} \\ \dfrac{m_1 v_{1x_i}^2}{2} + \dfrac{m_2 v_{2x_i}^2}{2} = \dfrac{m_1 v_{1x_f}^2}{2} + \dfrac{m_2 v_{2x_f}^2}{2} \end{cases}$$

Let's assume $m_1 = m_2 = m$

we know that everything happens along 'X' so we drop it

$v_{2i} = 0$

$$\begin{cases} \cancel{m} v_{1i} + 0 = \cancel{m} v_{1f} + \cancel{m} v_{2f} & \text{eq. 1} \\ \cancel{m} \dfrac{v_{1i}^2}{\cancel{2}} + 0 = \cancel{m} \dfrac{v_{1f}^2}{\cancel{2}} + \cancel{m} \dfrac{v_{2f}^2}{\cancel{2}} & \text{eq 2} \end{cases}$$

$$v_{1i}^2 = (v_{1f} + v_{2f})^2 \Leftarrow (\text{eq.1})^2$$

eq1 $\Rightarrow$

$$v_{1i}^2 = v_{1f}^2 + v_{2f}^2 + \underbrace{2 v_{1f} v_{2f}}_{\text{must be } 0}$$

eq.2

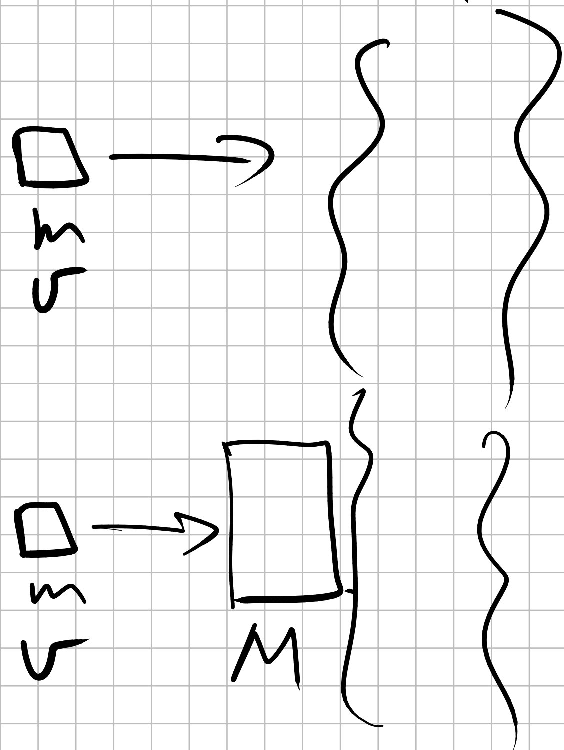
$$v_{1i}^2 = v_{1f}^2 + v_{2f}^2$$

$\Downarrow$

either $v_{1f} = 0$ or $v_{2f} = 0$

if $v_{1f} = 0 \iff v_{2f} = v_{1f}$

Hit Professor Demo



Energy to absorb

Spring Energy

$$\frac{mv^2}{2} = \frac{KX^2}{2}$$

$$m v_i + M\cdot 0 = \overbrace{m v_f + M v_f}^{\text{Stick together}}$$

$$v_f = \frac{m}{m+M} v_i$$

New kinetic energy $K = (M+m)\frac{v_f^2}{2}$

$$= \frac{(M+m)}{2}\left(\left(\frac{m}{m+M}\right) v_i\right)^2 = \frac{m}{m+M}\cdot\frac{m v_i^2}{2} \ll \frac{m v_i^2}{2}$$