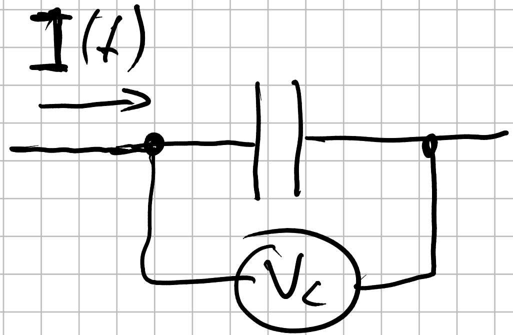
Reactive elements: capacitors and inductors

Capacitors

C

$$Z_C = \frac{1}{i\omega C}$$



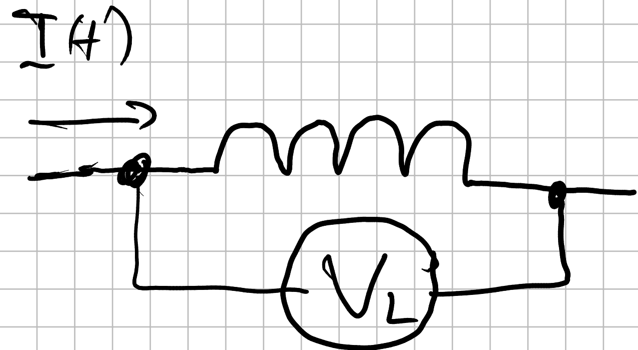
$$V_C = \frac{q}{C} = \frac{\int_{-\infty}^{t} I(t')\,dt'}{C}$$

Capacitance

Inductors

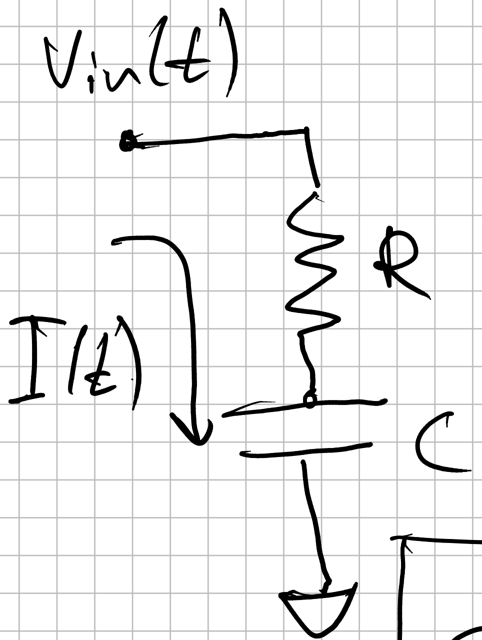
L

$$Z_L = i\omega L$$



$$V_L = L \cdot \frac{dI(t)}{dt}$$

Induction



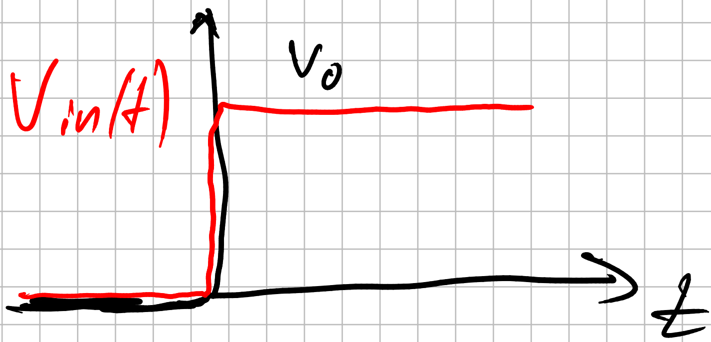
$$V_{in} = V_R + V_C$$

$$= I \cdot R + \frac{q(t)}{C} =$$

$$= I \cdot R + \frac{\int^{t} I(t')\,dt'}{C}$$

$$\boxed{\frac{dV_{in}}{dt} = \frac{dI(t)}{dt} \cdot R + I(t)\,\frac{1}{C}}$$

Example: Charging capacitor



$$\frac{dV_{in}}{dt} = 0 \quad (\text{except } t = 0)$$

$$0 = \frac{dI}{dt} R + I \frac{1}{C} \Rightarrow \frac{dI}{I} = -\frac{dt}{R \cdot C}$$

$$d(\ln I) = -d\left(\frac{t}{RC}\right) \Rightarrow \ln I = -\frac{t}{RC} + \underset{\ln I_0}{\underset{\shortparallel}{Const}}$$

$$\ln \frac{I}{I_0} = -\frac{t}{RC} \Rightarrow \boxed{I(t) = I_0\, e^{-t/RC}}$$

$$V_C = \frac{1}{C}\int I(t)\,dt = \frac{1}{C}\underbrace{I_0 (-RC)\cdot e^{-t/RC}}_{=I(t)} + V_{const}$$

$$\boxed{V_c(t) = -R\,I(t) + V_{const}}$$

Boundary conditions:

$$V_{in}(t=\infty) = R\underbrace{I(t=\infty)}_{0} - R\underbrace{I(t=\infty)}_{0} + V_{const}$$

$$V_{const} = V_0$$

What about I_0 - ?

$\leq 0, t=0$

Recall

at $t=0$

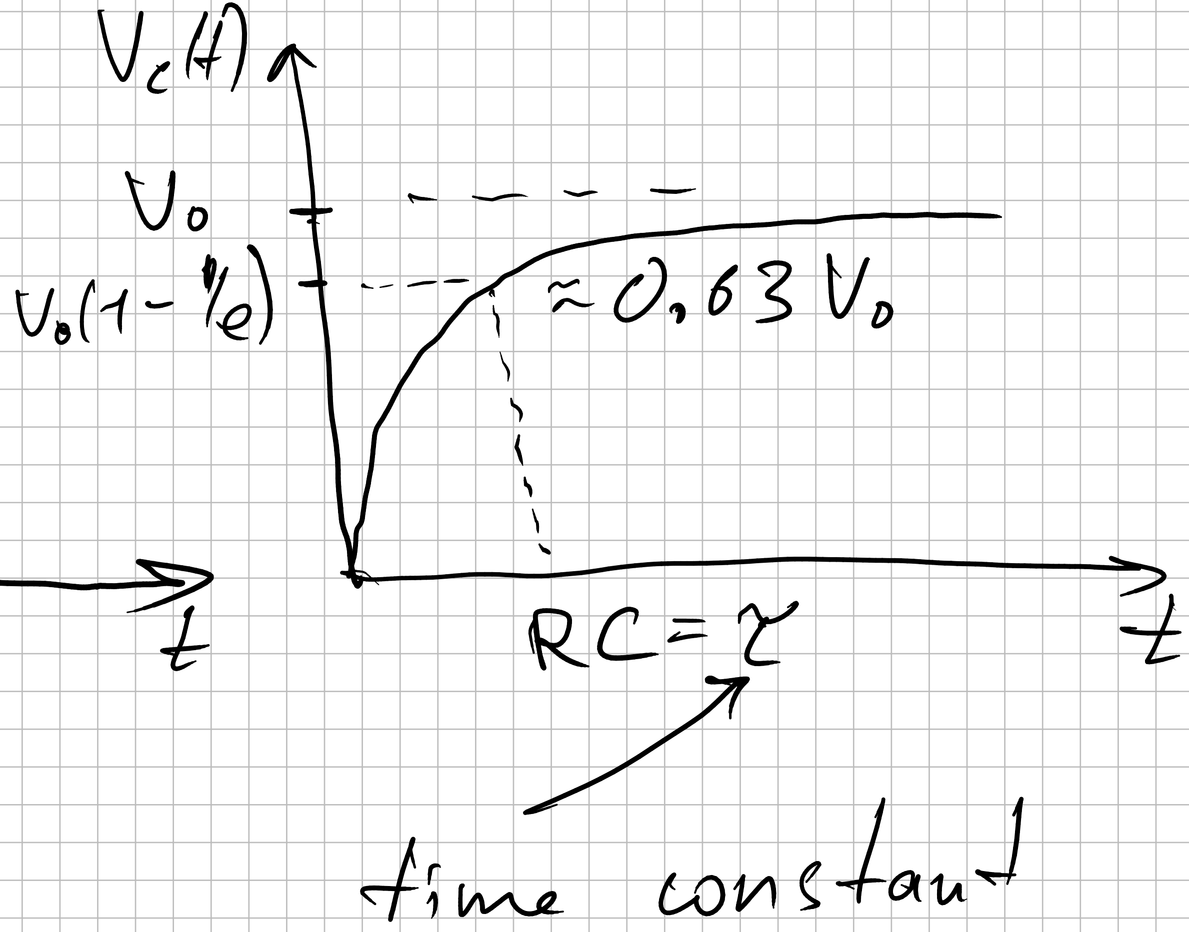
$$V_{in} = I(t)R + \frac{q(t)}{C}$$

$$V_{in} = V_0 = I_0 \underbrace{e^{-t/RC}}_{1,\ t=0} \cdot R$$

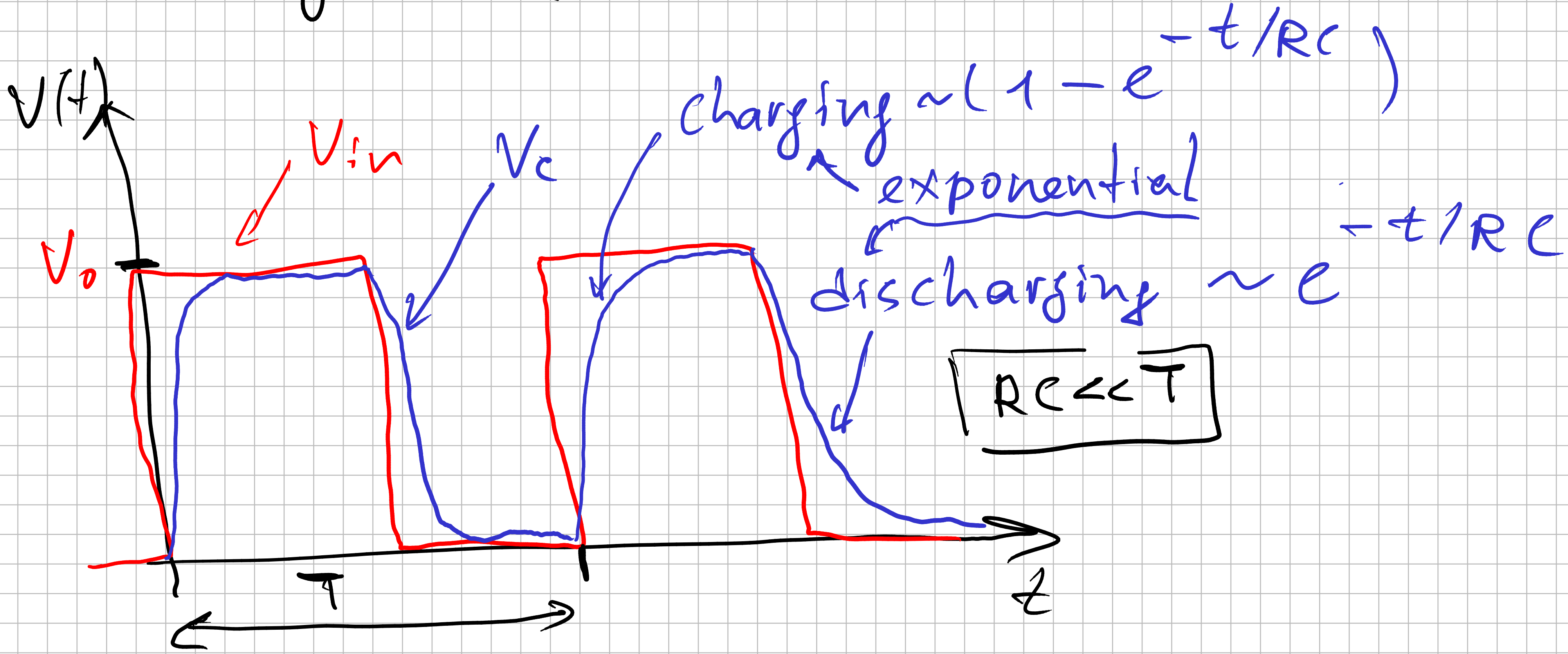
$$I_0 = \frac{V_0}{R} \Rightarrow$$

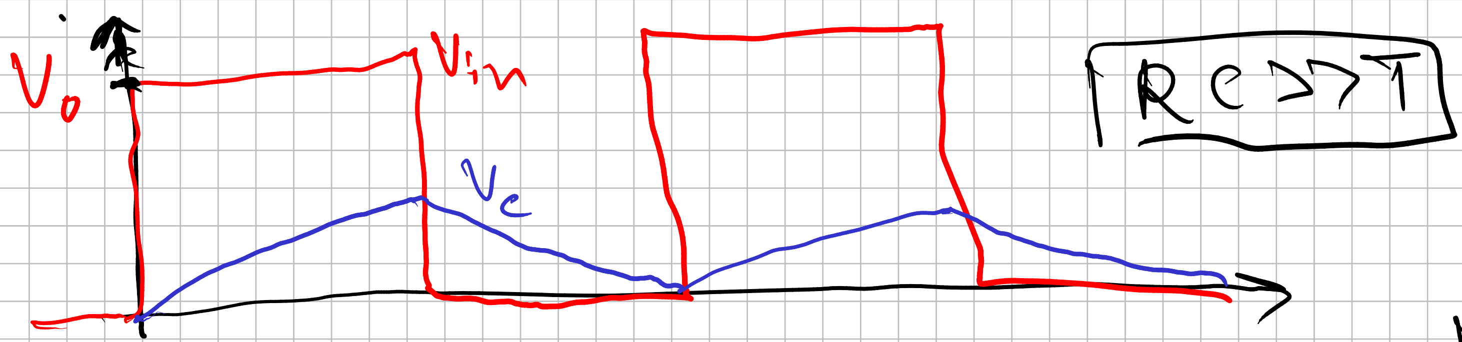
Capacitor is equivalent to a <u>short</u> ("R_c" $= 0$) for <u>abruptly</u> changing signals.

$$I(t) = \frac{V_0}{R} e^{-t/RC}, \quad V_c = V_0 \left(1 - e^{-t/RC}\right)$$



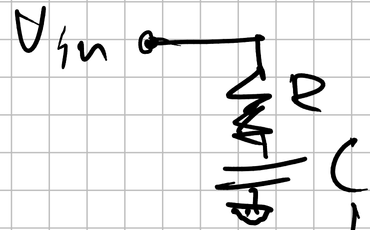
RC driven by periodic square signals.





Capacitor is never fully charged
$V_c(t) < V_0$

Complex impedance



$$V_{in}(t) = I(t)R + \frac{1}{C}\int I(t')dt'$$ This is hard

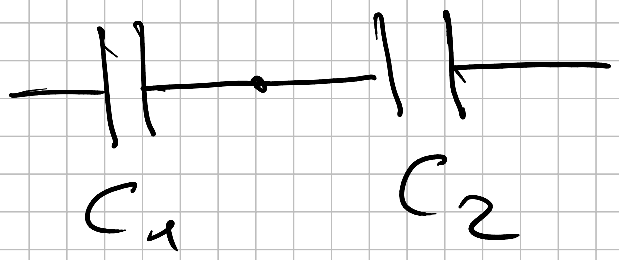
What if our input is in the form
$$V_{in}(t) = V_\omega \cdot e^{i\omega t}$$
We will seek solution in the form $I(t) = I_\omega e^{i\omega t}$

$$V_\omega \cancel{e^{i\omega t}} = I_\omega \cancel{e^{i\omega t}} R + \frac{1}{C}\frac{I_\omega}{i\omega}\cancel{e^{i\omega t}}$$

$$V_\omega = I_\omega \underbrace{\boxed{R}}_{Z_R} + \underbrace{\boxed{\frac{1}{i\omega C}}}_{Z_c} I_\omega \Rightarrow V_\omega = (Z_R + Z_C) I_\omega$$

$$\boxed{Z_c = \frac{1}{i\omega C}}$$

Simple network analysis



$$Z_{C_1} = \frac{1}{i\omega C_1} \qquad Z_{C_2} = \frac{1}{i\omega C_2}$$

! Impedaces follows the same reles as resistors. !

$$Z_{total} = Z_{C_1} + Z_{C_2} \quad \text{(serial connection)}$$

$$= \frac{1}{i\omega C_1} + \frac{1}{i\omega C_2} = \frac{1}{i\omega}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) =$$

$$= \frac{1}{i\omega C_T} \Rightarrow \boxed{C_T = \frac{C_1 \cdot C_2}{C_1 + C_2}}$$