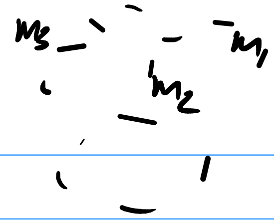
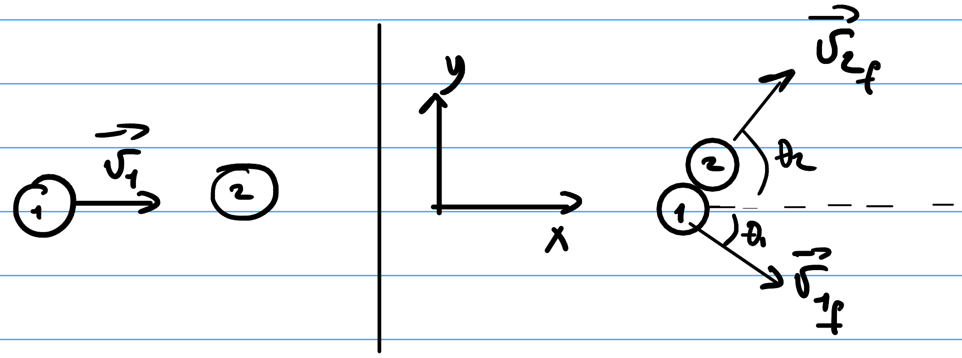
$$\vec{F}_{ext_{net}} = \frac{d\vec{P}}{dt} = \frac{d}{dt}\sum_i \vec{P}_i$$





$$0 = \vec{F}_{ext} = m_1\vec{v}_1 + m_2\cancel{\vec{v}_2}^{\,0} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

x: $m_1 v_1 = m_1 v_{1fx} + m_2 v_{2fx}$

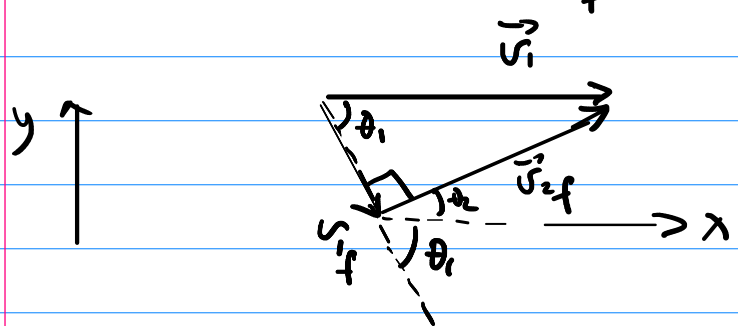
y: $0 = m_1 v_{1fy} + m_2 v_{2fy}$

Elastic collision $\iff$ Energy conserves

$$\frac{m_1 v_1^2}{2} = \frac{m_1 v_{1f}^2}{2} + \frac{m_2 v_{2f}^2}{2}$$

Simplification $m_1 = m_2$

$$v_1^2 = v_{1f}^2 + v_{2f}^2 \qquad \Big| \qquad a^2 = b^2 + c^2$$



$\theta_1 + \theta_2 = 90°$

$m_1 = m_2$

x: $\cancel{m_1} v_1 = \cancel{m_1} v_{1f} \cdot \cos\theta_1 + \cancel{m_2} v_{2f} \cos\theta_2$

y: $0 = -\cancel{m_1} v_{1f} \sin\theta_1 + \cancel{m_2} v_{2f} \sin\theta_2$

$$x: \quad v_1 = v_{1f}\cos\theta_1 + v_{2f}\cos\theta_2$$

$$y: \quad 0 = -v_{1f}\sin\theta_1 + v_{2f}\sin\theta_2$$

$\theta_2 = 90-\theta_1$

$\cos(90-\theta) = \sin\theta$

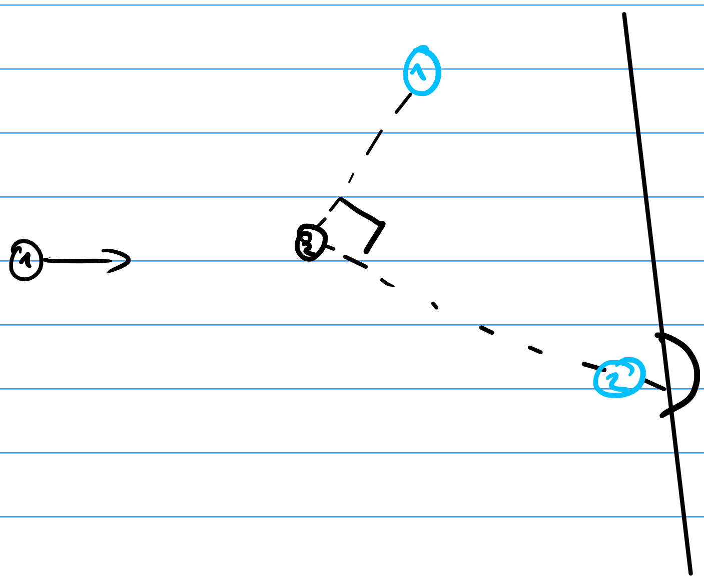
$\sin(90-\theta) = \cos\theta$

$$x: \quad v_1 = v_{1f}\cos\theta_1 + v_{2f}\sin\theta_1$$

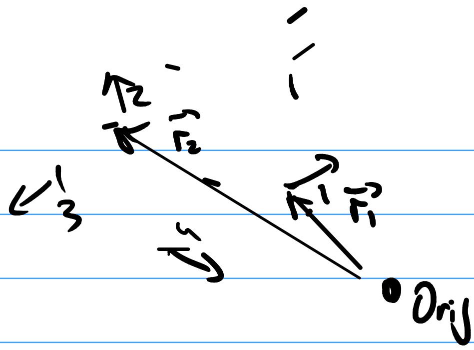
$$y: \quad 0 = -v_{1f}\sin\theta_1 + v_{2f}\cos\theta_1 \implies \underline{v_{2f} = v_{1f}\frac{\sin\theta_1}{\cos\theta_1}}$$

$$v_1 = v_{1f}\frac{\cos^2\theta_1}{\cos\theta_1} + v_{1f}\frac{\sin^2\theta_1}{\cos\theta_1} = v_{1f}\frac{1}{\cos\theta_1}$$

$$\boxed{\begin{aligned} v_{1f} &= v_1\cos\theta_1 \\ v_{2f} &= v_1\sin\theta_1 \end{aligned}}$$



Center of mass

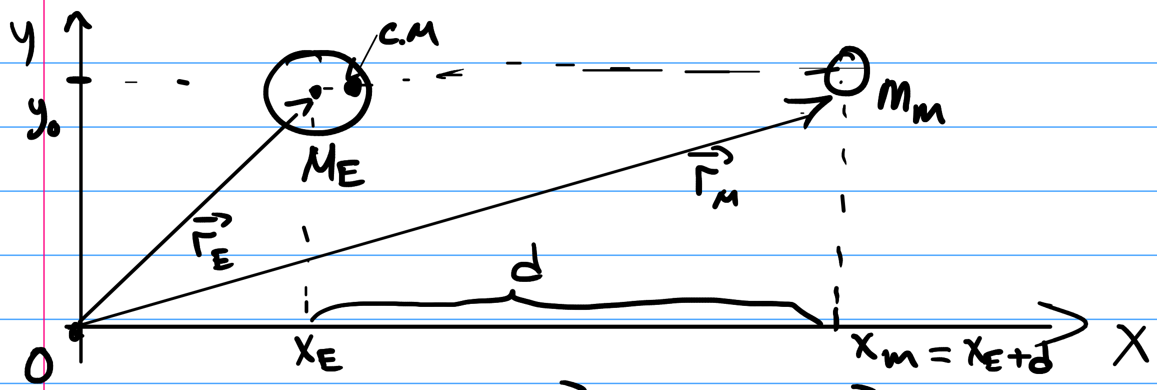


$$\vec{F}_{\substack{tot\\ext}} = \frac{d\vec{P}_t}{dt} = \frac{d}{dt}\sum_i \vec{p}_i = \frac{d}{dt}\sum_i m_i \vec{v}_i$$

$$= \frac{d}{dt}\sum_i m_i \frac{d\vec{r}_i}{dt} = \frac{d}{dt}\left(\frac{d}{dt}\underbrace{\sum_i m_i\vec{r}_i}_{M\vec{r}_{cm}}\right)$$

$$\vec{r}_{cm} = \frac{\sum m_i\vec{r}_i}{\sum m_i} = \frac{\sum m_i \vec{r}_i}{M_{tot}}$$

$$\vec{F}_{\substack{ext\\tot}} = \frac{d^2}{dt^2} M_{tot}\vec{r}_{cm} = M_{tot}\frac{d^2}{dt^2}\vec{r}_{cm} = M_{tot}\vec{a}_{cm}$$



$$\vec{r}_{cm} = \frac{M_E \cdot \vec{r}_E + m_M \vec{r}_m}{M_E + m_M}$$

$$x: \quad x_{cm} = \frac{M_E \cdot x_E + m_m (x_E + d)}{M_E + m_m} = x_E + \frac{m_m}{M_E + m_m} \cdot d$$

$$y: \quad y_{cM} = \frac{M_E y_0 + m_m y_0}{M_E + m_m} = y_0$$