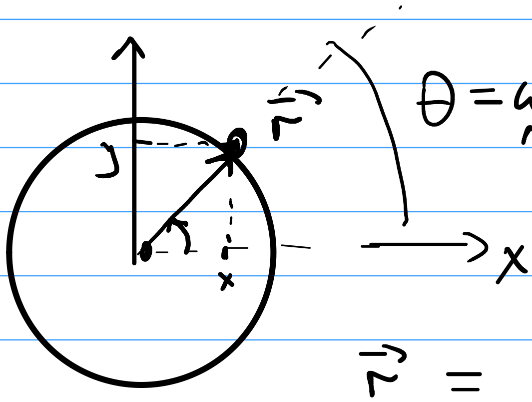
Uniform circular motion



$\theta = \omega \cdot t$

↑ angular speed [omega]

$\vec{r} = x \cdot \hat{i} + y\hat{j} =$

$= r \cdot \cos\theta \cdot \hat{i} + r \cdot \sin\theta \cdot \hat{j}$

$= r \cdot \cos(\omega t) \cdot \hat{i} + r \cdot \sin(\omega t) \cdot \hat{j}$

ω – need to be in radians

$\vec{v} = \frac{d\vec{r}}{dt} = r \cdot \frac{d}{dt}\left(\cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}\right) =$

$= r \cdot \left[\hat{i}\,(-\sin\omega t)\frac{d\omega t}{dt} + \hat{j}\cos(\omega t)\frac{d\omega t}{dt}\right]$

$= r\left[\hat{i}\,(-\sin(\omega t))\,\omega + \hat{j}\cos(\omega t)\,\omega\right]$

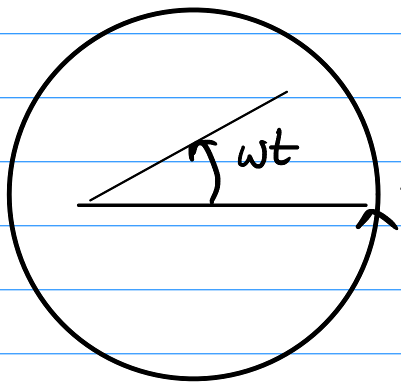
$\vec{a} = \frac{d\vec{v}}{dt} = r \cdot \left(\hat{i}\,(-\cos(\omega t))\,\omega^2 + \hat{j}\,(-\sin(\omega t))\,\omega^2\right)$

$= -\omega^2\left(\underbrace{r\cos(\omega t)\cdot\hat{i} + r\sin(\omega t)\cdot\hat{j}}_{\vec{r}}\right)$

$\boxed{\vec{a} = -\omega^2\vec{r}}$ centripetal acceleration

$\boxed{a = +\omega^2 r = \frac{v^2}{r^2}r = \frac{v^2}{r}}$

$360^0 = 2\pi$ radians



T ↖ period

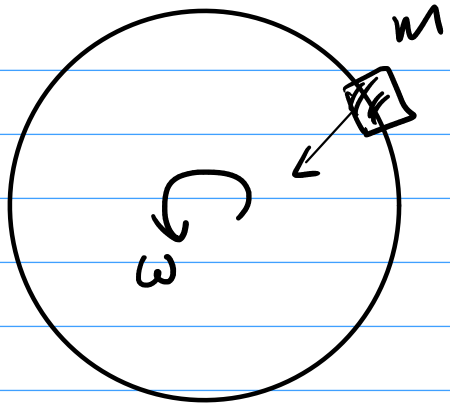
$\omega = \frac{2\pi}{T}$

$v = \frac{2\pi r}{T}$

$\boxed{v = \omega r}$

linear speed → v

angular speed → ω

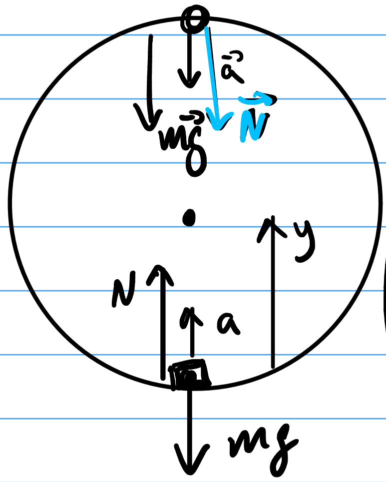


$\vec{F}_{Net} = m \cdot a = m \cdot \omega^2 r$

provided by friction

$F_{fr} \leq \mu m g$

$a = \omega^2 r = \frac{v^2}{r}$



Top: $m a_y = -m\frac{v^2}{r} = -N - mg$

$N = \frac{mv^2}{r} - mg \geq 0$

Bottom:

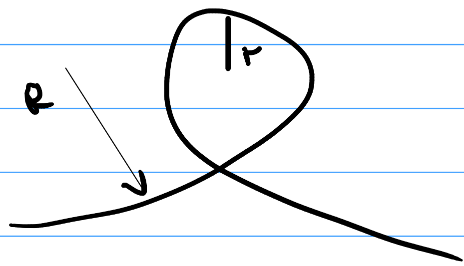
$ma = m a_y = m\frac{v^2}{r} = N - mg$

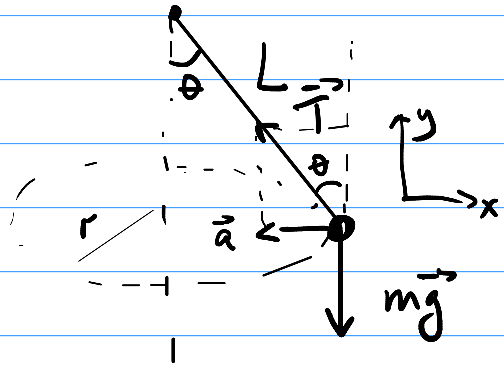
$N = \frac{mv^2}{r} + mg = 2mg$

↑ scale reading

smallest speed ⟺ $N = 0$

$\frac{\cancel{m}v^2}{r} = \cancel{m}g$





$$m\vec{a} = \vec{T} + m\vec{g}$$

$$\vec{a} = \left(-\frac{v^2}{r}\hat{i} + 0\hat{j}\right)$$

$$y: \quad m \cdot 0 = T\cos\theta - mg$$

$$T = \frac{mg}{\cos\theta}$$

$$x: -m\frac{v^2}{r} = -T\sin\theta + 0 \quad (mg_x)$$

$$+\cancel{m}\frac{v^2}{r} = +\frac{\cancel{m}g}{\cos\theta}\sin\theta = \cancel{m}g\tan\theta$$

$$\boxed{\tan\theta = \frac{v^2}{r}}$$