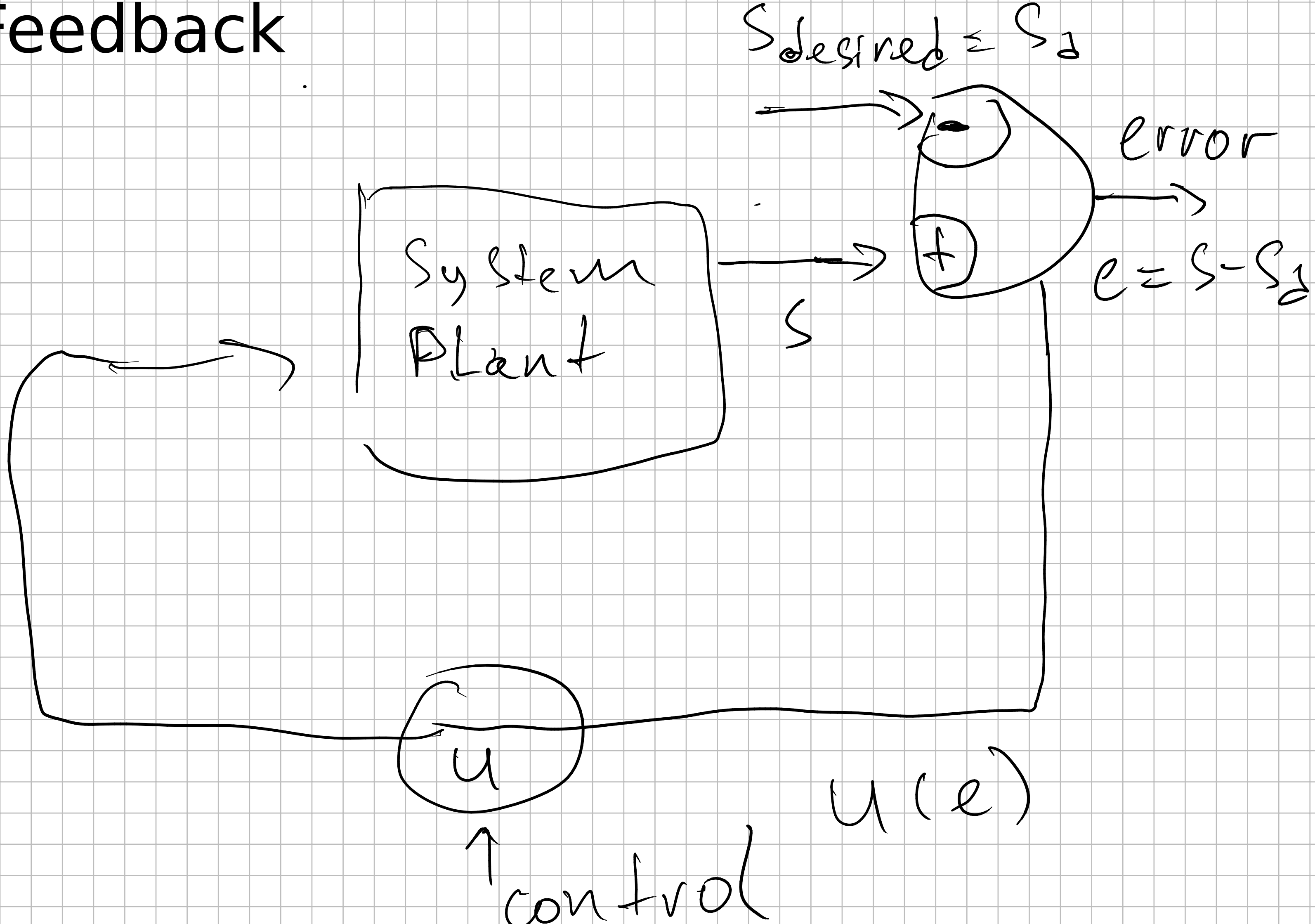
Feedback



Feedback theory is concerned with shapes/functional dependences of 'u' as function of 'e'

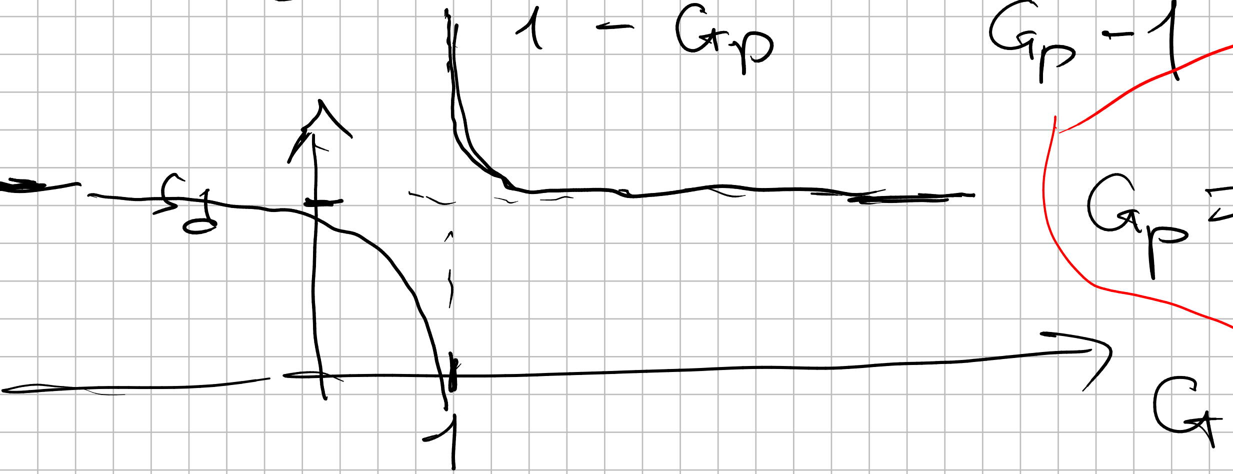
Proportional feedback

$$e \longrightarrow U(e) = G_p \cdot e$$

proportional

$$S = U(e) = G_p(S - S_d)$$

$$S = \frac{-G_p \cdot S_d}{1 - G_p} = \frac{G_p S_d}{G_p - 1}$$



$G_p = 1$ No No.

Real life complication: there is a delay between control and response. i.e. feedback is not instant.

$$S(t+\underset{\text{delay}}{\underline{\tau}}) = U(e) = G_p \cdot e(t)$$

$$S(t) + \tau \frac{dS}{dt} = G_p (S(t) - S_d)$$

$$\frac{dS}{dt} = \underbrace{\frac{G_p - 1}{\tau}} \cdot S - \frac{G_p}{\tau} \cdot S_d \quad **$$

$$S(t) = C \cdot e^{\frac{G_p-1}{\tau} t} + S_c \quad *$$

$$S(0) = S_0 = C \cdot \underbrace{e^{\dots(t=0)}}_{1} + S_c \quad (3)$$

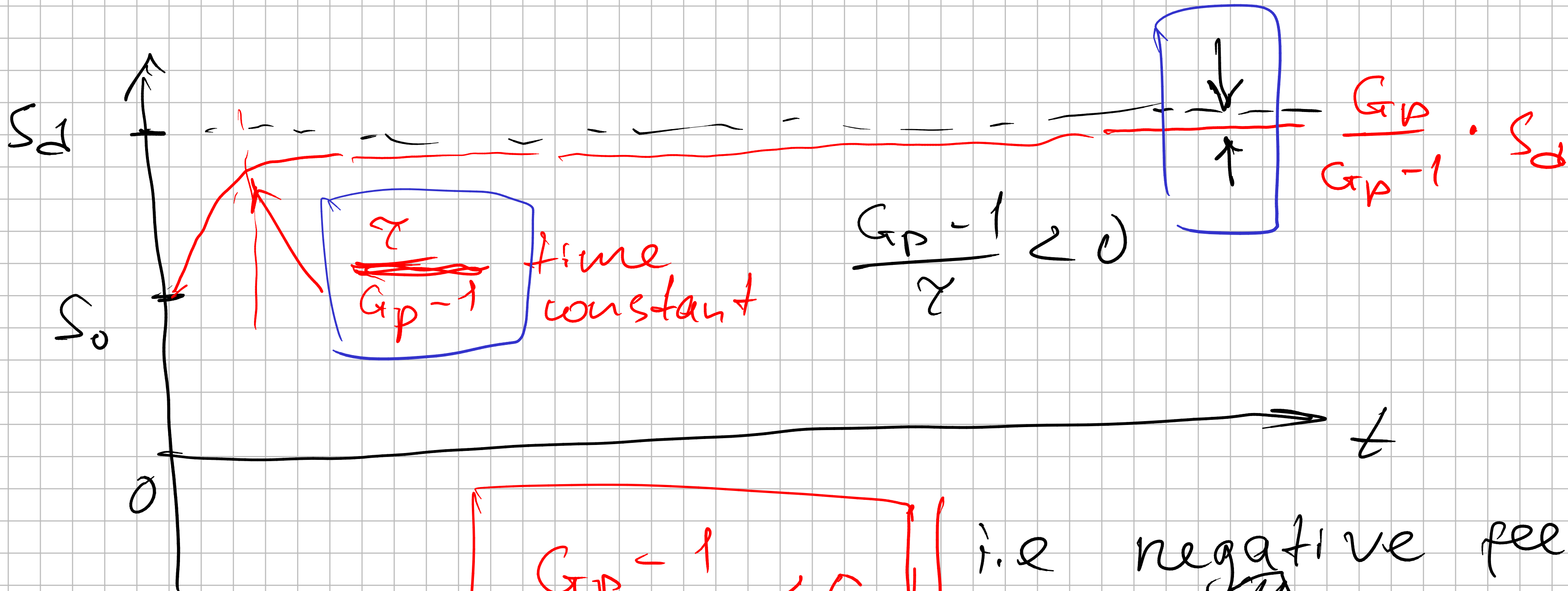
$$\frac{dS}{dt}(t=0) = \frac{G_p - 1}{\cancel{\tau}} S_0 - \frac{G_p}{\cancel{\tau}} S_d = C \cdot \frac{G_p - 1}{\cancel{\tau}} \underbrace{e^{\frac{G_p-1}{\tau} t}}_{=1} \quad {}^{''}0$$

$$* (t=0) \Rightarrow \underbrace{\left(S_0 - \frac{G_p}{G_p-1} S_d\right)}_{C} + S_c = S_0$$

$$\boxed{C = S_0 - \frac{G_p}{G_p - 1} S_d}$$

$$\boxed{S_c = \frac{G_p}{G_p - 1} S_d}$$

$$\boxed{s(t) = \left(s_0 - \frac{G_p}{G_p-1}\cdot s_d\right) e^{\frac{G_p-1}{\tau}t} + \frac{G_p}{G_p-1}\cdot s_d}$$



$$\boxed{\frac{G_p-1}{\tau} < 0} \;!!$$

i.e negative feedback

corrective

if $\frac{G_p-1}{\tau} > 0 \iff$ positive feedback

$\iff$ exponential run away!

P I D controller

proportional → P, integral → I, derivative → D

$$u(e) = \underbrace{G_p \cdot e}_{P} + \underbrace{G_I \int e(t)\,dt}_{I} + \underbrace{G_d \frac{de(t)}{dt}}_{D}$$ eq5

$$s(t+\tau) = s(t) + \tau \frac{ds}{dt} = u(e) = G_p(s-s_d) + G_I \int (s-s_d)\,dt + G_d \frac{ds}{dt} + n(t)$$

In real life we also have environment acting on the system $\Longrightarrow$ disturbance, noise

$$s(t) \to \int_{-\infty}^{+\infty} S_\omega \, e^{i\omega t} \, d\omega$$

Moving to Fourier space

$$\begin{cases} s(t) \Rightarrow S_\omega \\ \dfrac{ds}{dt} \to i\omega S_\omega \\ \displaystyle\int_{-\infty}^{t} (s - s_d)\, dt \to \dfrac{S}{i\omega}, \quad \omega \neq 0 \end{cases}$$

put it to eq. 5

$$S_\omega = \frac{n(\omega)}{1 - G_p - \frac{G_I}{i\omega} - (G_d - \gamma) i\omega}$$

noise transfer gain.

The smaller the better.

log-log plot