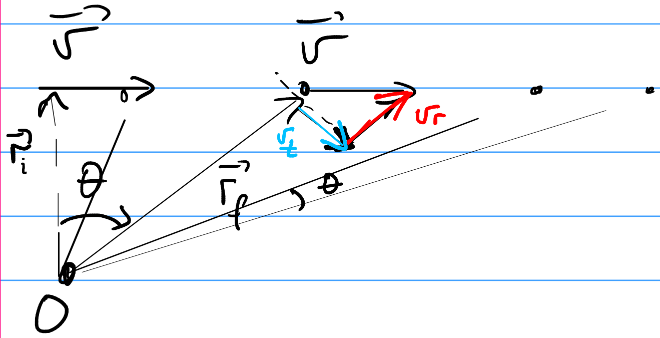
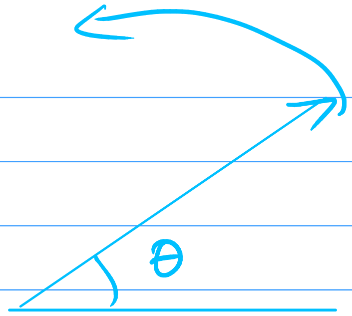
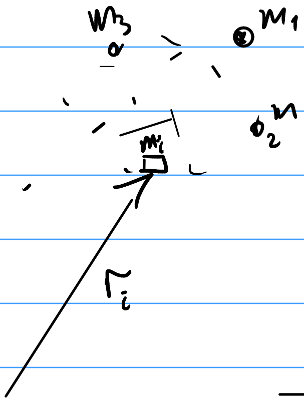




$$\vec{\omega} = \frac{d\vec{\theta}}{dt}$$

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$$



$$K = \sum_i \frac{m_i \vec{v}_i^{\,2}}{2} =$$

$$= \sum_i m_i \frac{(\vec{v}_{r_i} + \vec{v}_{t_i})^2}{2} =$$

$$= \sum_i m_i \frac{\left(\vec{v}_{r_i}^{\,2} + \vec{v}_{t_i}^{\,2} + \cancel{2\vec{v}_{r_i} \cdot \vec{v}_{t_i}}\right)}{2}$$

Obs. $\vec{v}_r \perp \vec{v}_t$

$$= \sum_i \frac{m_i v_{r_i}^2}{2} + \sum \frac{m_i v_{t_i}^2}{2} =$$

$$= \underbrace{\sum_i \frac{m_i v_{r_i}^2}{2}}_{K_{translational}} + \underbrace{\sum \frac{m_i (\omega_i \cdot r_i)^2}{2}}_{K_{rotational}}$$

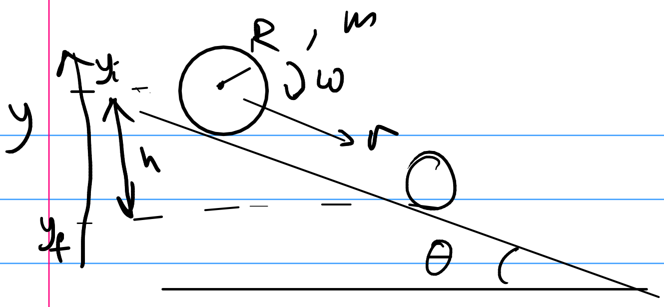
= / We will talk about rigid bodies only / $\Rightarrow \omega_i = \omega$

$$K_{rot} = \underbrace{\left(\sum_i m_i r_i^2\right)}_{I_A} \frac{\omega^2}{2} = I \frac{\omega^2}{2}$$

I_A — moment of inertia with respect to axis of rotation

$$K_{rot} = I \frac{\omega^2}{2} \quad \Bigg| \quad K_{transl} = m \frac{v^2}{2}$$

Linear	Rotational
m	I_A
position $\longrightarrow \vec{r}$	$\vec{\theta}$
$\frac{d\vec{r}}{dt} = \vec{v}$	$\frac{d\vec{\theta}}{dt} = \vec{\omega}$
$\vec{a} = \frac{d\vec{v}}{dt}$	$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$
iff $\vec{a} = \text{const} \Rightarrow \vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{\vec{a}t^2}{2}$	$\vec{\theta}_f = \vec{\theta}_i + \vec{\omega}_i t + \frac{\vec{\alpha}t^2}{2}$
$K_t = \frac{mv^2}{2}$	$K_r = I\frac{\omega^2}{2}$
$m\vec{a} = \vec{F}$	$I\vec{\alpha} = \vec{\tau} = \vec{r} \times \vec{F}$ torque (tau)
$\vec{p} = m\vec{v}$ momentum	$\vec{L} = I\vec{\omega}$ angular momentum
$\frac{d\vec{p}}{dt} = \vec{F}$	$\frac{d\vec{L}}{dt} = \vec{\tau}$
$dW = \vec{F} \cdot d\vec{r}$	$dW = \vec{\tau} \cdot d\vec{\theta}$
iff $\vec{F} = \text{const} \Rightarrow P = \vec{F} \cdot \vec{v}$ power	$P = \vec{\tau} \cdot \vec{\omega}$



$$E = \frac{mv^2}{2} + I\frac{\omega^2}{2} + mgy = const$$

$$E_f - E_i = 0$$

$$\frac{mv_f^2}{2} + I\frac{\omega_f^2}{2} + mgy_f -$$

$$- \cancel{\frac{mv_i^2}{2}}^{\,0} - I\cancel{\frac{\omega_i^2}{2}}^{\,0} - mgy_i = 0$$

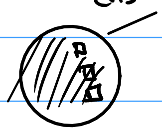
$$\omega = \frac{v}{R}$$

$$m\frac{v_f^2}{2} + I\frac{\omega_f^2}{2} = mg(y_i - y_f) = mgh$$

$$m\frac{v^2}{2} + \frac{I}{R^2}\frac{v^2}{2} = mgh$$

Ring $\rightarrow$ $I = \sum m_i r_i^2 = \left(\sum m_i\right) R^2 = MR^2$

disk



$$I = \sum m_i r_i^2 = \frac{1}{2} MR^2$$

$$v^2 = \frac{2 \cdot mgh}{m + \frac{I}{R^2}}$$