Useful numbers

$G = 6.67 \cdot 10^{-11} \left[\frac{N \cdot m^2}{kg^2}\right]$

$g = 9.8 \left[m/s^2\right]$

$R_E = 6.38 \cdot 10^6 \, [m]$ Earth radius

Earth - Moon distance $= 384 \cdot 10^6 \, [m]$

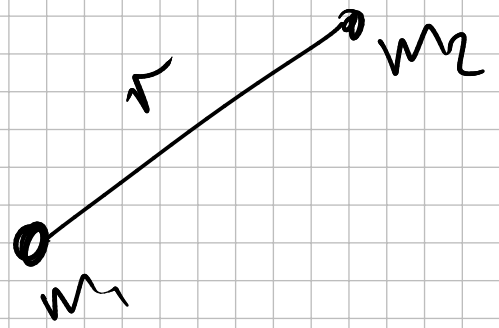
Earth - Sun distance $= 149.6 \cdot 10^9 \, [m]$

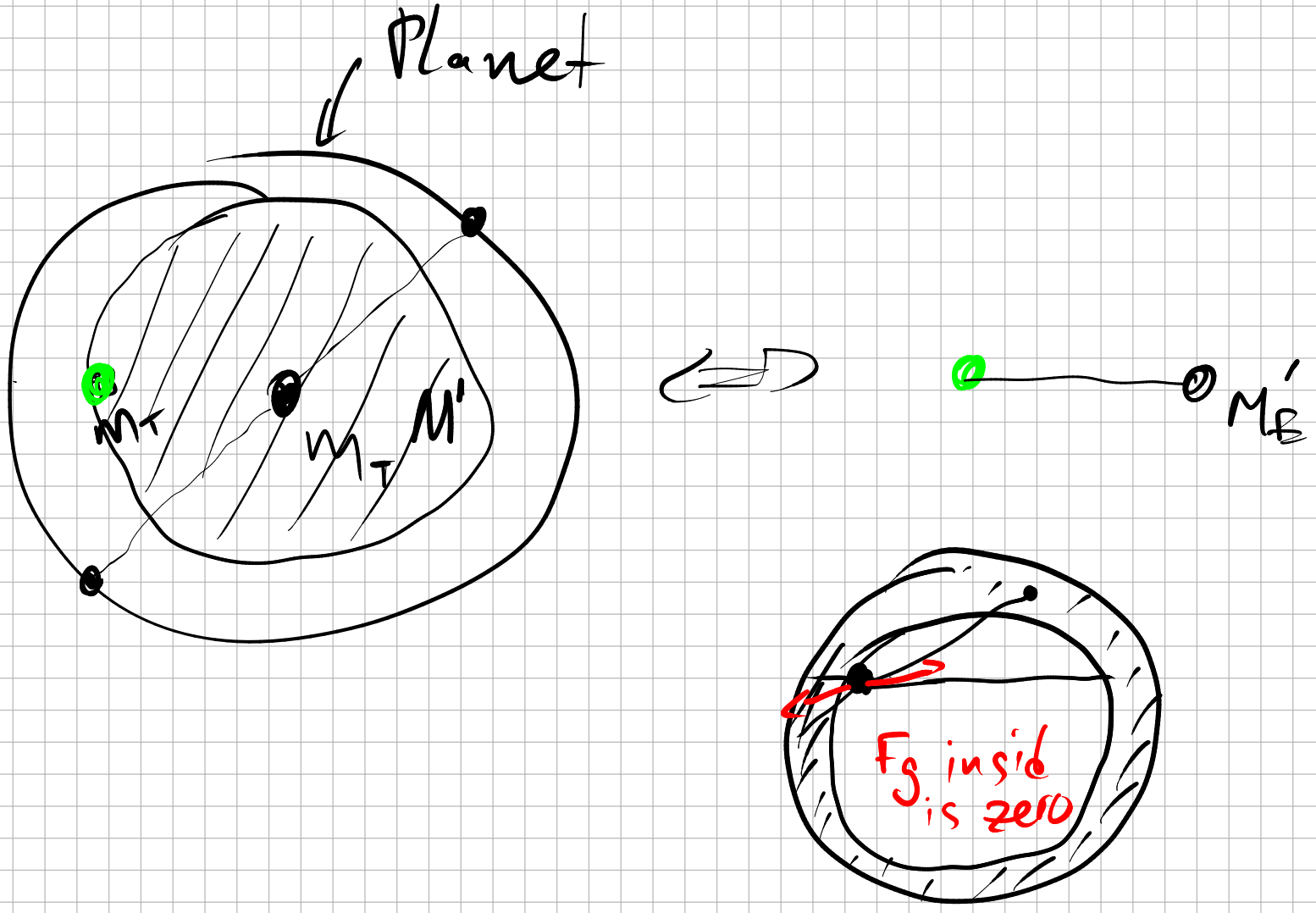
Moon - Earth period $= 27.3$ days

$M_E = 5.98 \cdot 10^{24} \, [kg]$ Earth mass

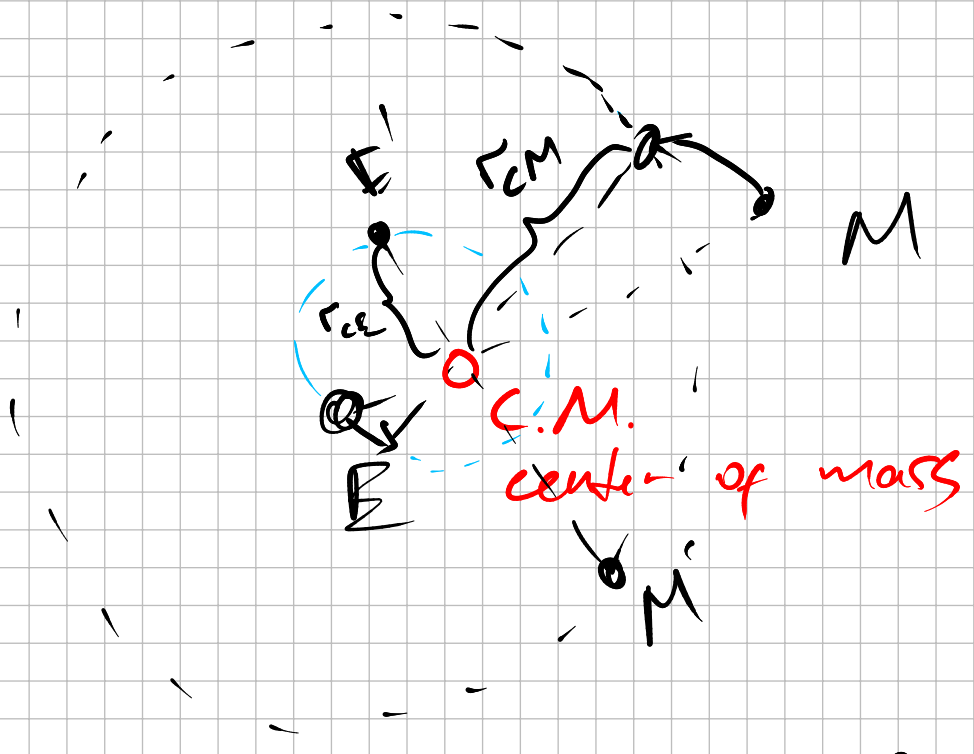
$M_M = 7.35 \cdot 10^{22} \, [kg]$ Moon mass

$$F_g = G \frac{m_1 m_2}{r^2}$$





Earth – Moon rotation

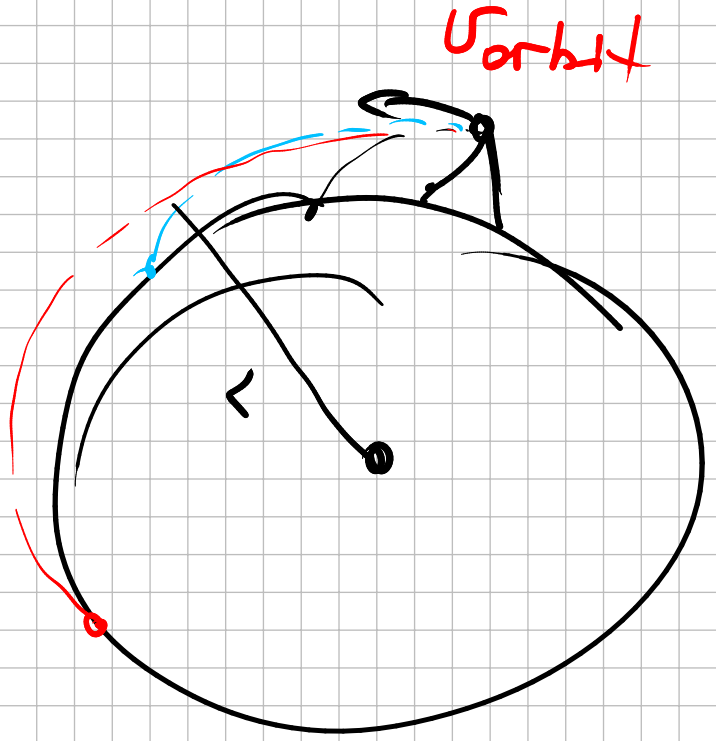


$$\omega^2 r_{MC} \;//\; m_M a_M = F_g = G\frac{m_M m_E}{r_{EM}^2}$$

$$m_E \cdot a_E = F_g = G\frac{m_M m_E}{r_{EM}^2} \;//\; \omega^2 r_{EC}$$

$$m_M \omega^2 r_{MC} = F_g = m_E \omega^2 r_{EC}$$

$$\boxed{\frac{r_{EC}}{r_{MC}} = \frac{M_E}{M_M}}$$

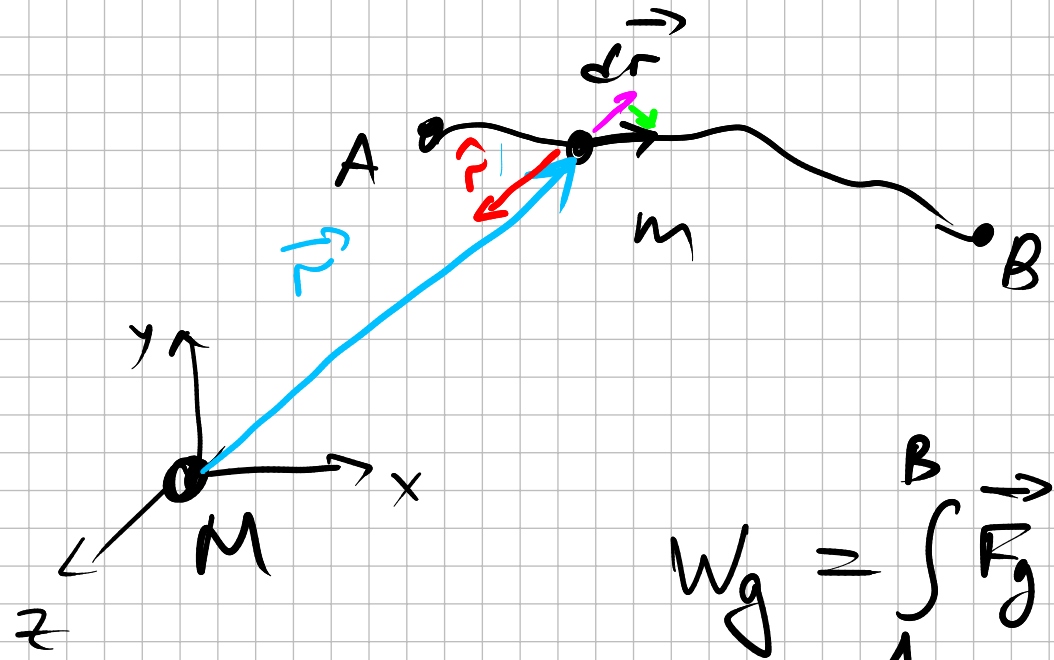


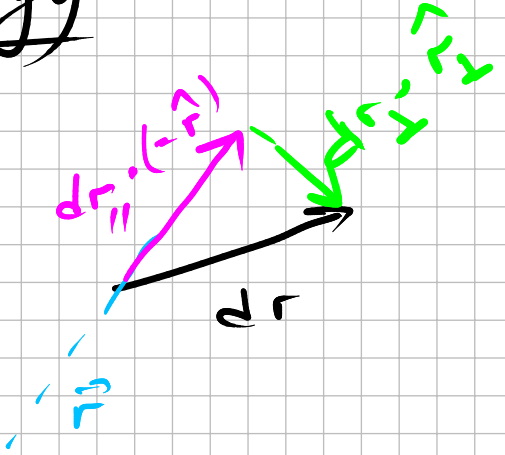
$$m a_c = \overset{v_{orb}^2/R_E}{\overset{\shortparallel}{m}} = F_g = G \frac{m M_E}{r^2}$$

$$= G \frac{m M_E}{R_E^2} = mg$$

$$v_{orb} = \sqrt{\frac{G M_E}{R_E}} \approx 8\ km/s$$

Potential Energy





$$W_g = \int_A^B \vec{F}_g \, d\vec{r} =$$

$$= \int_A^B G \frac{M m}{|\vec{r}|^2} \cdot \hat{r} \cdot d\vec{r}$$

$|\vec{r}|^2 = r^2$

$\hat{r} = -\frac{\vec{r}}{r}$

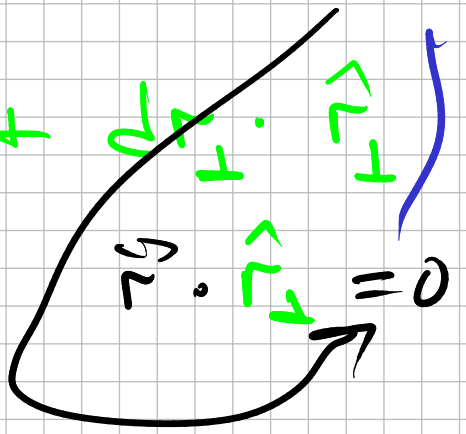
$d\vec{r}$ = change of position

$$= \int_A^B G \frac{M m}{r^3} \vec{r} \cdot d\vec{r}$$

$$\vec{r} \cdot d\vec{r} = \vec{r} \cdot \left(dr_{\parallel} \cdot (-\hat{r}) + d\vec{r}_{\perp} \cdot \hat{r}_{\perp} \right)$$

$$= -r \cdot dr$$

↑ change of distance



$$W_g = \int_A^B G \frac{Mm}{r^2} (-\hat{r}) dr = \int_A^B -G \frac{Mm}{r^2} dr$$

$$= + G \frac{mM}{r} \Big|_A^B = G \frac{Mm}{r_B} - G \frac{Mm}{r_A} = W_g$$

$$W_g = -\Delta U = -(U_B - U_A)$$

$$\boxed{U = -G \frac{Mm}{r}}$$