# Math with Operational amplifiers

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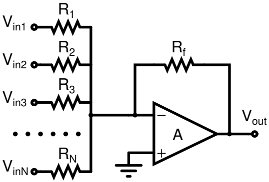
# Superposition principle

Op-Amps are linear to their inputs, thus we can apply **superposition principle**

If you have several inputs.

- Mentally put every but one input to the reference. Calculate the output of such circuit.
- Now put another input but put every other one to reference, and calculate the output.
- Keep doing this for every input.
- Sum all the results and you get the resulting output.

# Summing inverting amplifier

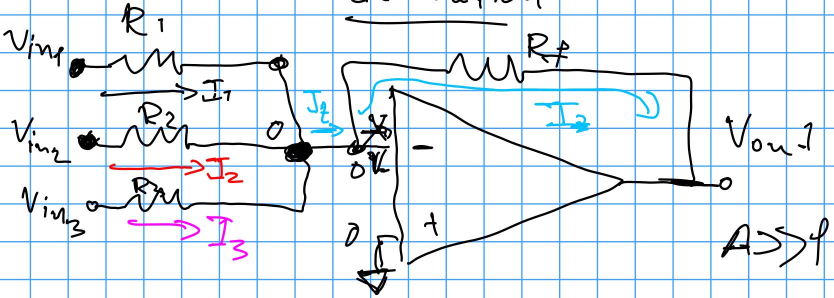



## for ideal Op-Amp ($A \gg 1$)

$$V_{out} = -\left(\frac{V_{in1}}{R_1} + \frac{V_{in2}}{R_2} + \frac{V_{in3}}{R_3} + \cdots + \frac{V_{inN}}{R_N}\right) R_f$$

$$Z_{inN} = R_N, Z_{out} = 0$$

# Summing inverting amp. derivation

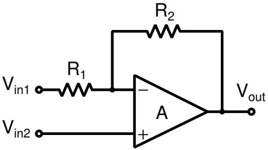



$$V_{out} = \underbrace{V_-}_{0} - I_t \cdot R_f = -(I_1 + I_2 + I_3) R_f$$

$$= -\left( \frac{V_{in_1} - \overset{V_-}{0}}{R_1} + \frac{V_{in_2} - \overset{V_-}{0}}{R_2} + \frac{V_{in_3} - \overset{V_-}{0}}{R_3} \right) R_f$$

$$= -\left( \frac{V_{in_1}}{R_1} + \frac{V_{in_2}}{R_2} + \frac{V_{in_3}}{R_3} \right) R_f$$

# Naive differential amplifier



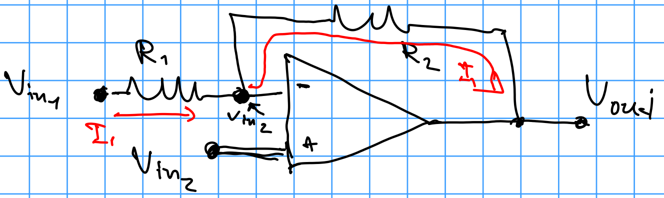

**for ideal Op-Amp ($A \gg 1$)**

$$V_{out} = -V_{in1}\frac{R_2}{R_1} + V_{in2}\left(1 + \frac{R_2}{R_1}\right)$$

$$Z_{in1} = R_1, Z_{in2} = \infty, Z_{out} = 0$$

Unfortunately, there is no way to get the output signal as the amplified difference of the inputs. i.e. $V_{out} \not\sim (V_{in1} - V_{in2})$,

# Derivation for Naive Diff. Amplifier




$V_+ = V_{in_2}$

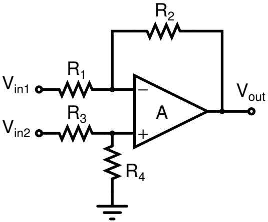
$A \gg 1 \Rightarrow V_+ = V_-$

$$V_{out} = \underbrace{V_-}_{V_+ = V_{in_2}} - V_2 = V_- - R_2 \cdot I_1$$

$$V_{out} = V_{in_2} - R_2 \frac{V_{in_1} - V_-}{R_1} = V_{in_2} - \frac{R_2}{R_1} \cdot (V_{in_1} - V_{in_2})$$

$$V_{out} = -\frac{R_2}{R_1} V_{in_1} + \left(1 + \frac{R_2}{R_1}\right) V_{in_2}$$

# Differential amplifier




## for ideal Op-Amp ($A \gg 1$)

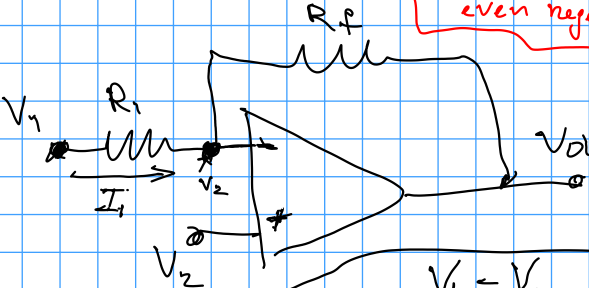
$$V_{out} = \frac{R_4}{R_1}\frac{R_1 + R_2}{R_3 + R_4} V_{in2} - \frac{R_2}{R_1} V_{in1}$$

$$Z_{out} = 0$$

## Important note about input impedances for differential connections

$A >> 1$

$$Z_{in_1} = \frac{V_1}{I_1}$$




Zin can be even negative

**Case 1**

$V_2 = 0$

$Z_{in} = R_1$

**Case 2**

$V_2 = V_1$

$Z_{in} = \infty$

(eq. 1) $$I_1 = \frac{V_1 - V_-}{R_1} = \frac{V_1 - V_2}{R_1}$$

**Case 3**

$V_2 > V_1$ , $I < 0 \Rightarrow Z_{in} \leq 0$

By now we know how with a help from Op-Amps do summing and subtracting signals. We also know how to multiply them by any constant. I.e. we can do elementary math arithmetic with our input signals.

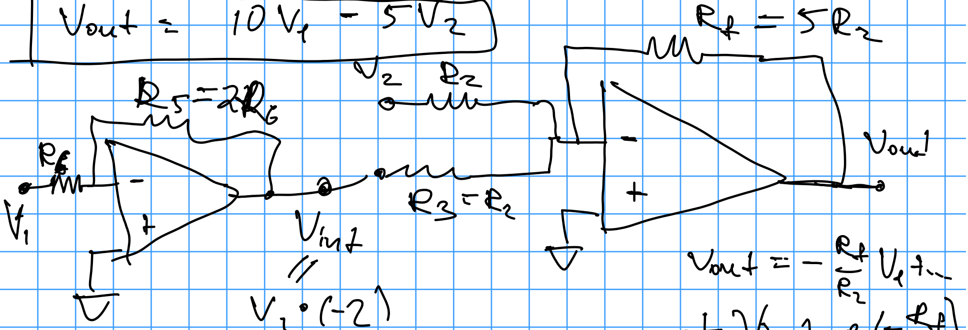
Later we will learn how to integrate and take derivative of a signal.

Now you see why they are called **Op**erational-**Amp**lifier.

Note: multiplying two inputs together is not trivial task, though there are specialized chip which can do it. There are also chips which can do exponents and logarithm.

# An example of OpAmp arithmetics

$$\boxed{V_{out} = 10V_1 - 5V_2}$$




$$V_{out} = -\frac{R_f}{R_2}V_1 + \dots$$

$$+ V_{int} \cdot \left(-\frac{R_f}{R_2}\right)$$

$$V_{out} = \underbrace{\left(-\frac{R_5}{R_6}\right)}_{=2}\underbrace{\left(-\frac{R_f}{R_2}\right)}_{=5}V_1 - \frac{R_f}{R_2}V_2$$

$$= 10V_1 - 5V_2$$