Full network analysis.

Eugeniy E. Mikhailov

WILLIAM & MARY
CHARTERED 1693

Lecture 02

Kirchhoff's Laws

Kirchhoff's Current Law

The algebraic sum of currents entering and exiting a node equals zero

Convention (quite arbitrary): currents going into the nodes are positive, the ones which go out of the node are negative.

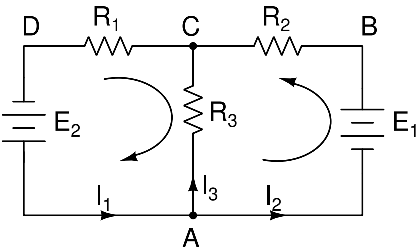
Kirchhoff's Voltage Law

The algebraic sum of all voltage changes (aka voltage drops) in a loop equals zero

Notes:

- chose a direction along which you travel a network. If you go over a resistor and current runs the same way then voltage change is negative, otherwise its positive.
- If you go over a voltage source from the negative terminal to the positive terminal the voltage change is positive, otherwise negative.

Example



our goal is to find I_1, I_2, and I_3

We chose $V_A = 0$ for node A.

It is our reference/grounded node

$$I_1 - I_2 - I_3 = 0 \quad (1)$$

We need 2 more independent equations.

For this we will go over 2 small loops as indicated by arrows.

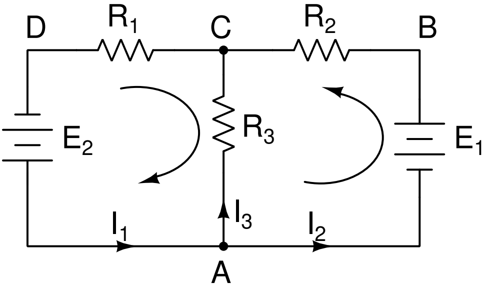
$$V_{DC} + V_{CA} + V_{AD} = 0 \quad (2)$$

$$V_{AB} + V_{BC} + V_{CA} = 0 \quad (3)$$

Notice:

$V_{AB} = +E_1$, $V_{BC} = -R_2 \times I_2$, $V_{CA} = +R_3 \times I_3$,
$V_{DC} = +R_1 \times I_1$, $V_{AD} = -E_2$.

Example (continued)



$$\begin{aligned} I_1 - I_2 - I_3 &= 0 \\ V_{DC} + V_{CA} + V_{AD} &= 0 \\ V_{AB} + V_{BC} + V_{CA} &= 0 \end{aligned} \quad \rightarrow \quad \begin{aligned} I_1 - I_2 - I_3 &= 0 \\ R_1 \times I_1 + R_3 \times I_3 - E_2 &= 0 \\ E_1 - R_2 \times I_2 + R_3 \times I_3 &= 0 \end{aligned}$$

Solving system of equation

So we need to find the unknowns I_1, I_2, and I_3 from the system of equations

$$\begin{aligned} I_1 - I_2 - I_3 &= 0 \\ R_1 \times I_1 + R_3 \times I_3 - E_2 &= 0 \\ E_1 - R_2 \times I_2 + R_3 \times I_3 &= 0 \end{aligned}$$

Let's reshape above equations to the canonical linear algebra form (move terms without unknown variables to the right)

$$\begin{aligned} I_1 - I_2 - I_3 &= 0 \\ R_1 \times I_1 + R_3 \times I_3 &= E_2 \\ -R_2 \times I_2 + R_3 \times I_3 &= -E_1 \end{aligned}$$

Equations form

$$\begin{bmatrix} 1 & -1 & -1 \\ R_1 & 0 & R_3 \\ 0 & -R_2 & R_3 \end{bmatrix} \times \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ E_2 \\ -E_1 \end{bmatrix}$$

Matrix form

Computer assisted symbolic solvers: WolframAlpha (Mathematica)

$$I_1 - I_2 - I_3 = 0$$
$$R_1 \times I_1 + R_3 \times I_3 - E_2 = 0$$
$$E_1 - R_2 \times I_2 + R_3 \times I_3 = 0$$

With WolframAlpha we can input

```
Solve[
{I_1- I_2 - I_3 == 0, R_1*I_1 + R_3*I_3 - E_2 == 0, E_1 - R_2*I_2 + R_3*I_3 == 0},
{I_1,I_2,I_3}
]
```

the results can be seen by following **WolframAlpha link**, which gives solution

$$I_1 = (E_1 R_3 + E_2(R_2 + R_3))/(R_2 R_3 + R_1(R_2 + R_3))$$
$$I_2 = (E_2 R_3 + E_1(R_1 + R_3))/(R_2 R_3 + R_1(R_2 + R_3))$$
$$I_3 = (E_2 R_2 - E_1 R_1)/(R_2 R_3 + R_1(R_2 + R_3))$$

Computer assisted symbolic solvers: Matlab

I would not recommend to use Matlab as symbolic solver, but it can be done

```
>> syms I_1 I_2 I_3 E_1 E_2 R_1 R_2 R_3; % declare symbols
>> S = solve( [I_1- I_2 - I_3 == 0,
       R_1*I_1 + R_3*I_3 - E_2 == 0,
       E_1 - R_2*I_2 + R_3*I_3 == 0],
       [I_1, I_2, I_3]);
>> S.I_1
ans=(E_1*R_3 + E_2*R_2 + E_2*R_3)/(R_1*R_2 + R_1*R_3 + R_2*R_3)
>> S.I_2
ans=(E_1*R_1 + E_1*R_3 + E_2*R_3)/(R_1*R_2 + R_1*R_3 + R_2*R_3)
>> S.I_3
ans=-(E_1*R_1 - E_2*R_2)/(R_1*R_2 + R_1*R_3 + R_2*R_3)
```

Linear algebra solution for system of equatons

Let's look at the Matrix form

$$\begin{bmatrix} 1 & -1 & -1 \\ R_1 & 0 & R_3 \\ 0 & -R_2 & R_3 \end{bmatrix} \times \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ E_2 \\ -E_1 \end{bmatrix}$$

Short notation $A \times I = B$, where I is a vector of unknown currents.
Currents can be found as $I = \text{inv}(A) * B$

Computer assisted **numerical** solvers: Matlab

Matlab is better suited for a numerical solutions, we will lose a general solution but a specific one is easy. In Matlab this is done as

```
>> R_1 = 10; R_2= 5; R_3= 15; E_1=10; E_2=15;
>> A = [ 1 -1 -1; R_1 0 R_3; 0 -R_2 R_2]
A =
     1    -1    -1
    10     0    15
     0    -5     5

>> B = [ 0; E_2; -E_1]
B =
     0
    15
   -10

>> I=inv(A)*B
I =
    1.7143
    1.8571
   -0.1429
```

Note that $I_3 = -0.1429$, i.e. it ends up negative. No need to worry, this means that direction of I_3 is opposite to the one which we selected in our schematic.

Computer assisted symbolic solvers: Maple

$solve(\{I1 - I2 - I3 = 0, E1 - R2 \cdot I2 + R3 \cdot I3 = 0, R1 \cdot I1 + R3 \cdot I3 - E2 = 0\}, [I1, I2, I3])$

$$\left[\left[I1 = \frac{R3\,E1 + R3\,E2 + R2\,E2}{R3\,R1 + R1\,R2 + R3\,R2}, I2 = \frac{R3\,E1 + R3\,E2 + R1\,E1}{R3\,R1 + R1\,R2 + R3\,R2}, I3 = -\frac{R1\,E1 - R2\,E2}{R3\,R1 + R1\,R2 + R3\,R2}\right]\right] \quad (1$$

Thévenin's and Norton's equivalent circuit theorems

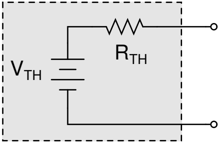
Any combination of voltage sources, current sources and resistors with two terminals is electrically equivalent

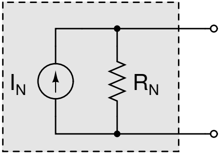
Thévenin's theorem

to a single voltage source V_{TH} and a single series resistor R_{TH} connected in series.

Norton's theorem

to a single current source I_N and a single series resistor R_N connected in parallel.





Note above circuits are equivalent to each other when

$$R_{TH} = R_N \text{ and } I_N = V_{TH}/R_{TH}$$