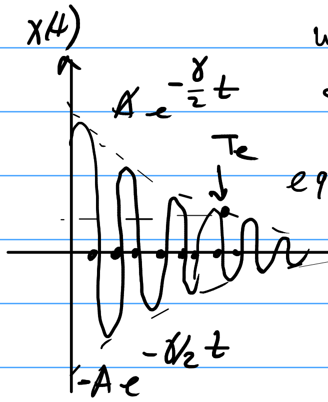
Damped oscillations

$$\ddot{x} = -\omega_0^2 x - \gamma \dot{x}$$

$$x(t) = \underbrace{A_0 \cdot e^{-\frac{\gamma}{2}t}}_{A(t)} \cdot \cos(\omega_0 t + \varphi)$$



weakly damped case

equidistant

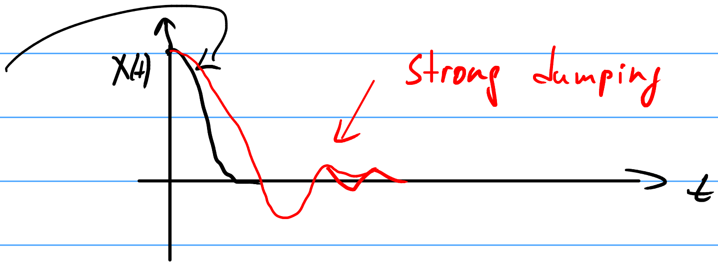
$$A(T_e) = \frac{A_0}{e} = A_0 e^{-\frac{\gamma}{2}T_e} \implies -1 = -\frac{\gamma}{2}T_e = -\frac{\gamma}{2}T\cdot N$$

$\gamma \ll \omega_0$

$$1 = \frac{\gamma}{2}T\cdot N = \frac{\gamma}{\cancel{2}}\frac{\cancel{2}\pi}{\omega_0}N = \frac{\gamma}{\omega_0}\pi N = 1$$

$$\boxed{N = \frac{1}{\pi}\frac{\omega_0}{\gamma} = \frac{1}{\pi}Q, \quad Q = \frac{\omega_0}{\gamma}}$$

Over damped

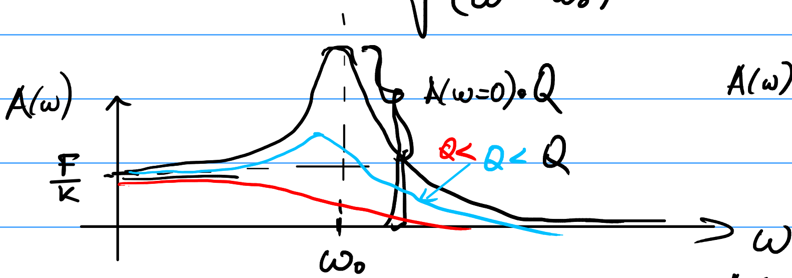


Driven oscillation

$$\ddot{x} = -\omega_0^2 x - \gamma\dot{x} + \frac{F}{m}\cdot\cos(\omega t)$$

$$x(t) = A(\omega)\cos(\omega t + \varphi)$$

$$A(\omega) = \frac{F}{m}\frac{1}{\sqrt{(\omega^2-\omega_0^2)^2 + (\gamma\cdot\omega)^2}}$$

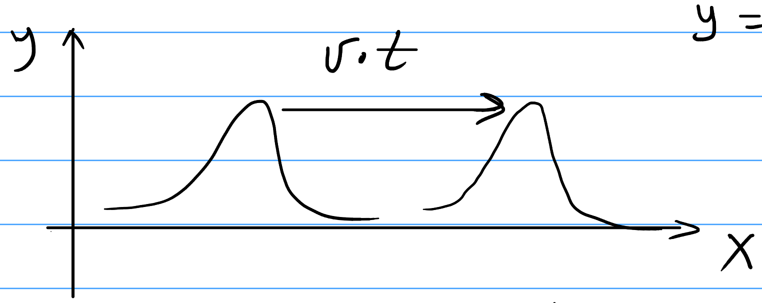


$$A(\omega) = \frac{F}{m}\frac{1}{\omega_0^2} = \frac{F}{\cancel{m}}\frac{1}{K/\cancel{m}}$$

$$A(\omega=\omega_0) = \frac{F}{m}\frac{1}{\sqrt{(\gamma\cdot\omega)^2}} = \frac{F\,Q}{m\omega_0^2}$$

$$\gamma = \frac{\omega_0}{Q}$$

Pulses



$$y = f(x \mp vt)$$
$$= f(x - v_x t)$$

$$y(x,t) = y(u) \quad , \quad u = x - v_x t$$

$$\frac{\partial y}{\partial t} = \underbrace{\left(\frac{d}{du} y\right)}_{y'} \cdot \frac{\partial u}{\partial t} = y' \cdot \frac{\partial (x - v_x t)}{\partial t} = y'(-v_x)$$

$$\frac{\partial}{\partial t}\left(\frac{\partial y}{\partial t}\right) = \underbrace{y''}_{= \frac{d^2 y}{du^2}} \cdot (-v_x)^2$$

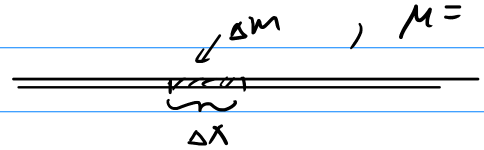
$$\frac{\partial y}{\partial x} = \underbrace{\left(\frac{d}{du} y\right)}_{\text{„}y'\text{"}}\left(\frac{\partial u}{\partial x}\right) = y' \, \frac{\partial (x - v_x t)}{\partial x} = y' \cdot 1$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x} y\right) = y''$$

$$\boxed{\frac{1}{v_x^2} \frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2} \qquad \text{Pulse propagation equation}}$$

$$v = \sqrt{\frac{F_T}{\mu}}$$

tension



$, \quad \mu = \frac{\Delta m}{\Delta x}$ linear mass density