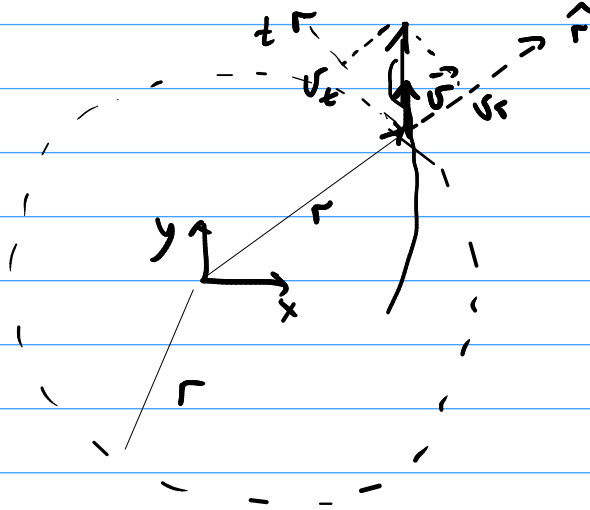


Cartesian $\vec{r} = \{x, y\}$

Polar $\vec{r} = \{r, \theta\}$

$$x = r \cdot \cos\theta, \quad y = r \cdot \sin\theta$$



$$\vec{v} = \{v_r, v_t\}$$

$$v_t = \omega \cdot r$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \left\{ \frac{d(v_r)}{dt}, \frac{d(v_t)}{dt} \right\}$$

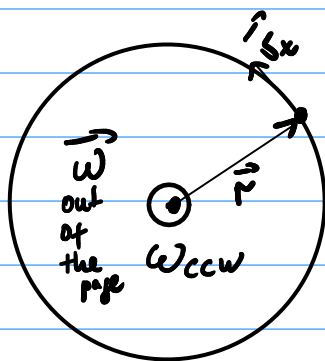
$$= \left\{ a_r, \left(\frac{d\omega}{dt} \right) \cdot r \right\} =$$

$$= \left\{ a_r, \alpha \cdot r \right\}$$

$$(-\hat{r}) \frac{v^2}{r} = \vec{a}_c$$

$$\vec{v}_t = v_t \cdot \hat{t} = \vec{\omega} \times \vec{r}$$

↑ vector product

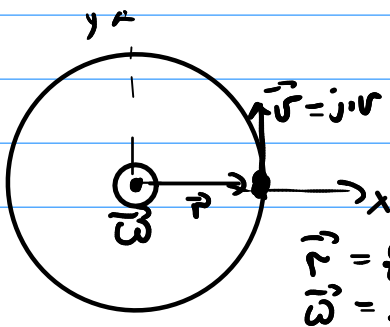


$$\vec{\omega} \times \vec{r} = \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_x & \omega_y & \omega_z \\ r_x & r_y & r_z \end{pmatrix} =$$

$$= \hat{i} (\omega_y r_z - \omega_z r_y) +$$

$$+ \hat{j} (\omega_z r_x - \omega_x r_z) +$$

$$+ \hat{k} (\omega_x r_y - \omega_y r_x)$$



$$\vec{r} = \{r, 0, 0\}$$

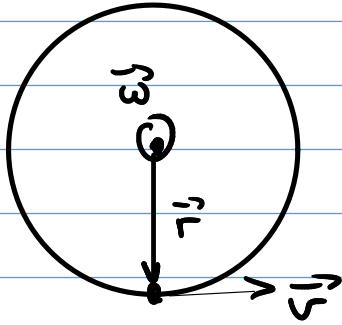
$$\vec{\omega} = \{0, 0, \omega\}$$

$$\vec{\omega} \times \vec{r} = \hat{i} \cdot 0 + \hat{j} (\omega \cdot r - 0) + \hat{k} (0) = \hat{j} \omega r$$

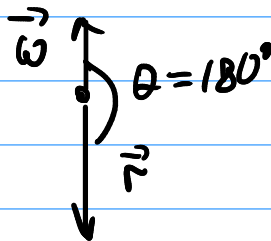
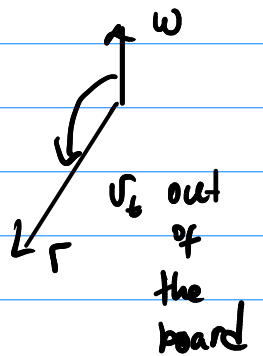
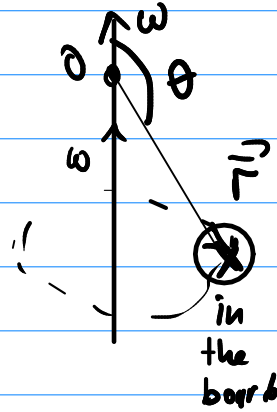
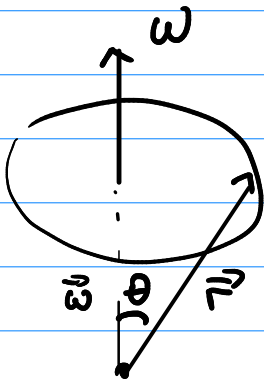
$$\hat{\omega} \times \hat{r} = \hat{\omega} + \hat{r} \times \hat{\omega}$$

$$\hat{\omega} = \{0, 0, \omega\}$$

$$\hat{r} = \{0, -r, 0\}$$



$$\left(\hat{\omega} \times \hat{r} \right)_t = \omega \cdot r \cdot \sin \theta$$



example

$$v_0 = 100 \text{ m/s}$$

$$r = 0.1 \text{ m}$$

$$t = 20 \text{ s to } v_f = 0$$

$$\omega_0 = \frac{v_0}{r} \quad - \omega^2 r - \text{centripetal}$$

$$\vec{a} = \{a_r, a_t\}$$

$$\frac{d\vec{\omega}}{dt} = \vec{\alpha} = \frac{d\omega}{dt} \hat{e}_3 = \frac{(v_f - v_0)}{\Delta t} \hat{e}_3 = -\frac{v_0}{r \Delta t} (\hat{\omega}) \quad \text{out of the board}$$

$$\omega_f = \omega_i + \alpha \Delta t$$

