

$$R_E = 6.4 \cdot 10^6 \text{ m}$$

$$h = 100 \text{ km} = 10^5$$

$$\Sigma \vec{F} = m \vec{a}$$

$$G \frac{M m_E}{r^2} = m \frac{v^2}{r}$$

$$v = \sqrt{\frac{G M_E}{r}}$$

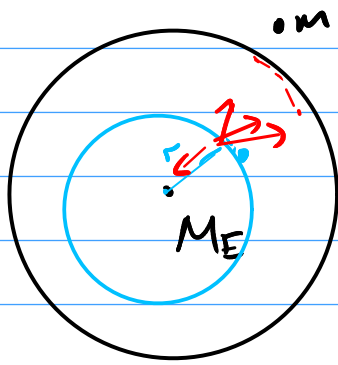
$$= \sqrt{\frac{6.67 \cdot 10^{-11} \cdot 6 \cdot 10^{24}}{6.4 \cdot 10^6}}$$

$$= \sqrt{\frac{6.67 \cdot 6}{6.4} \cdot 10^{2 \cdot 24 - 6}}$$

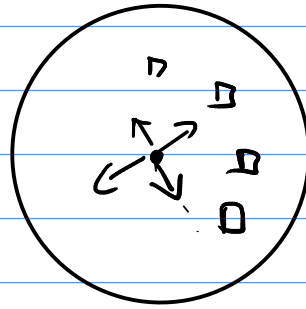
$$= \sqrt{\frac{66.7}{6.4} \cdot 10^6}$$

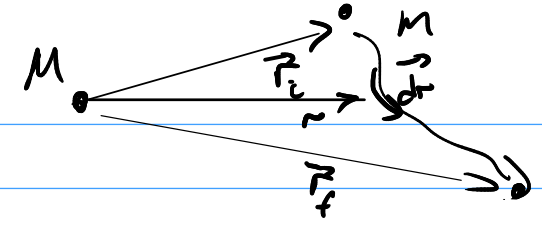
$$= \sqrt{\frac{66.7}{6.4} \cdot 6 \cdot 10^3} \frac{\text{m}}{\text{s}}$$

$$\approx 8 \cdot 10^3 \text{ m/s}$$



$$F_g = G \frac{M \cdot m}{r^2}$$



$$W_g = \int_{r_i}^{r_f} \vec{F}_g \cdot d\vec{r} =$$


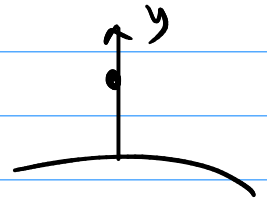
$$= \int_{r_i}^{r_f} \frac{GMm}{r^2} \cdot \hat{r} \cdot d\vec{r} = \int_{r_i}^{r_f} \frac{GMm}{r^2} (-1) dr$$

↑
element of path
↑
distance change

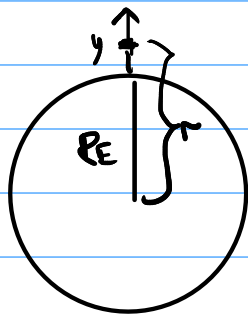
$$= -\frac{GMm}{1} \int_{r_i}^{r_f} \frac{dr}{r^2} = -\frac{GMm}{1} \left(-\frac{1}{r} \Big|_{r_i}^{r_f} \right)$$

$$= -\left(\frac{GMm}{r_f} - \left(-\frac{GMm}{r_i} \right) \right)$$

$$= -(U_f - U_i)$$



$$U(r) = -\frac{GMm}{r} \iff mgy$$



$$U(r) = U(R_E + y) =$$

$$= -\frac{GMm}{r} = -\frac{GMm}{R_E + y}$$

$$= -\frac{GMm}{R_E} \frac{1}{\left(1 + \frac{y}{R_E}\right)} = \left/ \frac{y}{R_E} = x \ll 1 \right/$$

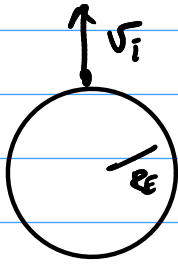
$$= -\frac{GMm}{R_E} \left(\frac{1}{1+x} \right) \approx -\frac{GMm}{R_E} (1-x)$$

$$= -\frac{GMm R_E}{R_E^2 R_E} \left(1 - \frac{y}{R_E} \right) = -mg \left(R_E - \frac{y R_E}{R_E} \right)$$

$$U(y) = \underbrace{-mg}_{\text{constant}} + \underbrace{mg \cdot y}_{\text{familiar}} \quad \text{iff } y \ll R_E$$

$$\Delta E = 0 = \Delta K + \Delta U$$

$$\vec{v}_i + \vec{v}_f = 0$$

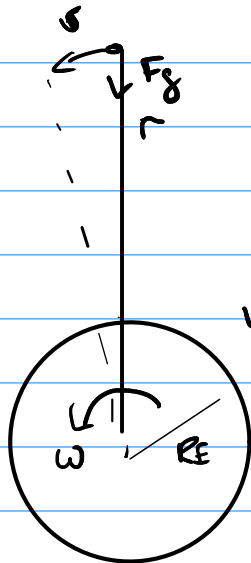


$$E_i = E_f$$

$$\underbrace{\frac{m v_i^2}{2}}_K - \underbrace{\frac{GMm}{r_i}}_U = K + U = \frac{m v_f^2}{2} - \frac{GMm}{r_f}$$

$$\frac{m v_i^2}{2} - \frac{GMm}{r_i} = 0$$

$$v_i = v_{\text{escape velocity}} = \sqrt{\frac{2GM_E}{R_E}} \approx 12 \frac{\text{km}}{\text{s}}$$



$$\frac{m v^2}{r} = G \frac{M m}{r^2}$$

$$v = \frac{2\pi r}{T} = \omega r$$

$$\frac{\omega^2 r^2}{r} = \frac{GM}{r^2}$$

$$\omega^2 = \frac{GM}{r^3}$$

$$\omega^2 = \left(\frac{2\pi}{24 \cdot 3600}\right)^2 = \frac{GM}{r^3} \Rightarrow \text{geo stat.}$$

$$\omega^2 = \frac{G \cdot M}{r^3} = \left(\frac{2\pi}{T}\right)^2 =$$

$$\frac{G \cdot M \cdot T^2}{(2\pi)^2} = r^3$$

$$\Rightarrow r^3 \sim T^2$$

3rd Kepler's law