Useful numbers

$G = 6.67 \cdot 10^{-11} \left[\frac{N \cdot m^2}{kg^2}\right]$

$g = 9.8 \left[m/s^2\right]$

$R_E = 6.38 \cdot 10^6 \, [m]$ Earth radius

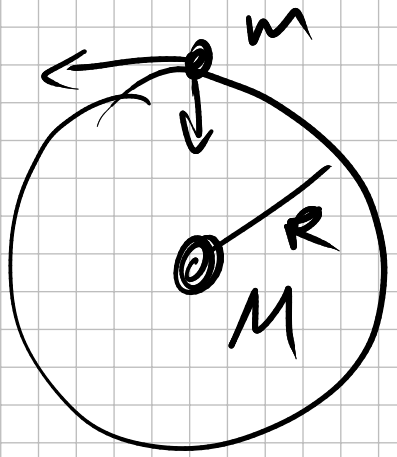
Earth - Moon distance $= 384 \cdot 10^6 \, [m]$

Earth - Sun distance $= 149.6 \cdot 10^9 \, [m]$

Moon - Earth period $= 27.3$ days

$M_E = 5.98 \cdot 10^{24} \, [kg]$ Earth mass

$M_M = 7.35 \cdot 10^{22} \, [kg]$ Moon mass



$$ma = F_g = G\frac{mM}{R^2}$$

$$\frac{F_g}{m} = a = \frac{v^2}{R} = \frac{(2\pi R/P)^2}{R} \leftarrow \text{period}$$

$$v = \sqrt{\frac{GM}{R}}$$

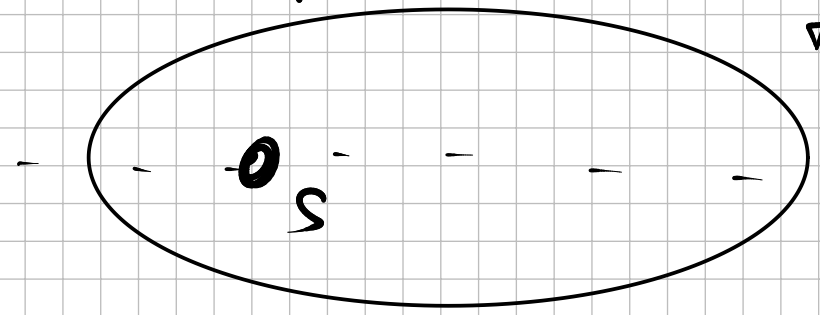
$$\frac{(2\pi)^2 R}{P^2} = G\frac{M}{R^2}$$

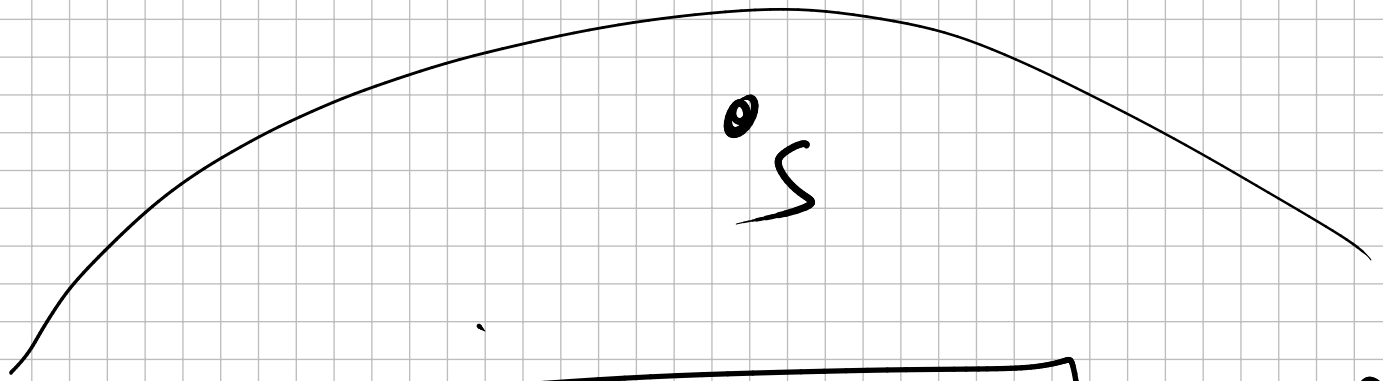
$$R^3 = \frac{G}{4\pi^2} M \cdot P^2$$

$$R^3 \sim P^2$$

2nd Kepler's Law

All planets move in ellipses around the sun

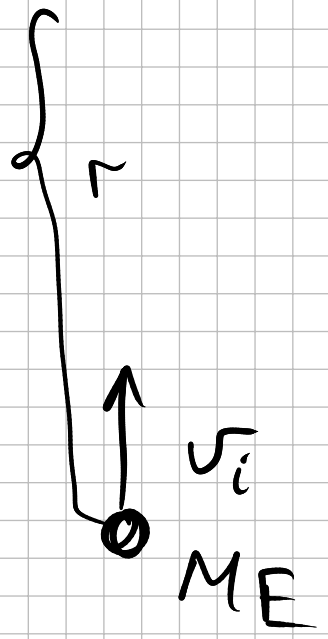




ellipses – closed

parabolic, hyperbolic } open

$$U_g = -G\frac{M_1 M_2}{r}$$



$$K + U = \text{const}$$

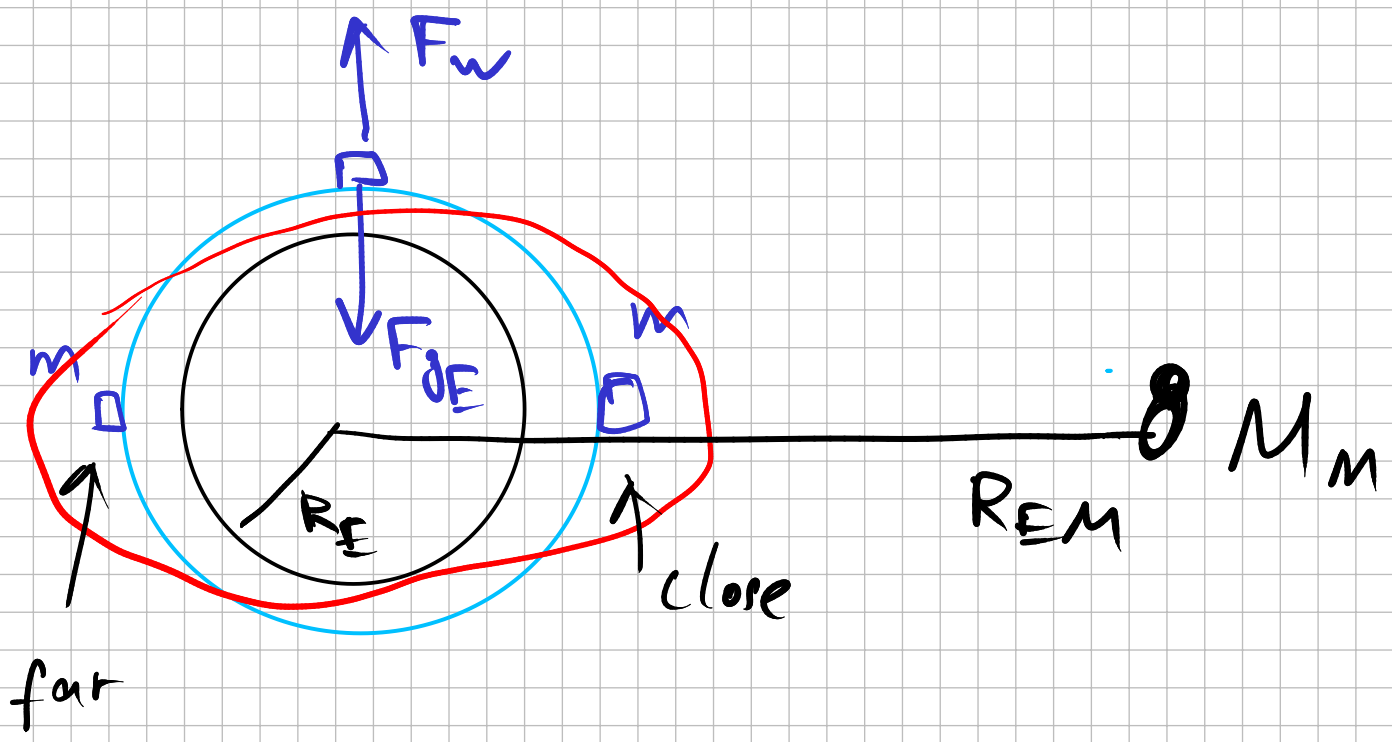
$$K_i + U_i = K_f + U_f$$

$$\frac{m v_i^2}{2} - \frac{G M_E m}{R_E} = \frac{m v_f^2}{2} - \frac{G M_E m}{r}$$

($v_f = 0$, $r \to \infty$, $\frac{G M_E m}{r} = 0$)

$$v_{escape} = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{2}\, v_{orbital}$$

Tides - water rise and fall



Close

$$F_{gM} + F_{water} = 0$$

$$F_{aver} + F_{gextra} + F_{water\ average} + F_{Wextra} = 0$$

close
far

$$G\frac{m M_M}{(R_{EM} \mp R_E)^2} + F_{water\ average} \pm F_{Wextra} = 0$$

$$G\frac{m M_m}{(R_{EM} \mp R_E)^2} = \frac{G m M_m}{R_{EM}^2 \left(1 \mp \frac{R_E}{R_{EM}}\right)^2}$$

($\frac{R_E}{R_{EM}} << 1$)

$$\simeq G\frac{m M_m}{R_{EM}^2}\left(1 \pm 2\frac{R_E}{R_{EM}}\right)$$

Forces:

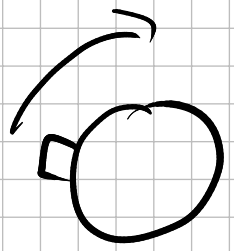
$$\frac{G m M_m}{R_{EM}^2} \mp 2\frac{G m M_m}{R_{EM}^2}\frac{R_E}{R_{EM}} + F_{w\,average} + F_{w\,extra} \neq 0$$

$\frac{G m M_m}{R_{EM}^2} \rightarrow 0$

close / far: $F_{w\,extra} = \mp 2G\frac{m M_m}{R_{EM}^3} R_E$

$\approx$ $h_{tide} \sim F_{w\,extra}$

Moon side lock in



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