

Physics 251 Atomic Physics Lab Manual

W. J. Kossler A. Reilly J. Kane (2006 edition)

I. Novikova (2009 edition)

M. Kordosky (2011-2012, 2015-2017 editions)

E. E. Mikhailov (2013-2014, 2019 editions)

J. R. Stevens (2016-2018 edition)

Fall 2018

Contents

1	Optical Interferometry	3
2	Blackbody Radiation	16
3	Electron Diffraction	27
4	Measurement of Charge-to-Mass (e/m) Ratio for the Electron	33
5	Photoelectric Effect	40
6	Single and Double-Slit Interference, One Photon at a Time	49
7	Faraday Rotation	68
8	Atomic Spectroscopy	75
9	Superconductivity	88

Chapter 1

Optical Interferometry

Experiment objectives: Assemble and align Michelson and Fabry-Perot interferometers, calibrate them using a laser of known wavelength, and then use them characterize the bright yellow emission line in sodium (Na).

Introduction

Optical interferometers are the instruments that rely on the interference of two or more superimposed reflections of the input laser beam. These are one of the most common optical tools, and are used for precision measurements, surface diagnostics, astrophysics, seismology, quantum information, etc. There are many configurations of optical interferometers, and in this lab you will become familiar with two of the more common setups.

The **Michelson interferometer**, shown in Fig. 1.1, is based on the interference of two beams: the initial light is split into two arms on a beam splitter, and then these resulting beams are reflected and recombined on the same beamsplitter again. The difference in optical paths in the two arms leads to a changing relative phase of two beams, so when overlapped the two light fields will interfere constructively or destructively. Such an interferometer was first used by Michelson and Morley in 1887 to determine that electromagnetic waves propagate in vacuum, giving the first strong evidence against the theory of a *luminiferous aether* (a fictitious medium for light wave propagation) and providing insight into the true nature of electromagnetic radiation. Michelson interferometers are widely used in many areas of physics and engineering, including the recent discovery of gravitational waves at the LIGO facility (see Sect. 1 at the end of this lab for more information).

Figure 1.1 shows the traditional setting for a Michelson interferometer. A beamsplitter (a glass plate which is partially silver-coated on the front surface and angled at 45 degrees) splits the laser beam into two parts of equal amplitude. One beam (that was initially transmitted by the beamsplitter) travels to a moveable mirror M_1 and back again. One-half of this amplitude is then reflected from the partially-silvered surface and directed at 90 degrees toward the observer (you will use a viewing screen). At the same time the second beam (reflected by the beamsplitter) travels at 90 degrees toward an adjustable mirror M_2 and

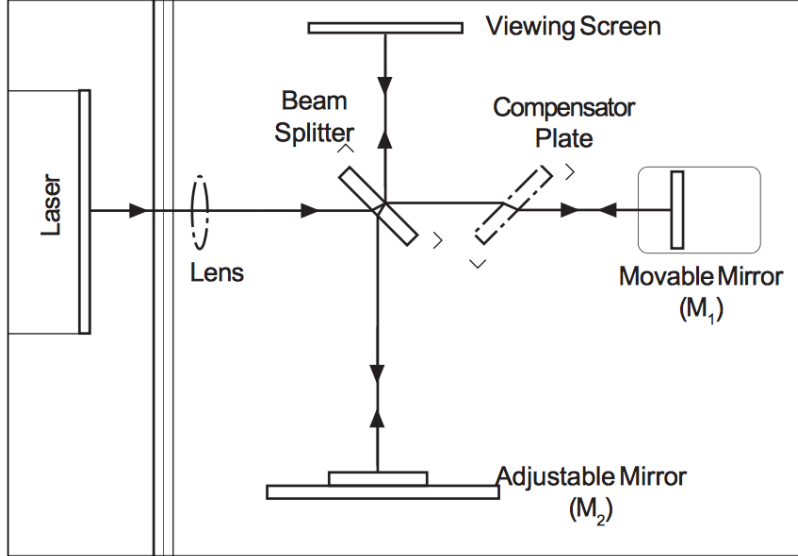


Figure 1.1: A Michelson Interferometer setup.

back. Since this beam travels through the glass beamsplitter plate 3 times, its optical path length is longer than the first beam which only passes through the beamsplitter 1 time. To compensate for that, the first beam passes twice through a clear glass plate called the compensator plate, that has the same thickness. At the beamsplitter one-half of this light is transmitted to an observer, overlapping with the beam reflected by M_2 , and the total amplitude of the light at the screen is a combination of amplitude of the two beams:

$$\mathbf{E}_{total} = \mathbf{E}_1 + \mathbf{E}_2 = \frac{1}{2}\mathbf{E}_0 + \frac{1}{2}\mathbf{E}_0 \cos(k\Delta l) \quad (1.1)$$

Here $k\Delta l$ is a phase shift (“optical delay”) between the two wavefronts caused by the difference in optical path lengths for the two beams Δl , $k = 2\pi n/\lambda$ is the wave number, λ is the wavelength of light in vacuum, and n is the refractive index of the optical medium (in our case - air).

Mirror M_1 is mounted on a precision traveling platform. Imagine that we adjust its position (by turning the micrometer screw) such that the distance traversed on both arms is exactly identical. Because the thickness of the compensator plate and the beamsplitter are the same, both wavefronts pass through the same amount of glass and air, so the path length of the light beams in both interferometer arms will be exactly the same. Therefore, the two fields will arrive in phase to the observer, and their amplitudes will add up constructively, producing a bright spot on the viewing screen. If now you turn the micrometer to offset the length of one arm by a half of light wavelength, $\Delta l = \lambda/2$, they will acquire a relative phase of π , and total destructive interference will occur:

$$\mathbf{E}_1 + \mathbf{E}_2 = 0 \text{ or } \mathbf{E}_1 = -\mathbf{E}_2.$$

It is easy to see that constructive interference happens when the difference between path lengths in the two interferometer arms is equal to the integer number of wavelengths $\Delta l =$

$m\lambda$, and destructive interference corresponds to a half-integer number of wavelengths $\Delta l = (m+1/2)\lambda$ (here m is an integer number). Since the wavelength of light is small (600–700 nm for a red laser), Michelson interferometers are able to measure distance variation with very good precision.

In **Fabry-Perot configuration** the input light field bounces between two closely spaced partially reflecting surfaces, creating a large number of reflections. Interference of these multiple beams produces sharp spikes in the transmission for certain light frequencies. Thanks to the large number of interfering rays, this type of interferometer has extremely high resolution, much better than a Michelson interferometer. For that reason Fabry-Perot interferometers are widely used in telecommunications, lasers and spectroscopy to control and measure the wavelengths of light. In this experiment we will take advantage of the high spectral resolution of the Fabry-Perot interferometer to resolve two very closely-spaced emission lines in Na spectra by observing changes in the overlapping interference fringes from the two lines.

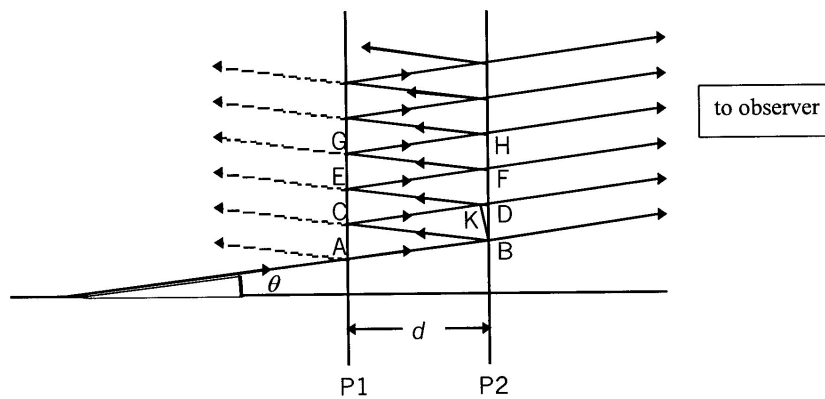


Figure 1.2: Sequence of Reflection and Transmission for a ray arriving at the treated inner surfaces P_1 & P_2 .

A Fabry-Perot interferometer consists of two parallel glass plates, flat to better than $1/4$ of an optical wavelength λ , and coated on the inner surfaces with a partially transmitting metallic layer. Such two-mirror arrangement is normally called an *optical cavity*. The light in a cavity by definition bounces back and forth many times before escaping; the idea of such a cavity is crucial for the construction of a laser. Any light transmitted through such cavity is a product of interference between beams transmitted at each bounce as diagrammed in Figure 1.2. When the incident ray arrives at interface point A, a fraction t is transmitted and the remaining fraction r is reflected, such that $t + r = 1$ (this assumes no light is lost inside the cavity). The same thing happens at each of the points A, B, C, D, E, F, G, H . . . , splitting the initial ray into parallel rays AB, CD, EF, GH, etc. Between adjacent ray pairs,

say AB and CD , there is a path difference of :

$$\delta = BC + CK \quad (1.2)$$

where BK is normal to CD . In a development similar to that used for the Michelson interferometer, you can show that:

$$\delta = 2d \cos \theta \quad (1.3)$$

If this path difference produces constructive interference, then δ is some integer multiple of λ , namely,

$$m\lambda = 2d \cos \theta \quad (1.4)$$

This applies equally to ray pairs CD and EF , EF and GH , etc, so that all parallel rays to the right of $P2$ will constructively interfere with one another when brought together.

Issues of intensity of fringes & contrast between fringes and dark background are addressed in Melissinos, *Experiments in Modern Physics*, pp.309-312.

Laser Safety

Never look directly at the laser beam! Align the laser so that it is not at eye level. Even a weak laser beam can be dangerous for your eyes.

Alignment of Michelson interferometer

Equipment needed: Pasco precision interferometry kit, a laser, Na lamp, adjustable-height platform (or a few magazines or books).

To simplify the alignment of a Michelson interferometer, it is convenient to work with diverging optical beams. In this case an interference pattern will look like a set of concentric bright and dark circles, since the components of the diverging beam travel at slightly different angles, and therefore acquire different phase, as illustrated in Figure 1.3. Suppose that the actual path length difference between two arms is d . Then the path length difference for two off-axis rays arriving at the observer is $\Delta l = a + b$ where $a = d / \cos \theta$ and $b = a \cos 2\theta$:

$$\Delta l = \frac{d}{\cos \theta} + \frac{d}{\cos \theta} \cos 2\theta \quad (1.5)$$

Recalling that $\cos 2\theta = 2(\cos \theta)^2 - 1$, we obtain $\Delta l = 2d \cos \theta$. The two rays interfere constructively for any angle θ_c for which $\Delta l = 2d \cos \theta = m\lambda$ (m =integer); at the same time, two beams traveling at the angle θ_d interfere destructively when $\Delta l = 2d \cos \theta = (m + 1/2)\lambda$ (m =integer). Because of the symmetry about the normal direction to the mirrors, this will mean that interference (bright and dark fringes) appears in a circular shape. If the fringes are not circular, it means simply that the mirrors are not parallel, and additional alignment of the interferometer is required.

When the path length difference is varied, by using the micrometer to move one of the mirrors a distance Δl along the horizontal axis of Figure 1.3, the fringes appear to “move”.

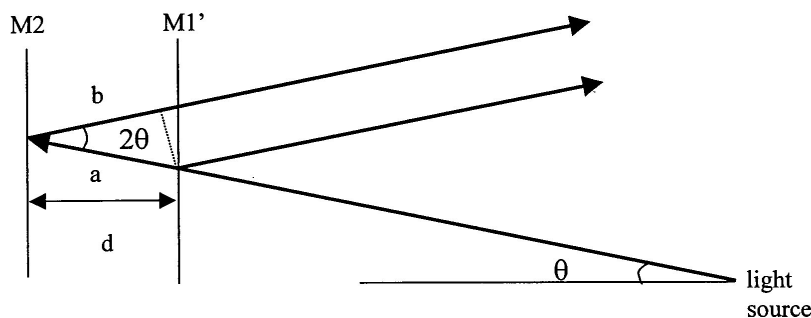


Figure 1.3: Explanation the interference pattern. Notice that to simplify the figure we have “unfold” the interferometer by neglecting the reflections on the beamsplitter.

As the micrometer is turned, the condition for constructive and destructive interference is alternately satisfied at any given angle. If we fix our eyes on one particular spot and count how many bright fringes pass that spot as we move mirror M_1 by a known distance, we can determine the wavelength of light in the media using the condition for constructive interference

$$2\Delta l \cos \theta = m\lambda. \quad (1.6)$$

The factor of two comes from the fact that if I move the mirror Δl light has to go an additional distance $\Delta l \cos \theta$ to get to it and then the same distance return from it. For simplicity, we might concentrate on the center of the fringe bull's eye at $\theta = 0$. Equation 1.6 for constructive interference then reduces to $2\Delta l = m\lambda$ ($m = \text{integer}$). If X_1 is the initial position of the mirror M_1 (as measured on the micrometer) and X_2 is the final position after a number of fringes Δm has been counted, we have $2\Delta l = 2(X_2 - X_1) = \lambda\Delta m$. Then the laser wavelength λ is then given as:

$$\lambda = 2(X_2 - X_1)/\Delta m. \quad (1.7)$$

Set up the interferometer as shown in Figure 1.1 using components from the PASCO interferometry kit. The mirrors $M_{1,2}$ are, correspondingly, a movable and an adjustable mirror from the kit. Alignment is not too difficult:

1. Begin by installing the movable mirror, adjustable mirror, component holder with viewing screen (magnetically attaches), and the lens holder. Do not install the lens yet. Make sure screws are firmly tightened as we don't want components to move around.
2. Level the table by adjusting its legs and/or adjust the laser height, location and direction to get the beam spot at the center of the movable mirror. It is a good idea to tape your laser down at this point, or make sure it's securely mounted in the optical breadboard (if your station has one).

3. Tweak the laser direction using the two knobs on its back so that the beam reflects back into its aperture.
4. Install the beam splitter, orient it to center the new beam spot on the adjustable mirror.
5. Tweak the adjustable mirror using the two knobs on its back. You want to make the two sets of dots visible on the viewing screen come into alignment. When they are properly aligned you should see interference fringes appear in the dot.
6. It is possible to see interference without the compensator (ask yourself why). But, it can potentially make the pattern better and more distinct. Try inserting it and aligning to improve the pattern.
7. Install the lens to blow up the pattern.
8. Further tweak the adjustable mirror to bring the bullseye on screen. You will find it's very sensitive and a little tweaking goes a long way.

Be patient, make small changes, think about what you are doing, and get some help from the instructor and TA. It helps to start the alignment procedure with the micrometer and the movable mirror near the center of their ranges. Also, the alignment knobs on the laser and adjustable mirror should be at the middle of their ranges, so that the plane of each is perpendicular to the table.

A word of caution: sometimes dust on a mirror or imperfections on optical surfaces may produce similar intensity patterns. True interference disappears if you block one arm of the interferometer. Try it!

Note: before starting the measurements, make sure you understand how to read the micrometer properly!



Figure 1.4: Micrometer readings. The coarse division equals to $100 \mu\text{m}$, and smallest division on the rotary dial is $1 \mu\text{m}$ (same as 1 micron). The final measurements is the sum of two. So, from left to right, the figures above show $211 \mu\text{m}$, $345 \mu\text{m}$, and $166 \mu\text{m}$.

Wavelength measurements using Michelson interferometer

Calibration of the interferometer

Record the initial reading on the micrometer. Focus on the central fringe and begin turning the micrometer. You will see that the fringes move. For example, the central spot will change from bright to dark to bright again, that is counted as one fringe. A good method: pick out a reference line on the screen and then softly count fringes as they pass the point. Count a total of about $\Delta m = 50$ fringes and record the new reading on the micrometer.

Each lab partner should make at least two independent measurements, starting from different initial positions of the micrometer. For each trial, approximately 50 fringes should be accurately counted and tabulated with the initial X_1 and final X_2 micrometer settings. Do this at least five times (e.g., 5×50 fringes). Consider moving the mirror both forward and backward. Make sure that the difference $X_2 - X_1$ is consistent between all the measurements. Calculate the average value and standard deviation of the micrometer readings $\langle X_2 - X_1 \rangle$. What value do you obtain for the laser wavelength, and what are your uncertainties on that measurement? Does this match what you expect for a red laser?

Experimental tips

1. Avoid touching the face of the front-surface mirrors, the beamsplitter, and any other optical elements!
2. The person turning the micrometer should also do the counting of fringes. It can be easier to count them in bunches of 5 or 10 (*i.e.* 100 fringes = 10 bunches of 10 fringes).
3. Use a reference point or line and count fringes as they pass.
4. Before the initial position X_1 is read make sure that the micrometer has engaged the drive screw (There can be a problem with “backlash”). Just turn it randomly before counting.
5. Avoid hitting the table which can cause a sudden jump in the number of fringes.

Measurement of the Na lamp wavelength

A calibrated Michelson interferometer can be used as a **wavemeter** to determine the wavelength of different light sources. In this experiment you will use it to measure the wavelength of strong yellow sodium fluorescent light, produced by the discharge lamp.¹

¹Actually, sodium might be a better calibration source than a HeNe laser, since it has well known lines, whereas a HeNe can lase at different wavelengths. Perhaps an even better calibration source might be a line from the Hydrogen Balmer series, which can be calculated from the Standard Model.

Without changing the alignment of the interferometer (i.e. without touching any mirrors), remove the focusing lens and carefully place the interferometer assembly on top of an adjustable-height platform such that it is at the same level as the output of the lamp. Since the light power in this case is much weaker than for a laser, you won't be able to use the viewing screen. You will have to observe the interference looking directly to the output beam - unlike laser radiation, the spontaneous emission of a discharge is not dangerous². However, your eyes will get tired quickly! Placing a diffuser plate in front of the lamp will make the observations easier. Since the interferometer is already aligned, you should see the interference picture. Make small adjustments to the adjustable mirror to make sure you see the center of the bull's eye.

Repeat the same measurements as in the previous part by moving the mirror and counting the number of fringes. Each lab partner should make at least two independent measurements, recording initial and final position of the micrometer, and you should do at least five trials. Calculate the wavelength of the Na light for each trial. Then calculate the average value and its experimental uncertainty. Compare with the expected value of 589 nm.

In reality, the Na discharge lamp produces a doublet - two spectral lines that are very close to each other: 589 nm and 589.59 nm. Do you think your Michelson interferometer can resolve this small difference? Hint: the answer is no - we will use a Fabry-Perot interferometer for that task.

²In the "old days" beams in high energy physics were aligned using a similar technique. An experimenter would close his eyes and then put his head in a collimated particle beam. Cerenkov radiation caused by particles traversing the experimenter's eyeball is visible as a blue glow or flashes. This is dangerous but various people claim to have seen it done... when a radiation safety officer wasn't around.

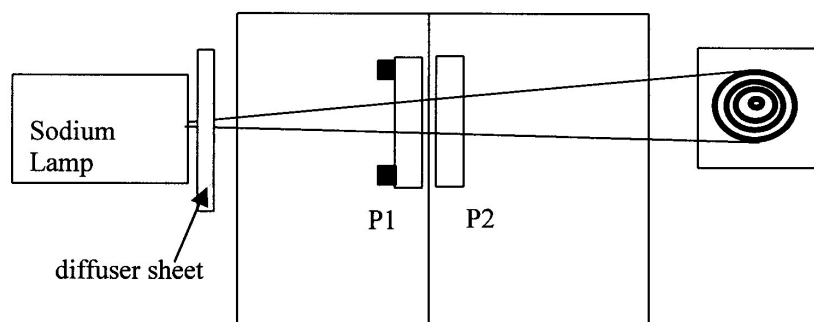


Figure 1.5: The Fabry-Perot Interferometer. **For initial alignment the laser and the convex lens are used instead of the Na lamp.**

Alignment of the Fabry-Perot interferometer

Disassemble the Michelson Interferometer, and assemble the Fabry-Perot interferometer as shown in Figure 1.5, initially using the laser for alignment instead of the sodium lamp. First, place the viewing screen behind the two partially-reflecting mirrors ($P1$ and $P2$), and adjust the mirrors such that the multiple reflections on the screen overlap. Then place a convex lens after the laser to spread out the beam, and make small adjustments until you see the concentric circles. Is there any difference between the thickness of the bright lines for the two different interferometers? Why?

Loosen the screw that mounts the movable mirror and change the distance between the mirrors. Realign the interferometer again, and comment on the difference in the interference picture. Can you explain it?

Align the interferometer one more time such that the distance between the two mirrors is 1.0 – 1.5 mm, but make sure the mirrors do not touch!

Sodium doublet measurements

1. Turn off the laser, remove the viewing screen and the lens, and place the interferometer on the adjustable-height platform, or alternatively place the Na lamp on its side and plan to adjust its height with books or magazines. With the diffuser sheet in front of the lamp, check that you see the interference fringes when you look directly at the lamp through the interferometer. If necessary, adjust the knobs on the adjustable mirror to get the best fringe pattern.
2. Because the Na emission consists of two lights at two close wavelengths, the interference picture consists of two sets of rings, one corresponding to fringes of λ_1 , the other to those for λ_2 . Move the mirror back and forth (by rotating the micrometer) to identify two

sets of rings. Notice that they move at slightly different rates (due to the wavelength difference).

3. Seek the START condition illustrated in Fig.(1.6), such that all bright fringes are evenly spaced. Note that alternate fringes may be of somewhat different intensities. Practice going through the fringe conditions as shown in Fig.(1.6) by turning the micrometer and viewing the relative movement of fringes. Do not be surprised if you have to move the micrometer quite a bit to return to the original condition again.
4. Turn the micrometer close to zero reading, and then find a place on the micrometer (d_1) where you have the “START” condition for fringes shown in Fig.(1.6). Now advance the micrometer rapidly while viewing the fringe pattern (NO COUNTING OF FRINGES IS REQUIRED). Note how the fringes of one intensity are moving to overtake those of the other intensity (in the manner of Fig.(1.6)). Keep turning until the “STOP” pattern is achieved (the same condition you started with). Record the micrometer reading as d_2 .
5. Each lab partner should repeat this measurement at least one time, and each group should have at least three independent measurements.
6. Make sure to tabulate all the data taken by your group.

We chose the “START” condition (the equally spaced two sets of rings) such that for the given distance between the two mirrors, d_1 , the bright fringes of λ_1 occur at the points of destructive interference for λ_2 . Thus, the bull’s eye center ($\theta = 0$) we can write this down as:

$$2d_1 = m_1\lambda_1 = \left(m_1 + n + \frac{1}{2}\right)\lambda_2 \quad (1.8)$$

Here the integer n accounts for the fact that $\lambda_1 > \lambda_2$, and the $1/2$ for the condition of destructive interference for λ_2 at the center. The “STOP” condition corresponds to the similar situation, but the net action of advancing by many fringes has been to increment the fringe count of λ_2 by one more than that of λ_1 :

$$2d_2 = m_2\lambda_1 = \left(m_2 + n + \frac{3}{2}\right)\lambda_2 \quad (1.9)$$

Try to estimate your uncertainty in identifying the START and STOP positions by turning the micrometer back and forth, identifying the points at which you can begin to see doublet, rather than equally spaced, lines. The variance from multiple measurements should agree, at least approximately, with this estimate.

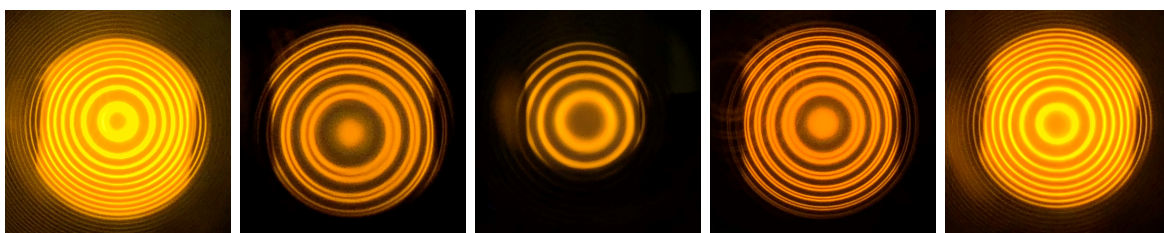
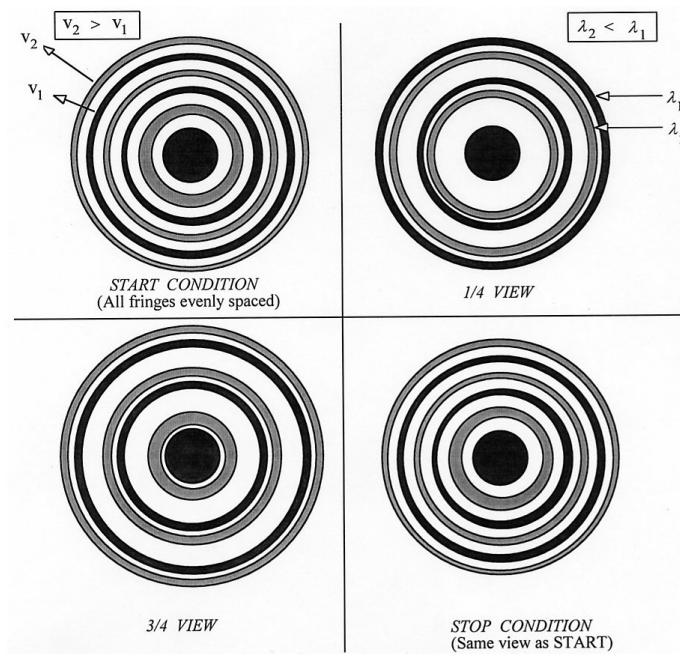
Subtracting the two interference equations, and solving for the distance traveled by the mirror $d_2 - d_1$ we obtain:

$$2(d_2 - d_1) = \frac{\lambda_1\lambda_2}{(\lambda_1 - \lambda_2)} \quad (1.10)$$

Solving this for $\Delta\lambda = \lambda_1 - \lambda_2$, and accepting as valid the approximation that $\lambda_1\lambda_2 \approx \lambda^2$ (where λ is the average of λ_1 and $\lambda_2 \approx 589.26$ nm), we obtain:

$$\Delta\lambda \approx \frac{\lambda^2}{2(d_2 - d_1)} \quad (1.11)$$

Use this equation and your experimental measurements to calculate average value of Na doublet splitting and its standard deviation, as well as any experimental uncertainties. Compare your result with the established value of $\Delta\lambda_{Na} = 0.598$ nm.



(a) Start (b) 1/4 view (c) 1/2 view (d) 3/4 view (e) Stop

Figure 1.6: The sequence of fringe patterns encountered in the course of the measurements. Note false colors in the upper picture: the colors for λ_1 and λ_2 are indistinguishable for a human eye.

Observation of Gravitational Waves (LIGO)

A Michelson interferometer can help to test the theory of general relativity! Gravity waves, predicted by the theory of relativity, are ripples in the fabric of space and time produced by violent events in the distant universe, such as the collision of two black holes. Gravitational waves are emitted by accelerating masses much as electromagnetic waves are produced by accelerating charges, and often travel to Earth. Until recently the only indirect evidence for these waves was the observation of the rotation of a binary pulsar (for which the 1993 Nobel Prize was awarded).



Figure 1.7: For more details see <http://www.ligo.caltech.edu/>

The Laser Interferometry Gravitational-wave Observatory (LIGO) Michelson interferometer built with the goal of directly detecting gravitational wave. Two mirrors hang 2.5 mi apart, forming one “arm” of the interferometer, and two more mirrors make a second arm perpendicular to the first. Laser light enters the arms through a beam splitter located at the corner of the L, dividing the light between the arms. The light is allowed to bounce between the mirrors repeatedly before it returns to the beam splitter. If the two arms have identical lengths, then interference between the light beams returning to the beam splitter will direct all of the light back toward the laser. But if there is any difference between the lengths of the two arms, some light will travel to where it can be recorded by a photodetector.

The space-time ripples cause the distance measured by a light beam to change as the gravitational wave passes by. These changes are minute: just 10^{-16} centimeters, or one-hundred-millionth the diameter of a hydrogen atom over the 2.5 mile length of the arm. Yet, they are enough to change the amount of light falling on the photodetector, which produces a signal defining how the light falling on the photodetector changes over time. LIGO requires at least two widely separated detectors, operated in unison, to rule out false signals and confirm that a gravitational wave has passed through the earth. Three interferometers were built for LIGO – two near Richland, Washington, and the other near Baton Rouge, Louisiana, shown in Fig. 1.7.

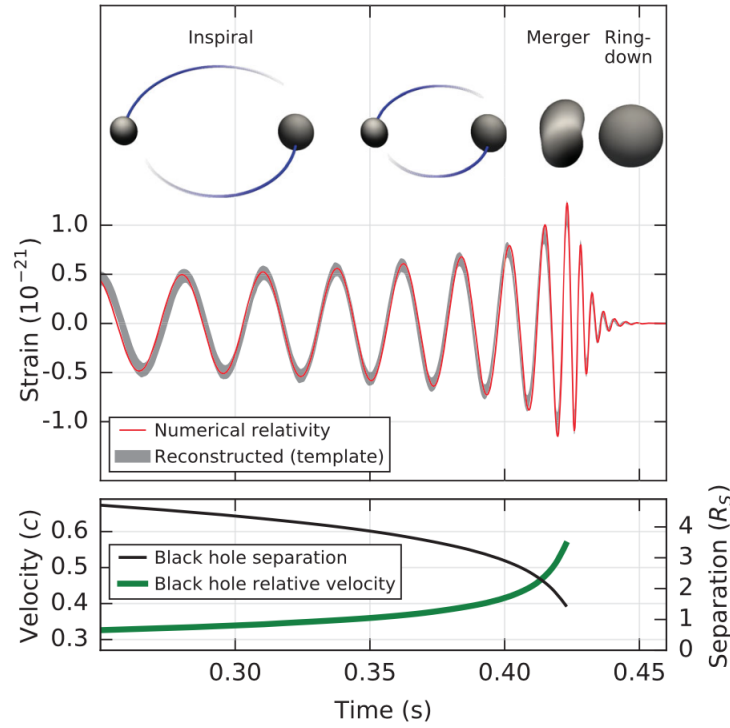


Figure 1.8: Data from LIGO's two sites from the first detection of gravitational waves caused by the collision of two black holes (from [Physical Review Letters 116, 061102](#)).

In 2015, LIGO observed the first direct detection of gravitational waves from the collision of two black holes. Figure 1.8 shows data from the black hole merger event where the 'strain' reflects the observed change in distance in the interferometer arm due to the passing gravitation wave, which oscillates as a function of time. For a more information you can browse the [LIGO webpage](#) and view a [simulation of a black hole merger](#).

Chapter 2

Blackbody Radiation

Experiment objectives: explore radiation from objects at certain temperatures, commonly known as “blackbody radiation”; make measurements testing the Stefan-Boltzmann law in high- and low-temperature ranges; measure the inverse-square law for thermal radiation.

A brief writeup: Your instructor may tell you to do a brief writeup of this experiment. If so you will take data as instructed in the pages that follow. However when it comes time to do the analysis and write the report you will follow the *Instructions for a brief writeup* on page [25](#).

Theory

A familiar observation to us is that dark-colored objects absorb more thermal radiation (from the sun, for example) than light-colored objects. You may have also observed that a good absorber of radiation is also a good emitter (like dark-colored seats in an automobile). Although we observe thermal radiation (“heat”) mostly through our sense of touch, the range of energies at which the radiation is emitted can span the visible spectrum (thus we speak of high-temperature objects being “red hot” or “white hot”). For temperatures below about 600°C , however, the radiation is emitted in the infrared, and we cannot see it with our eyes, although there are special detectors (like the one you will use in this lab) that can measure it.

An object which absorbs all radiation incident on it is known as an “ideal blackbody”. In 1879 Josef Stefan found an empirical relationship between the power per unit area radiated by a blackbody and the temperature, which Ludwig Boltzmann derived theoretically a few years later. This relationship is the **Stefan-Boltzmann law**:

$$S = \sigma T^4 \tag{2.1}$$

where S is the radiated power per unit area (W/m^2), T is the temperature (in Kelvins), and $\sigma = 5.6703 \times 10^{-8} \text{W}/\text{m}^2\text{K}^4$ is the Stefan’s constant.

Most hot, opaque objects can be approximated as blackbody emitters, but the most ideal blackbody is a closed volume (a cavity) with a very small hole in it. Any radiation entering

the cavity is absorbed by the walls, and then is re-emitted out. Physicists first tried to calculate the spectral distribution of the radiation emitted from the ideal blackbody using *classical thermodynamics*. This method involved finding the number of modes of oscillation of the electromagnetic field in the cavity, with the energy per mode of oscillation given by kT . The classical theory gives the **Rayleigh-Jeans law**:

$$u(\lambda, T) = \frac{8\pi kT}{\lambda^4}$$

where $u(\lambda)(J/m^4)$ is the spectral radiance (energy radiated per unit area at a single wavelength or frequency), and λ is the wavelength of radiation. This law agrees with the experiment for radiation at long wavelengths (infrared), but predicts that $u(\lambda)$ should increase infinitely at short wavelengths. This is not observed experimentally (Thank heaven, or we would all be constantly bathed in ultraviolet light—a true ultraviolet catastrophe!). It was known that the wavelength distribution peaked at a specific temperature as described by **Wien’s law**:

$$\lambda_{max}T = 2.898 \times 10^{-3}m \cdot K$$

and went to zero for short wavelengths.

The breakthrough came when Planck assumed that the energy of the oscillation modes can only take on discrete values rather than a continuous distribution of values, as in classical physics. With this assumption, Planck’s law was derived:

$$u(\lambda, T) = \frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT} - 1}$$

where c is the speed of light and $h = 6.626076 \times 10^{-34} J \cdot s$ is the Planck’s constant. This proved to be the correct description.

Sometimes physicists have to have crazy ideas!

“The problem of radiation-thermodynamics was solved by Max Planck, who was a 100 percent classical physicist (for which he cannot be blamed). It was he who originated what is now known as modern physics. At the turn of the century, at the December 14, 1900 meeting of the German Physical Society, Planck presented his ideas on the subject, which were so unusual and so grotesque that he himself could hardly believe them, even though they caused intense excitement in the audience and the entire world of physics.”

From George Gamow, *“Thirty Years that Shook Physics, The Story of Quantum Physics”*, Dover Publications, New York, 1966.

Safety

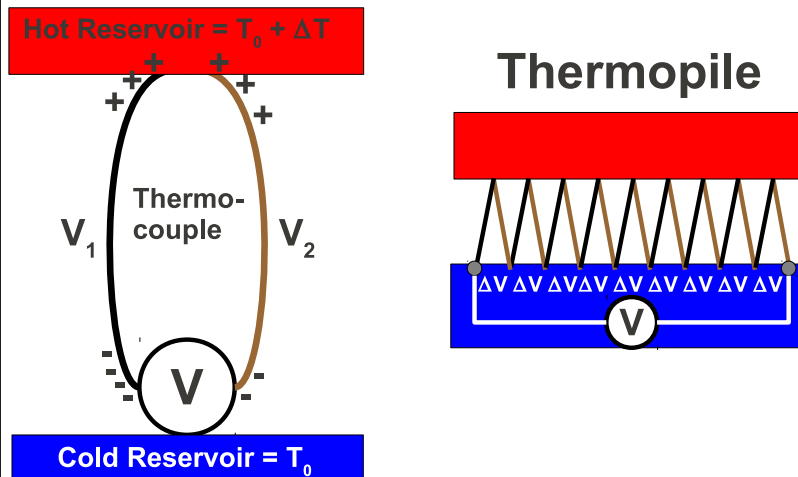
The Stefan lamp and thermal cube will get very hot - be careful!!!

Thermal radiation rates from different surfaces

Equipment needed: Pasco Radiation sensor, Pasco Thermal Radiation Cube, two multi-meters, window glass.

Before starting actual experiment take some time to have fun with the thermal radiation sensor. Can you detect your lab partner? What about people across the room? Point the sensor in different directions and see what objects affect the readings. **These exercises are fun, but you will also gain important intuition about various factors which may affect the accuracy of the measurements!**

How does the radiation sensor work?



Imagine a metal wire connected to a cold reservoir at one end and a hot reservoir at the other. Heat will flow between the ends of the wire, carried by the electrons in the conductor, which will tend to diffuse from the hot end to the cold end. Vibrations in the conductor's atomic lattice can also aid this process. This diffusion causes a potential difference between the two ends of the wire. The size of the potential difference depends on the temperature gradient and on details of the conductive material, but is typically in the few 10s of $\mu V/K$. A thermocouple, shown on the left, consists of two different conductive materials joined together at one end and connected to a voltmeter at the other end. The potential is, of course, the same on either side of the joint, but the difference in material properties causes $\Delta V = V_1 - V_2 \neq 0$. This ΔV is measured by the voltmeter and is proportional to ΔT . Your radiation sensor is a thermopile, simply a "pile" of thermocouples connected in series, as shown at the right. This is done to make the potential difference generated by the temperature gradient easier to detect.

1. Connect the two multimeters and position the sensor as shown in Fig. 2.1. The multimeter attached to the cube should be set to read resistance while the one attached to the infrared radiation sensor will monitor the potential (in the millivolt range). Make sure the shutter on the sensor is pushed all the way open!
2. Before turning on the cube, measure the resistance of the thermistor at room tem-

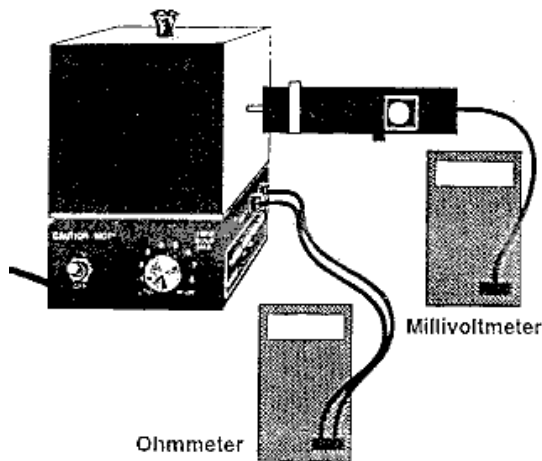


Figure 2.1: Thermal radiation setup

perature, and obtain the room temperature from the instructor. You will need this information for the data analysis.

3. Turn on the thermal radiation cube and set the power to “high”. When the ohmmeter reading decreases to 40 k Ω (5-20 minutes) set power switch to “8”. (If the cube has been preheated, immediately set the switch to “8”.)

Important: when using the thermal radiation sensor, make each reading quickly to keep the sensor from heating up. Use both sheets of white isolating foam (with the silvered surface facing the lamp) to block the sensor between measurements.

Sensor calibration: To obtain the radiation sensor readings for radiated power per unit area S in the correct units (W/m^2), you need to use the voltage-to-power conversion factor 22 mV/mW , and the area of the sensor $2mm \times 2mm$:

$$S[W/m^2] = \frac{S[mV]}{22[mV/mW]} \cdot 10^{-3} \cdot \frac{1}{4 \cdot 10^{-6}[m^2]}$$

4. When the cube has reached thermal equilibrium the ohmmeter will be fluctuating around a constant value. Record the resistance of the thermistor in the cube and determine the approximate value of the temperature using the data table in Fig 2.2. Use the radiation sensor to measure the radiation emitted from the four surfaces of the cube. Place the sensor so that the posts on its end are in contact with the cube surface (this ensures that the distance of the measurement is the same for all surfaces) and record the sensor reading. Each lab partner should make an independent measurement.
5. Place the radiation sensor approximately 5 cm from the black surface of the radiation cube and record its reading. Place a piece of glass between the sensor and the cube. Record again. Does window glass effectively block thermal radiation? Try observing the effects of other objects, recording the sensor reading as you go.

Therm. Res. (Ω)	Temp. ($^{\circ}\text{C}$)	Therm. Res. (Ω)	Temp. ($^{\circ}\text{C}$)	Therm. Res. (Ω)	Temp. ($^{\circ}\text{C}$)	Therm. Res. (Ω)	Temp. ($^{\circ}\text{C}$)	Therm. Res. (Ω)	Temp. ($^{\circ}\text{C}$)	Therm. Res. (Ω)	Temp. ($^{\circ}\text{C}$)
207,850	10	66,356	34	24,415	58	10,110	82	4,615.1	106	2,281.0	130
197,560	11	63,480	35	23,483	59	9,767.2	83	4,475.0	107	2,218.3	131
187,840	12	60,743	36	22,590	60	9,437.7	84	4,339.7	108	2,157.6	132
178,650	13	58,138	37	21,736	61	9,120.8	85	4,209.1	109	2,098.7	133
169,950	14	55,658	38	20,919	62	8,816.0	86	4,082.9	110	2,041.7	134
161,730	15	53,297	39	20,136	63	8,522.7	87	3,961.1	111	1,986.4	135
153,950	16	51,048	40	19,386	64	8,240.6	88	3,843.4	112	1,932.8	136
146,580	17	48,905	41	18,668	65	7,969.1	89	3,729.7	113	1,880.9	137
139,610	18	46,863	42	17,980	66	7,707.7	90	3,619.8	114	1,830.5	138
133,000	19	44,917	43	17,321	67	7,456.2	91	3,513.6	115	1,781.7	139
126,740	20	43,062	44	16,689	68	7,214.0	92	3,411.0	116	1,734.3	140
120,810	21	41,292	45	16,083	69	6,980.6	93	3,311.8	117	1,688.4	141
115,190	22	39,605	46	15,502	70	6,755.9	94	3,215.8	118	1,643.9	142
109,850	23	37,995	47	14,945	71	6,539.4	95	3,123.0	119	1,600.6	143
104,800	24	36,458	48	14,410	72	6,330.8	96	3,033.3	120	1,558.7	144
100,000	25	34,991	49	13,897	73	6,129.8	97	2,946.5	121	1,518.0	145
95,447	26	33,591	50	13,405	74	5,936.1	98	2,862.5	122	1,478.6	146
91,126	27	32,253	51	12,932	75	5,749.3	99	2,781.3	123	1,440.2	147
87,022	28	30,976	52	12,479	76	5,569.3	100	2,702.7	124	1,403.0	148
83,124	29	29,756	53	12,043	77	5,395.6	101	2,626.6	125	1,366.9	149
79,422	30	28,590	54	11,625	78	5,228.1	102	2,553.0	126	1,331.9	150
75,903	31	27,475	55	11,223	79	5,066.6	103	2,481.7	127		
72,560	32	26,409	56	10,837	80	4,910.7	104	2,412.6	128		
69,380	33	25,390	57	10,467	81	4,760.3	105	2,345.8	129		

Figure 2.2: Resistance vs. temperature for the thermal radiation cube

Use your data to address the following questions in your lab report:

1. Is it true that good absorbers of radiation are good emitters?
2. Is the emission from the black and white surface similar?
3. Do objects at the same temperature emit different amounts of radiation?
4. Does glass effectively block thermal radiation? Comment on the other objects that you tried.

Tests of the Stefan-Boltzmann Law

High temperature regime

Equipment needed: Radiation sensor, 3 multimeters, Stefan-Boltzmann Lamp, Power supply.

1. **Before turning on the lamp**, measure the resistance of the filament of the Stefan-Boltzmann lamp at room temperature. Record the room temperature, visible on the wall thermostat and on the bench mounted thermometers in the room.

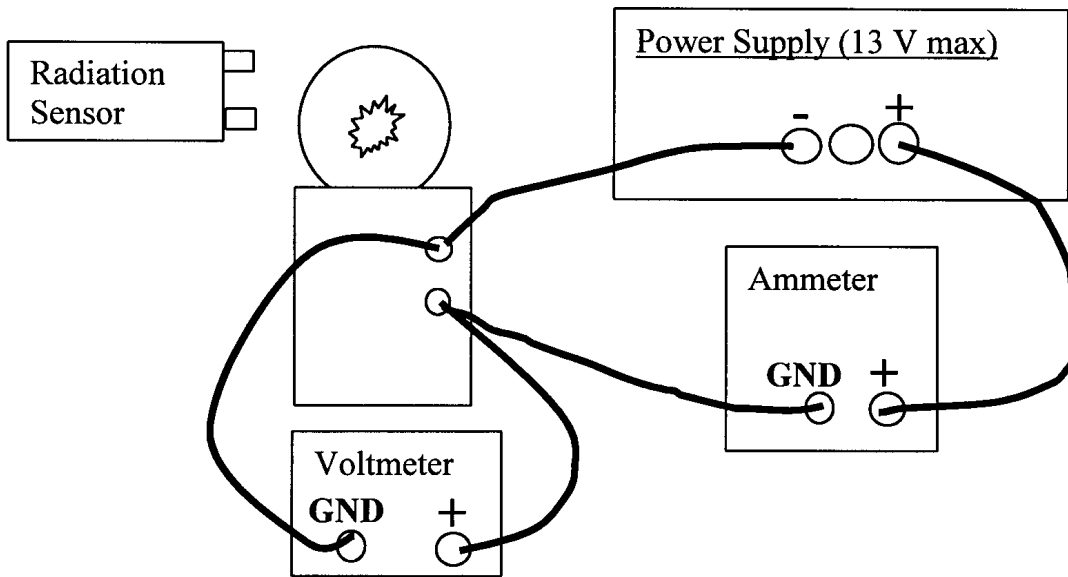


Figure 2.3: Lamp connection for high-temperature Stefan-Boltzmann setup

2. Connect a multimeter as voltmeter to the output of the power supply. **Important:** make sure it is in the **voltmeter mode**. Compare voltage readings provided by the power supply and the multimeter. Which one is the correct one? Think about your measurement uncertainties.
3. Set up the equipment as shown in Fig. 2.3. **VERY IMPORTANT:** make all connections to the lamp when the power is off. Turn the power off before changing/removing connections. The voltmeter should be directly connected to the binding posts of the Stefan-Boltzmann lamp. In this case the multimeter voltmeter has direct access to the voltage drop across the bulb, while the power supply voltmeter reads an extra voltage due to finite resistance of the current meter. Compare readings on the multimeter and the power supply current meters. Which one is the correct one? Think about your measurement uncertainties.
4. Place the thermal sensor at the same height as the filament, with the front face of the sensor approximately 6 cm away from the filament (this distance will be fixed throughout the measurement). Make sure no other objects are viewed by the sensor other than the lamp.
5. Turn on the lamp power supply. Set the voltage, V , in steps of one volt from 1-12 volts. At each V , record the ammeter (current) reading from the lamp and the voltage from the radiation sensor. Calculate the resistance of the lamp using Ohm's Law and determine the temperature T of the lamp from the table shown in Fig. 2.4. You can

use a table to record your data similar to the sample table 2.1.

R/R _{300K}	Temp K	Resistivity μΩ cm	R/R _{300K}	Temp K	Resistivity μΩ cm	R/R _{300K}	Temp K	Resistivity μΩ cm	R/R _{300K}	Temp K	Resistivity μΩ cm
1.0	300	5.65	5.48	1200	30.98	10.63	2100	60.06	16.29	3000	92.04
1.43	400	8.06	6.03	1300	34.08	11.24	2200	63.48	16.95	3100	95.76
1.87	500	10.56	6.58	1400	37.19	11.84	2300	66.91	17.62	3200	99.54
2.34	600	13.23	7.14	1500	40.36	12.46	2400	70.39	18.28	3300	103.3
2.85	700	16.09	7.71	1600	43.55	13.08	2500	73.91	18.97	3400	107.2
3.36	800	19.00	8.28	1700	46.78	13.72	2600	77.49	19.66	3500	111.1
3.88	900	21.94	8.86	1800	50.05	14.34	2700	81.04	26.35	3600	115.0
4.41	1000	24.93	9.44	1900	53.35	14.99	2800	84.70			
4.95	1100	27.94	10.03	2000	56.67	15.63	2900	88.33			

Figure 2.4: Table of tungsten’s resistance as a function of temperature.

Data(± error)			Calculations		
V((volts)	I(amps)	Rad(mV)	R(ohms)	T(K)	T ⁴ (K ⁴)
1.00					
...					

Table 2.1: Sample table for recording your data.

In the lab report plot the reading from the radiation sensor (convert to W/m^2) (on the y axis) versus the temperature T on the x axis. According to the Stefan-Boltzmann Law, the data should show a $S \propto T^4$ behavior. Do a fit to $S = CT^n$ where C and n are free parameters. Report the value of each parameter and its uncertainty. Does $n = 4$, within uncertainty? How does it compare to the accepted value of Stefan’s constant?

Examine the fit. How many points are within 1 uncertainty (σ) of the line? How many are between 1 and 2 σ ? For a good fit, about 2/3 of the points should be within 1σ of the line, and the other 1/3 between $1-2\sigma$, with perhaps one point between 2 and 3σ , if we have about 20 points. That latter number scales up with the number of points. Unless you have a few 100 datapoints, a good fit should not have any points further than 3σ . Based on this criteria, was is your fit good? Do the points seem to systematically differ from the fit line in a particular region? Can you think of a reason why that would be?

The parameter C is equal to $A\sigma$. For an ideal black body $A = 1$, but in your case it will be very different. $A < 1$ means the object does not absorb (or emit) all the radiation incident on it (this object only radiates a fraction of the radiation of a true blackbody). The material lampblack has $A = 0.95$ while tungsten wire has $A = 0.032$ (at $30^\circ C$) to 0.35 (at $3300^\circ C$). Using the result of your fit, and assuming we know the Stephan-Boltzman constant σ by some other means, what is A and what is the uncertainty on it? Is it consistent with tungsten? What else could be affecting this measurement?

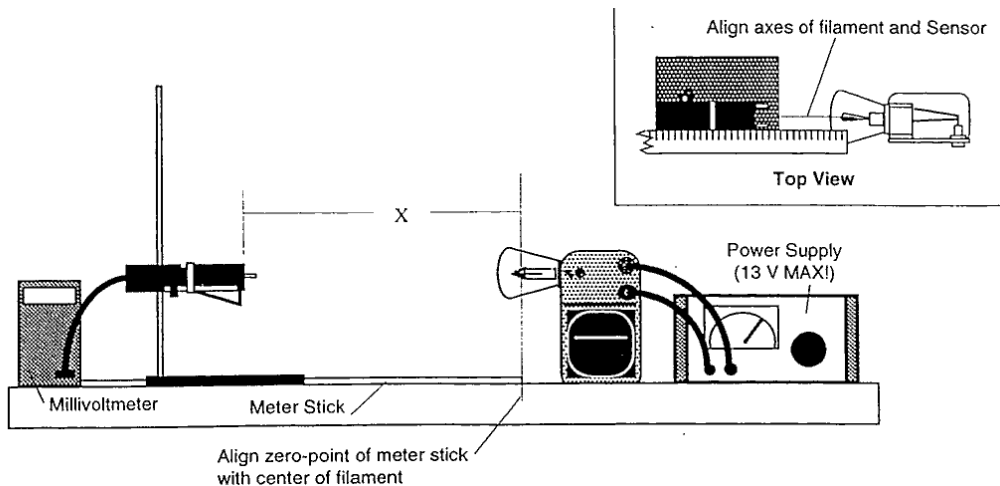


Figure 2.5: Inverse square law setup

Test of the inverse-square law

Equipment needed: Radiation sensor, Stefan-Boltzmann lamp, multimeter, power supply, meter stick. A point source of radiation emits that radiation according to an inverse square law: that is, the intensity of the radiation in (W/m^2) is proportional to the inverse square of the distance from that source. You will determine if this is true for a lamp.

1. Set up the equipment as shown in Fig. 2.5. Tape the meter stick to the table. Place the Stefan-Boltzmann lamp at one end, and the radiation sensor in direct line on the other side. The zero-point of the meter stick should align with the lamp filament (or, should it?). Adjust the height of the radiation sensor so it is equal to the height of the lamp. Align the system so that when you slide the sensor along the meter stick the sensor still aligns with the axis of the lamp. Connect the multimeter (reading millivolts) to the sensor and the lamp to the power supply.
2. With the **lamp off**, slide the sensor along the meter stick. Record the reading of the voltmeter at 10 cm intervals. Average these values to determine the ambient level of thermal radiation. You will need to subtract this average value from your measurements with the lamp on.
3. Turn on the power supply to the lamp. Set the voltage to approximately 10 V. **Do not exceed 13 V!** Adjust the distance between the sensor and lamp from 2.5-100 cm and record the sensor reading. **Before the actual experiment think carefully about at what distances you want to take the measurements. Is taking them at constant intervals the optimal approach? At what distances would you expect the sensor reading change more rapidly?**
4. Make a plot of the corrected radiation measured from the lamp versus the distance from the lamp to the sensor x . Fit the data to $S = B + Cx^n$ where B , C and n are

- free parameters. Based on the criteria in the previous section, how good is the fit?
5. What are the values of B , C and n (and, of course, their uncertainties)? How do they agree with what you expect from the inverse square law?
 6. Can the lamp be considered a point source? If not, how could this affect your measurements?

Your instructor may have told you to do a “brief writeup” of this experiment. If so, please see the instructions starting on the next page (page [25](#)).

Instructions for a brief writeup

This lab is reasonably straightforward. There is much data to take and several fits, but describing the setup and providing an introduction seems less necessary than for some of the others. Therefore, just answer the following questions/bullet points in your report. **Please restate each one so it's easy for us to follow along.** *Refer to the relevant section earlier in the manual for additional discussion. This is just a bullet point summary of the deliverables.*

Thermal radiation rates from different surfaces

First, tabulate your data and include it (with label and caption). Then use your data to address the following questions:

1. Is it true that good absorbers of radiation are good emitters? *Hint:* use what you know about absorption of visible light for these surfaces
2. Is emission from the black and white surfaces similar?
3. Do objects at the same temperature emit different amounts of radiation?
4. Is glass effective in blocking thermal radiation? What about other objects that you tried?

Tests of the Stephan-Boltzmann Law

1. Tabulate your data as in Tab 4.1 and include it.
2. Estimate and report uncertainties on the voltage, current, and radiation sensor.
3. Plot the readings from the radiation sensor (y-axis) vs T (x-axis) with uncertainties and fit to $S = CT^n$. Include the plot in your report.
4. Is it a good fit?
5. Does the Stephan-Boltzmann equation seem to hold? Does $n = 4$, within uncertainties?
6. What value (and uncertainty) do your data imply for A ? Is it consistent with tungsten? What else could be affecting your measurement?

Test of the inverse square law

1. Tabulate your data and include it in the report. Be sure to include the lamp-off data, used to correct the lamp-on data for ambient radiation.
2. Estimate and report uncertainties on the distance between the sensor and lamp, and on the readings from the radiation sensor.

3. Make a plot of the corrected radiation (y-axis, with uncertainty) vs distance (x-axis) and fit it to $S = B + Cx^n$. Include it in your report.
4. How good is the fit?
5. Does $n = -2$ within uncertainty? Do the data confirm the inverse square law?
6. What are B and C ? What did you expect? Comment on any discrepancies.

Chapter 3

Electron Diffraction

Experiment objectives: observe diffraction of the beam of electrons on a graphitized carbon target and calculate the intra-atomic spacings in the graphite.

History

A primary tenet of quantum mechanics is the wavelike properties of matter. In 1924, graduate student Louis de Broglie suggested in his dissertation that since light has both particle-like **and** wave-like properties, perhaps all matter might also have wave-like properties. He postulated that the wavelength of objects was given by $\lambda = h/p$, where h is Planck's constant and $p = mv$ is the momentum. *This was quite a revolutionary idea*, since there was no evidence at the time that matter behaved like waves. In 1927, however, Clinton Davisson and Lester Germer discovered experimental proof of the wave-like properties of matter — particularly electrons. This discovery was quite by mistake: while studying electron reflection from a nickel target, they inadvertently crystallized their target, while heating it, and discovered that the scattered electron intensity as a function of scattering angle showed maxima and minima. That is, electrons were “diffracting” from the crystal planes much like light diffracts from a grating, leading to constructive and destructive interference. Not only was this discovery important for the foundation of quantum mechanics (Davisson and Germer won the Nobel Prize for their discovery), but electron diffraction is an extremely important tool used to study new materials. In this lab you will study electron diffraction from a graphite target, measuring the spacing between the carbon atoms.

Theory

Consider planes of atoms in a **crystal** as shown in Fig. 3.1 separated by distance d . Electron “waves” reflect from each of these planes. Since the electron is wave-like, the combination of the reflections from each interface produces to an interference pattern. This is completely analogous to light interference, arising, for example, from different path lengths in the Fabry-Perot or Michelson interferometers. The de Broglie wavelength for the electron is given by

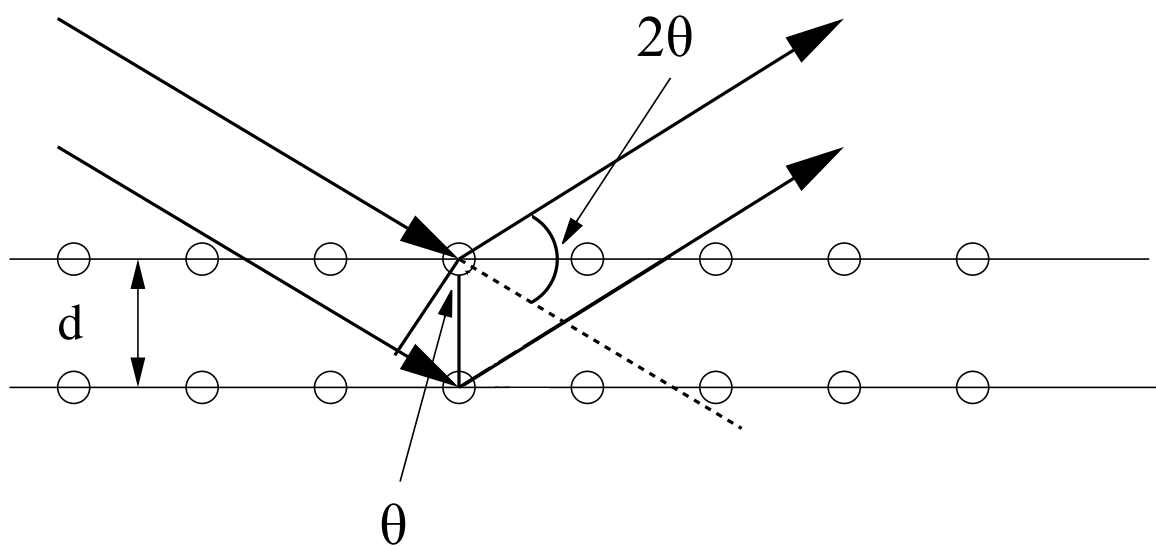


Figure 3.1: Electron Diffraction from atomic layers in a crystal.

$\lambda = h/p$, where p can be calculated by knowing the energy of the electrons when they leave the “electron gun”:

$$\frac{1}{2}mv^2 = \frac{p^2}{2m} = eV_a, \quad (3.1)$$

where V_a is the accelerating potential. The condition for constructive interference is that the path length difference for the two waves shown in Fig. 3.1 be a multiple of a wavelength. This leads to Bragg’s Law:

$$n\lambda = 2d \sin \theta \quad (3.2)$$

where $n = 1, 2, \dots$ is integer. In this experiment, only the first order diffraction $n = 1$ is observed. Therefore, the intra-atomic distance in a crystal can be calculated by measuring the angle of electron diffraction and their wavelength (*i.e.* their momentum):

$$d = \frac{\lambda}{2 \sin \theta} = \frac{1}{2 \sin \theta} \frac{h}{\sqrt{2em_e V_a}} \quad (3.3)$$

where h is Planck’s constant, e is the electronic charge, m_e is the electron’s mass, and V_a is the accelerating voltage.

Experimental Procedure

Equipment needed: Electron diffraction apparatus and power supply, ruler and thin receipt paper or masking tape.

Safety

You will be working with high voltage. The power supply will be connected for you, but inspect the apparatus when you arrive **before** turning the power on. If any wires are unplugged, ask an instructor to reconnect them. Also, **before** turning the power on, identify the high voltage contacts on the electron diffraction tube, make sure these connections are well-protected and cannot be touched by accident while taking measurements.

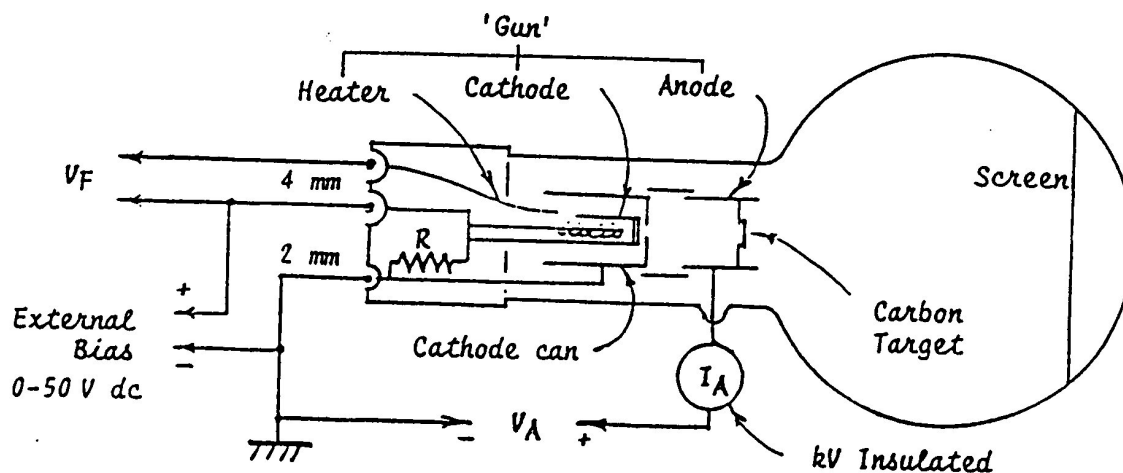


Figure 3.2: Electron Diffraction Apparatus.

Setup

The diagram of the apparatus is given in Fig.3.2. An electron gun (consisting of a heated filament to boil electrons off a cathode and an anode to accelerate them, similar to the e/m experiment) “shoots” electrons at a carbon (graphite) target.

The electrons diffract from the carbon target and the resulting interference pattern is viewed on a phosphorus screen.

The graphitized carbon is not completely crystalline but consists of crystal sheets in random orientations. Therefore, the constructive interference patterns will be seen as bright circular rings. For the carbon target, two rings (an outer and inner, corresponding to different crystal planes) will be seen, corresponding to two spacings between atoms in the graphite arrangement (see Fig. 3.3).

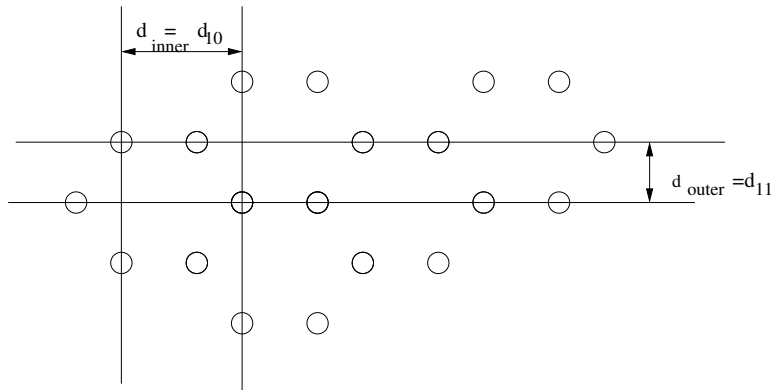


Figure 3.3: Spacing of carbon atoms. Here subscripts 10 and 11 correspond to the crystallographic directions in the graphite crystal.

Data acquisition

Acceptable power supply settings:

Filament Voltage	V_F	6.3 V ac/dc (8.0 V max.)
Anode Voltage	V_A	1500 - 5000 V dc
Anode Current	I_A	0.15 mA at 4000 V (0.20 mA max.)

1. Switch on the heater and wait one minute for the oxide cathode to achieve thermal stability.
2. Slowly increase V_a until you observe two rings appear around the direct beam. Slowly change the voltage and determine the highest achievable accelerating voltage, and the lowest voltage when the rings are visible.
3. Measure the diffraction angle θ for both inner and outer rings for 5-10 voltages from that range, using the same thin receipt paper (see procedure below). Each lab partner should repeat these measurements (using an individual length of the thin paper).
4. Calculate the average value of θ from the individual measurements for each voltage V_a . Calculate the uncertainties for each θ .

Measurement procedure for the diffraction angle θ

To determine the crystalline structure of the target, one needs to carefully measure the diffraction angle θ . It is easy to see (for example, from Fig. 3.1) that the diffraction angle θ is 1/2 of the angle between the beam incident on the target and the diffracted beam to a ring, hence the 2θ appearing in Fig. 3.4. You are going to determine the diffraction angle θ for a given accelerated voltage from the approximate geometrical ratio

$$L \sin 2\theta = R \sin \phi, \quad (3.4)$$

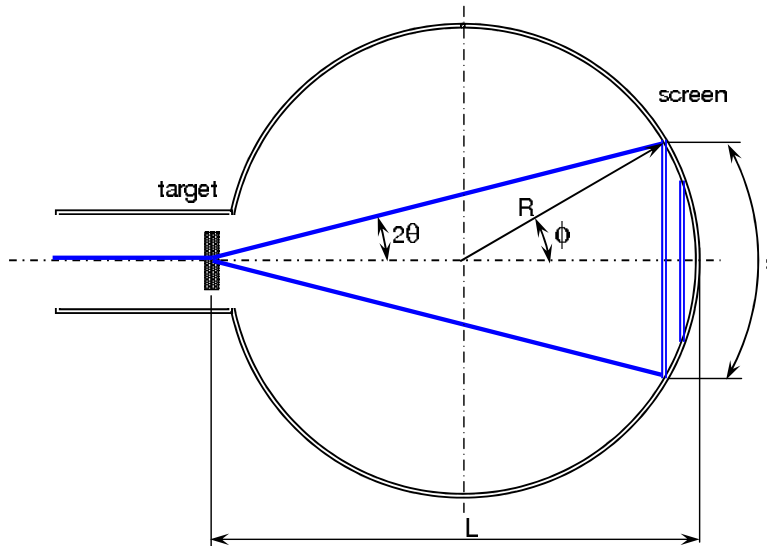


Figure 3.4: Geometry of the experiment.

where the distance between the target and the screen $L = 0.130$ m is controlled during the production process to have an accuracy better than 2%. *Note, this means that the electron tubes are not quite spherical.*

The ratio between the arc length s and the radius of the curvature for the screen $R = 0.066$ m gives the angle ϕ in radians: $\phi = s/2R$. To measure ϕ carefully place a piece of thin receipt paper on the tube so that it crosses the ring along the diameter. Mark the position of the ring for each accelerating voltage, and then remove the paper and measure the arc length s corresponding to each ring. You can also make these markings on masking tape placed gently on the tube.

Data analysis

Use the graphical method to find the average values for the distances between the atomic planes in the graphite crystal d_{11} (outer ring) and d_{10} (inner ring). To determine the combination of the experimental parameters that is proportional to d , one needs to substitute the expression for the electron's velocity Eq.(3.1) into the diffraction condition given by Eq.(3.2):

$$\sin \theta = \frac{h}{2\sqrt{2}m_e e} \frac{1}{d} \frac{1}{\sqrt{V_a}} \quad (3.5)$$

Make a plot of $\sin \theta$ (y-axis, with uncertainty) vs $1/\sqrt{V_a}$ (x-axis) for the inner and outer rings (both curves can be on the same graph). Fit the linear dependence and measure the slope for both lines. From the values of the slopes find the distances between atomic layers d_{inner} and d_{outer} .

Compare your measurements to the accepted values: $d_{inner} = d_{10} = .213$ nm and $d_{outer} = d_{11} = 0.123$ nm.

Seeing with Electrons

The resolution of ordinary optical microscopes is limited (the diffraction limit) by the wavelength of light (≈ 400 nm). This means that we cannot resolve anything smaller than this by looking at it with light (even if we had no limitation on our optical instruments). Since the electron wavelength is only a couple of angstroms (10^{-10} m), with electrons as your “light source” you can resolve features to the angstrom scale. This is why “scanning electron microscopes” (SEMs) are used to look at very small features. The SEM is very similar to an optical microscope, except that “light” in SEMs is electrons and the lenses used are made of magnetic fields, not glass.

Chapter 4

Measurement of Charge-to-Mass (e/m) Ratio for the Electron

Experiment objectives: measure the ratio of the electron charge-to-mass ratio e/m by studying the electron trajectories in a uniform magnetic field.

History

J.J. Thomson first measured the charge-to-mass ratio of the fundamental particle of charge in a cathode ray tube in 1897. A cathode ray tube basically consists of two metallic plates in a glass tube which has been evacuated and filled with a very small amount of background gas. One plate is heated (by passing a current through it) and “particles” boil off of the cathode and accelerate towards the other plate which is held at a positive potential. The gas in between the plates inelastically scatters the electrons, emitting light which shows the path of the particles. The charge-to-mass (e/m) ratio of the particles can be measured by observing their motion in an applied magnetic field. Thomson repeated his measurement of e/m many times with different metals for cathodes and also different gases. Having reached the same value for e/m every time, it was concluded that a fundamental particle having a negative charge e and a mass 2000 times less than the lightest atom existed in all atoms. Thomson named these particles “corpuscles” but we now know them as electrons. In this lab you will essentially repeat Thomson’s experiment and measure e/m for electrons.

Theory

The apparatus shown in Figure 4.1 consists of a glass tube that houses a small electron gun. This gun has a cathode filament from which electrons can be thermionically released (boiled off), and a nearby anode which can be set to a potential which is positive relative to the cathode. Electrons boiled off the cathode are accelerated to the anode, where most are collected. The anode contains a slit, however, which lets a fraction of the electrons into

the larger volume of the glass tube. Some of these electrons scatter inelastically with the background gas, thereby emitting tracer light to define the path of the electrons.

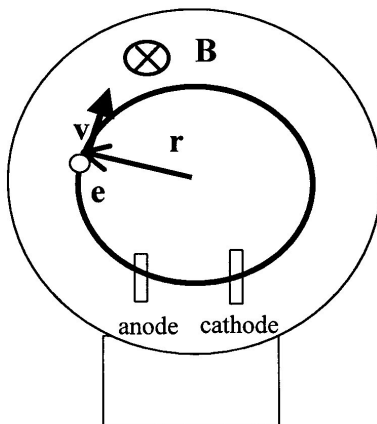


Figure 4.1: The schematic for the e/m apparatus.

To establish the uniform magnetic field a pair of circular Helmholtz coils are wound and the tube centered in the volume of the coils (see Appendix). The tube is oriented so that the beam which exits the electron gun is traveling perpendicular to the Helmholtz field. *We would like the field to be uniform, i.e., the same, over the orbit of the deflected electrons to the level of 1% if possible.*

An electron released thermionically at the cathode has on the order of 1 eV of kinetic energy. This electron “falls” through the positive anode potential V_a , gaining a kinetic energy of:

$$\frac{1}{2}mv^2 = eV_a \quad (4.1)$$

The magnetic field of the Helmholtz coils is perpendicular to this velocity, and produces a magnetic force which is transverse to both \mathbf{v} and \mathbf{B} : $\mathbf{F} = e\mathbf{v} \times \mathbf{B}$. This centripetal force makes an electron move along the circular trajectory; the radius of this trajectory r can be found from Newton’s Second Law:

$$m \left(\frac{v^2}{r} \right) = evB \quad (4.2)$$

From this equation we obtain the expression for the charge-to-mass ratio of the electron, expressed through the experimental parameters:

$$\frac{e}{m} = \frac{v}{rB} \quad (4.3)$$

We shall calculate magnetic field B using the Biot-Savart law for the two current loops of the Helmholtz coils (see Appendix):

$$B = \frac{8}{\sqrt{125}} \frac{\mu_0 N I_{hc}}{a}. \quad (4.4)$$

Here N is the number of turns of wire that form each loop, I_{hc} is the current (which is the same in both loops), a is the radius of the loops (in meters), and the magnetic permeability constant is $\mu_0 = 4\pi \cdot 10^{-7} \text{ T m/A}$.

Noting from Eq.(4.1) that the velocity is determined by the potential V_a as $v = \sqrt{2eV_a/m}$, and using Eq.(4.4) for the magnetic field B , we get:

$$\frac{e}{m} = 2 \frac{V_a}{r^2} \frac{1}{B^2} = 2 \frac{V_a}{r^2} \left[\frac{125}{64} \left(\frac{a}{\mu_0 N I_{hc}} \right)^2 \right] \quad (4.5)$$

The accepted value for the charge-to-mass ratio of the electron is $e/m = 1.7588196 \cdot 10^{11} \text{ C/kg}$.

Experimental Procedure

Equipment needed: Pasco e/m apparatus (SE-9638), Pasco High Voltage Power supply (for the accelerating voltage and the filament heater), GW power supply (for the Helmholtz coils), two digital multimeters.

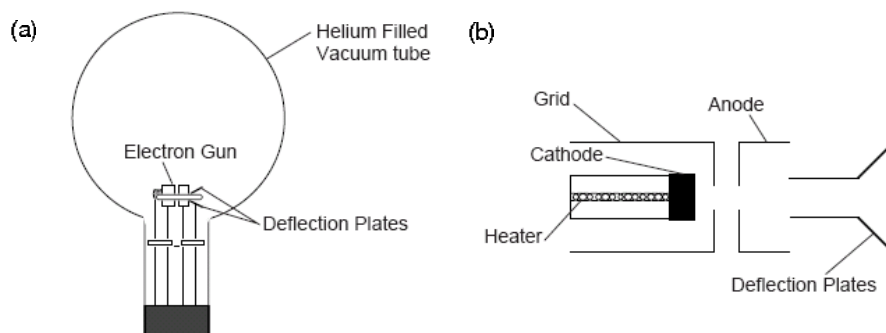


Figure 4.2: (a) e/m tube; (b) electron gun.

Pasco SE-9638 Unit:

The e/m tube (see Fig. 4.2a) is filled with helium at a pressure of 10^{-2} mm Hg, and contains an electron gun and deflection plates. The electron beam leaves a visible trail in the tube, because some of the electrons collide with helium atoms, which are excited and then radiate visible light. The electron gun is shown in Fig. 4.2b. The heater heats the cathode, which emits electrons. The electrons are accelerated by a potential applied between the cathode and the anode. The grid is held positive with respect to the cathode and negative with respect to the anode. It helps to focus the electron beam.

The Helmholtz coils of the e/m apparatus have a radius and separation of $a = 15$ cm. Each coil has $N = 130$ turns. The magnetic field (B) produced by the coils is proportional

to the current through the coils (I_{hc}) times $7.80 \cdot 10^{-4}$ Tesla/Ampere [$B(\text{tesla}) = (7.80 \cdot 10^{-4})I_{hc}$]. A mirrored scale is attached to the back of the rear Helmholtz coil. It is illuminated automatically when the heater of the electron gun is powered. By lining the electron beam up with its image in the mirrored scale, you can measure the radius of the beam path without parallax error. The cloth hood can be placed over the top of the e/m apparatus so the experiment can be performed in a lighted room.

Pasco High Voltage Power Supply

The Pasco High Voltage Power Supply in Fig. 4.3 provides up to 500 V DC accelerating voltage to the electrodes in the e/m tube, which is measured on the voltmeter shown in Fig. 4.4. The electron gun heater is also powered by this supply using the right panel, which should be set to a maximum of 6 V AC using the dial on the supply.

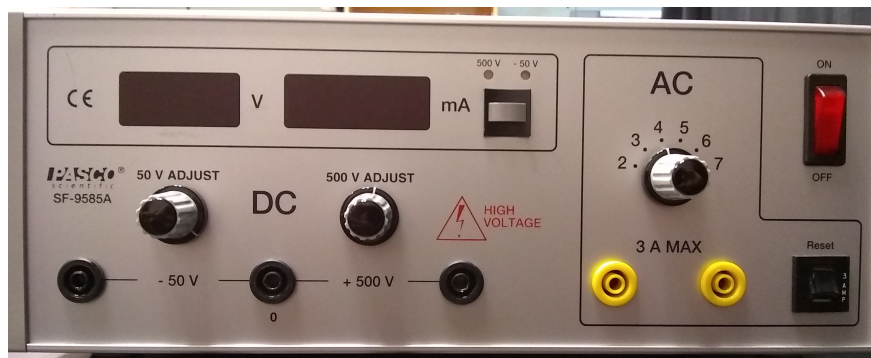


Figure 4.3: Pasco High Voltage Power Supply.

Safety

You will be working with high voltage. Make all connections when power is off. Turn power off before changing/removing connections. Make sure that there are no loose or open contacts.

Set up

The wiring diagram for the apparatus is shown in Fig. 4.4. **Important: Do not turn any equipment until an instructor has checked your wiring.**

Acceptable power supplies settings:

Electron Gun/filament Heater 6 V AC. Do not go to 7V!

Electrodes 150 to 300 V DC

Helmholtz Coils 6 – 9 V DC.

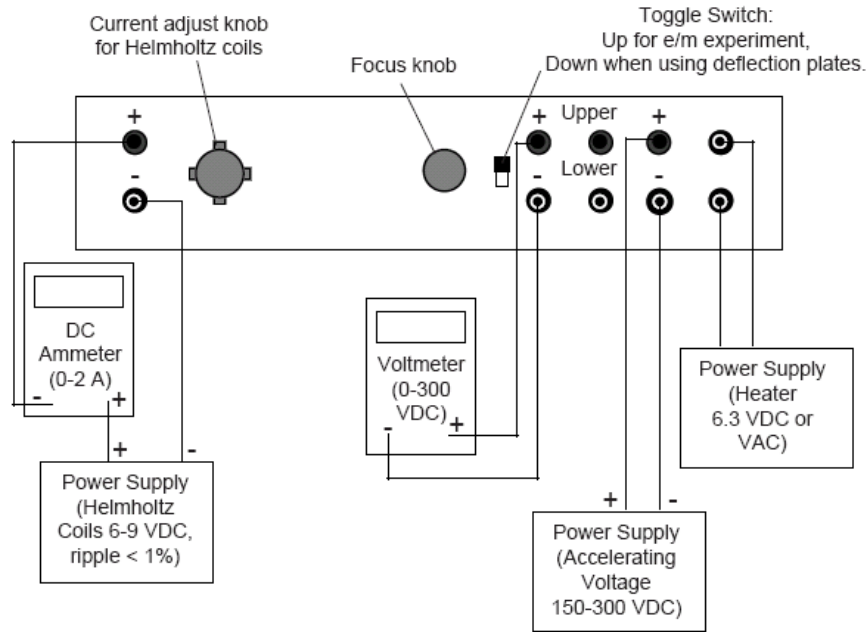


Figure 4.4: Connections for e/m Experiment.

Warning: The voltage for a filament heater should **never** exceed 6.3 VAC. Higher values can burn out filament.

The Helmholtz current should NOT exceed 2 amps. To avoid accidental overshoot run the power supply at a “low” setting in a *constant current* mode and ask the TA or instructor how to set the current limit properly.

Data acquisition

1. Slowly turn the current adjust knob for the Helmholtz coils clockwise. Watch the ammeter and take care that the current is less than 2 A.
2. Wait several minutes for the cathode to heat up. When it does, you will see the electron beam emerge from the electron gun. Its trajectory will be curved by the magnetic field.
3. Rotate the tube slightly if you see any spiraling of the beam. Check that the electron beam is parallel to the Helmholtz coils. If it is not, turn the tube until it is. Don't take it out of its socket. As you rotate the tube, the socket will turn.
4. Measurement procedure for the radius of the electron beam r :
For each measurement record:

Accelerating voltage V_a

Current through the Helmholtz coils I_{hc}

Look through the tube at the electron beam. To avoid parallax errors, move your head to align one side of the electron beam ring with its reflection that you can see on the mirrored scale. Measure the radius of the beam as you see it, then repeat the measurement on the other side, then average the results. Each lab partner should repeat this measurement, and estimate the uncertainty. Do this silently and tabulate results. After each set of measurements (e.g., many values of I_{hc} at one value of V_a) compare your results. This sort of procedure helps reduce group-think, can get at some sources of systematic errors, and is a way of implementing experimental skepticism.

5. Repeat the radius measurements for at least 4 values of V_a and for each V_a for 5-6 different values of the magnetic field.

Improving measurement accuracy

1. The greatest source of error in this experiment is the velocity of the electrons. First, the non-uniformity of the accelerating field caused by the hole in the anode causes the velocity of the electrons to be slightly less than their theoretical value. Second, collisions with the helium atoms in the tube further rob the electrons of their velocity. Since the equation for e/m is proportional to $1/r^2$, and r is proportional to v , experimental values for e/m will be greatly affected by these two effects.
2. To minimize the error due to this lost electron velocity, measure radius to the outside of the beam path.
3. To minimize the relative effect of collisions, keep the accelerating voltage as high as possible. (Above 250 V for best results.) Note, however, that if the voltage is too high, the radius measurement will be distorted by the curvature of the glass at the edge of the tube. Our best results were made with radii of less than 5 cm.
4. Your experimental values will be higher than theoretical, due to the fact that both major sources of error cause the radius to be measured as smaller than it should be.

Calculations and Analysis:

1. Calculate e/m for each of the readings using Eq. 4.5 and determine the uncertainty on e/m based on your uncertainty estimates for those readings. NOTE: Use MKS units for calculations.
2. For each of the four V_a settings calculate the mean $\langle e/m \rangle$ and the standard deviation σ . Are these means consistent with one another sufficiently that you can combine them? [Put quantitatively, are they within 2σ of each other?]
3. Finally, plot the data in the following way which should, (according to Eq. 4.5), reveal a linear relationship: plot V_a on the abscissa [x-axis] versus $r^2 B^2/2$ on the ordinate [y-axis]. The uncertainty in $r^2 B^2/2$ should come from the standard deviation of the

different measurements made by your group at the fixed V_a . The optimal slope of this configuration of data should be $\langle m/e \rangle$. Determine the slope from your plot and its error by doing a linear fit, and compare the slope parameter to the accepted value for $\langle m/e \rangle$. What is the value of the intercept? What should you expect it to be?

4. Comment on which procedure gives a better value of $\langle e/m \rangle$ (averaging or linear plot).

Appendix: Helmholtz coils

The term Helmholtz coils refers to a device for producing a region of nearly uniform magnetic field. It is named in honor of the German physicist Hermann von Helmholtz. A Helmholtz pair consists of two identical coils with electrical current running in the same direction that are placed symmetrically along a common axis, and separated by a distance equal to the radius of the coil a . The magnetic field in the central region may be calculated using the Biot-Savart law:

$$B_0 = \frac{\mu_0 I a^2}{(a^2 + (a/2)^2)^{3/2}}, \quad (4.6)$$

where μ_0 is the magnetic permeability constant, I is the total electric current in each coil, a is the radius of the coils, and the separation between the coils is equal to a .

This configuration provides very uniform magnetic field along the common axis of the pair, as shown in Fig. 4.5. The correction to the constant value given by Eq.(4.6) is proportional to $(x/a)^4$ where x is the distance from the center of the pair. However, this is true only in the case of precise alignment of the pair: the coils must be parallel to each other!

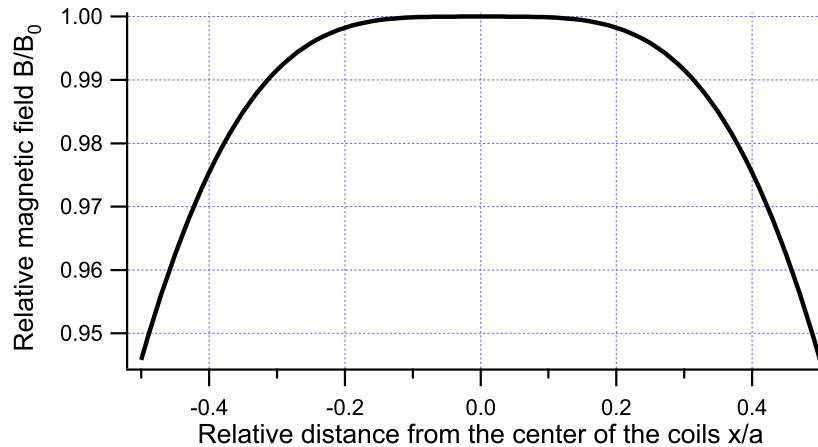


Figure 4.5: Dependence of the magnetic field produced by a Helmholtz coil pair B of the distance from the center (on-axis) x/a . The magnetic field is normalized to the value B_0 in the center.

Chapter 5

Photoelectric Effect

Experiment objectives: measure the ratio of Planck's constant to the electron charge h/e using the photoelectric effect.

History

The photoelectric effect and its understanding was an important step in the development of quantum mechanics. You probably know that Max Planck was the first to postulate that light was made up of discrete packages of energy. At that time it was a proposed hypothetical light property that allowed for the proper description of black body radiation. Several years after Planck made this suggestion, Albert Einstein refined the idea to explain the strange features of electrons which are emitted from metal surfaces after absorbing energy from light. Later he received a Nobel prize for this work.

Theory

Consider an apparatus as outlined in Figure 5.1 (it is the apparatus which Heinrich Hertz used to inadvertently discover the “photoelectric effect”). Light of a frequency ν strikes a cathode, causing electrons to be emitted with velocity v . A positive voltage applied between the anode and the cathode can accelerate emitted electrons towards the positive anode, producing an electrical current in the circuit. A reverse bias potential applied between the anode and cathode will slow down the electrons, and even stop them from reaching the anode by matching their kinetic energy. This is the way to carefully measure the kinetic energies of the photoelectrons.

$$KE_{max} = \left(\frac{1}{2}mv^2\right)_{max} = eV_0 \quad (5.1)$$

where KE_{max} is the maximum kinetic energy, m and e are the mass and the charge of an electron, and V_0 is a potential required to stop the electrons (known as the stopping potential).

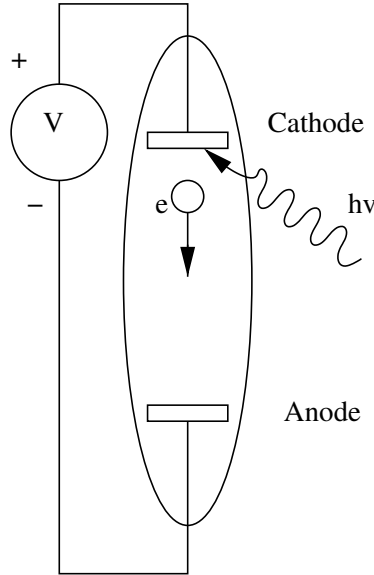


Figure 5.1: A simplified schematic of the photoelectric apparatus.

From the point of view of the wave theory of light, the energy carried by the light wave is proportional to its intensity and independent of light frequency. Thus, it was logical to expect that stronger light should increase the energy of photoelectrons. However, the experiments of Philipp Lenard in 1900 showed something different: although the maximum current increased with light intensity, the stopping potential V_0 was independent of light intensity. This meant that increasing the rate of energy falling onto the cathode does not increase the maximum energy of the electrons. This was quite a surprising result from the classical point of view.

This “controversy” was elegantly resolved five years later by Albert Einstein, who postulated that light arrived in discrete quanta known as “photons”, and each photon led to an emission of a single electron. The energy of each photon is determined by the frequency of light - $h\nu$. Thus the energy of an emitted photoelectron (and therefore the value of the stopping potential V_0) is determined by the frequency of an individual incident photon:

$$eV_0 = \frac{1}{2}mv^2 = h\nu - \phi = \frac{hc}{\lambda} - \phi \quad (5.2)$$

where ϕ is known as the “work function” - the amount of energy needed to free the electron from the cathode. The value of the work energy is a property of the cathode material. Also, it is now clear why the intensity of light does not affect the stopping potential: more intense light has higher photon flux and thus increases the number of emitted photoelectrons, but the energy of each electron is determined only by the energy of a single photon, and that depends only on light frequency ν .

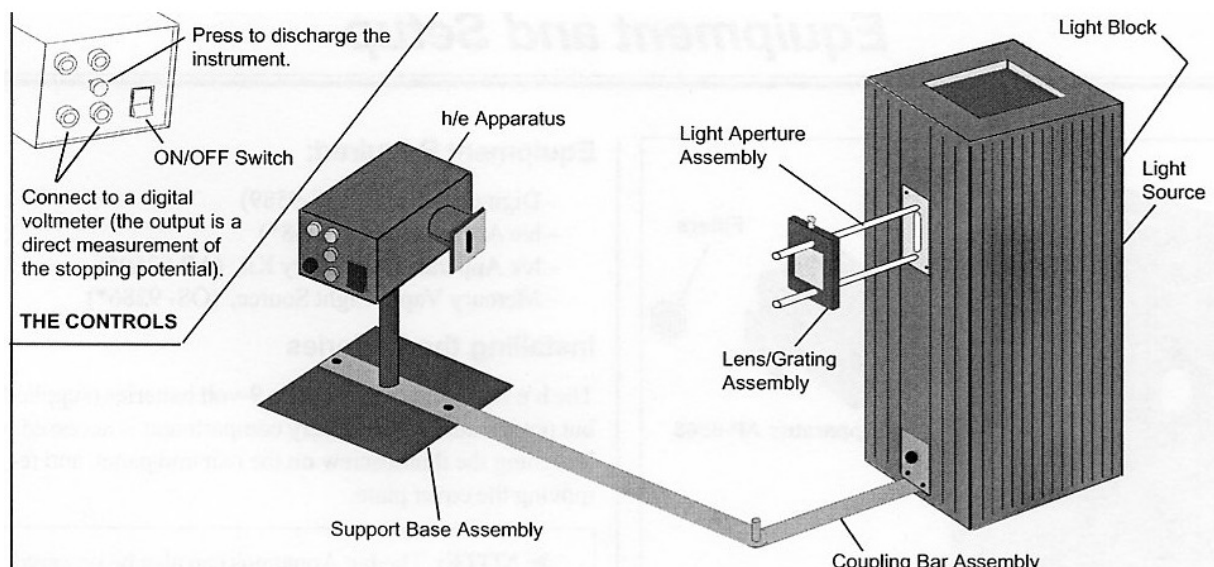


Figure 5.2: The Pasco photoelectric effect setup.

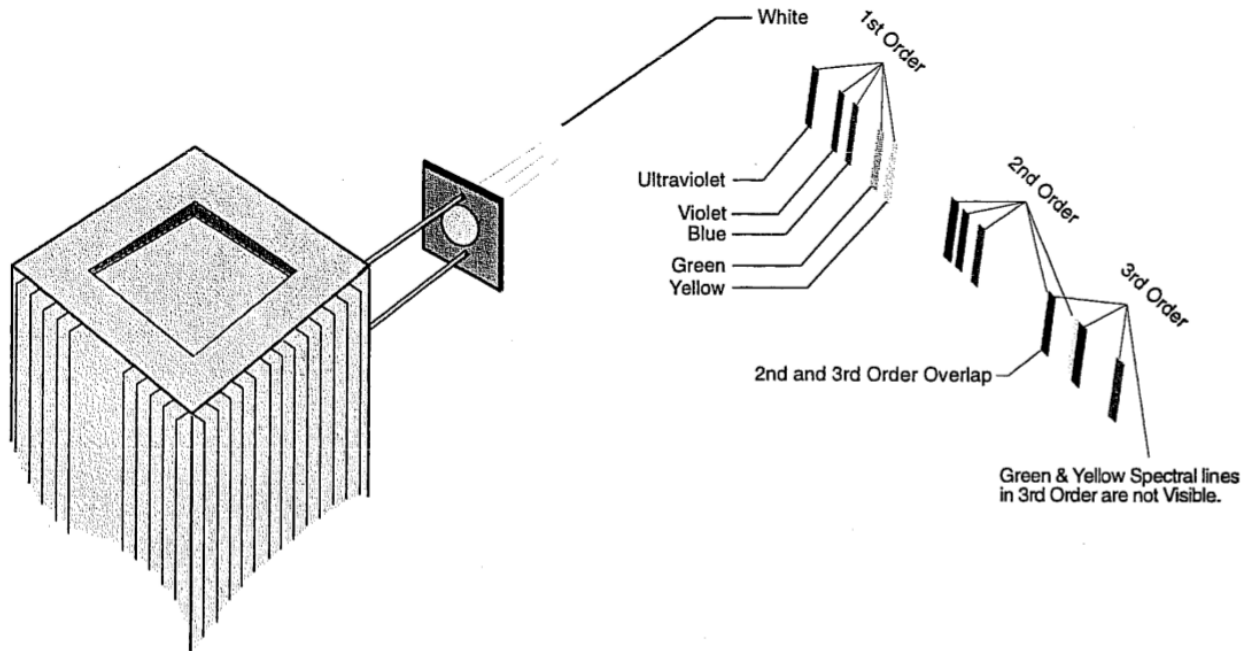
Where do all those electrons come from?

At Jefferson Lab in nearby Newport News, high-energy electrons (GeV range) are aimed at targets to probe the fundamental properties of quarks. But where do all those electrons come from? From the photoelectric effect, of course! A laser beam is aimed at a cathode consisting of a material like copper or GaAs. The frequency of the laser is such that electrons will be emitted from the cathode. In this way, high current electron beams are produced. One benefit of using a laser and the photoelectric effect is that if the laser is “pulsed” in time, the electron beam will be also (this allows synchronized timing in the experiments). Also, the polarization of the laser can be manipulated to allow for the emission of electrons with particular spin. See: www.jlab.org

Procedure

Equipment needed: Pasco photoelectric apparatus, Hg lamp, digital voltmeter. The Pasco apparatus contains a circuit which automatically determines the stopping potential, which you measure off of a voltmeter, so there is no need to adjust the stopping potential yourself or measure the current (lucky you!). Read the brief description of its operation in the appendix.

Set up the equipment as shown in Fig. 5.2. First, place a lens/grating assembly in front of the Mercury lamp, and observe a dispersed spectrum on a sheet of paper, as shown in Fig. 5.3. Identify spectral lines in both the first and the second diffraction orders on both sides, use Fig. 5.4 for the guidance. Keep in mind that the color “assignment” is fairly relative, and make sure you find all lines mentioned in the table in Fig. 5.3. Often the first/second order lines on one side are brighter than on the other - check your apparatus and determine what orders you will be using in your experiment.



All values except wavelength for yellow line are from *Handbook of Chemistry and Physics, 46th ed.* The wavelength of the yellow was determined experimentally using a 600 line/mm grating.

► **NOTE:** The yellow line is actually a doublet with wavelengths of 578 and 580 nm.

Color	Frequency (Hz)	Wavelength (nm)
Yellow	5.18672E+14	578
Green	5.48996E+14	546.074
Blue	6.87858E+14	435.835
Violet	7.40858E+14	404.656
Ultraviolet	8.20264E+14	365.483

Figure 5.3: The mercury lamp visible diffraction spectrum.

After that, install the h/e apparatus and focus the light from the mercury vapor light source onto the slot in the white reflective mask on the h/e apparatus. Tilt the light shield of the apparatus out of the way to reveal the white photodiode mask inside the apparatus. Slide the Lens/Grating assembly forward and back on its support rods until you achieve the sharpest image of the aperture centered on the hole in the photodiode mask. Secure the Lens/Grating by tightening the thumbscrew. Align the system by rotating the h/e apparatus on its support base so that the same color light that falls on the opening of the light screen falls on the window in the photodiode mask, with no overlap of color from other spectral lines. Return the light shield to its closed position.

Check the polarity of the leads from your digital voltmeter (DVM), and connect them to the OUTPUT terminals on the h/e apparatus.

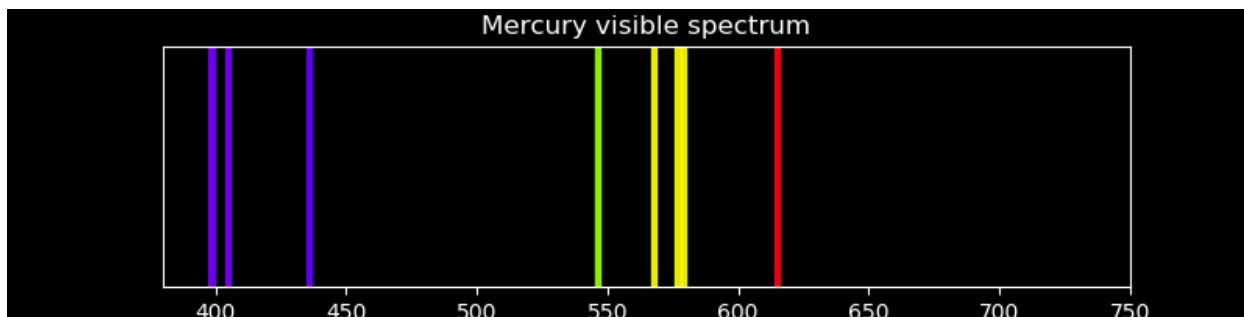


Figure 5.4: The mercury lamp visible spectrum. The spectrum is reconstructed according to NIST data provided at <https://physics.nist.gov/PhysRefData/Handbook/Tables/mercurytable2.htm>

Experimental procedure

Part A: The dependence of the stopping potential on the intensity of light

1. Adjust the h/e apparatus so that one of the blue first order spectral lines falls upon the opening of the mask of the photodiode.
2. Press the instrument discharge button, release it, and observe how much time¹ is required to achieve a stable voltage.
3. It's important to check our data early on to make sure we are not off in crazyland. This procedure is technically known as "sanity checking". According to equation Eq. 5.2 the frequency (or, better, wavelength) of light can be found from the measured voltage with knowledge of the work function ϕ . Unfortunately, we don't know the work function. You'll measure it in the second part of the lab.
 - For now, switch to violet color and measure the stopping potential for this color.
 - compute hc using ΔV_0 between the two measurements and values of light wavelength for these two cases. Does your data agree with the accepted value? For this computation, it might be helpful to realize that eV_0 is the electron charge times the voltage. If you measured $V_0 = 1$ V then $eV_0 = 1$ "electron-volt" or "eV". This is why electron-volt units are useful. Also, $hc = 1240$ nm eV.
4. Go back to the original spectral line.
5. Place the variable transmission filter in front of the white reflective mask (and over the colored filter, if one is used) so that the light passes through the section marked 100%

¹Use the stopwatch feature of your cell phone. You don't need a precise measurement.

and reaches the photodiode. Record the DVM voltage reading and time to recharge after the discharge button has been pressed and released.

6. Do the above measurements for all sections of the variable transmission filter.

Part B: The dependence of the stopping potential on the frequency of light

In this section you'll measure the stopping power for the five different emission lines of Mercury to demonstrate that the stopping power depends on, and is a measurement of, the wavelength (and frequency). Your measurements will consist of the stopping power for each of the five Mercury lines, measured for both first and second orders. This is a total of 10 data points. You'll then analyze the data by fitting stopping power vs frequency to extract Planck's constant and the work function. That procedure is described in more detail in the next section.

There is a particular issue with the yellow and green lines that, if left unaddressed, can corrupt your measurements. The procedure below walks you through the measurements of those two lines, highlighting a few places that systematic problems can creep in if we are not careful. In particular, you will be using yellow and green filters to mitigate one source of systematic bias.

1. Adjust the h/e apparatus so that the 1st order yellow colored band falls upon the opening of the mask of the photodiode. Take a quick measurement with the lights on and record the DVM voltage. Do the same for the green and blue lines.
2. Now, with the lights still on, repeat the yellow and green measurements with the yellow and green filters attached to the h/e detector. What do you see now? In order to explain your observations try holding the filters just in front of the diffraction grating and look at the pattern on the screen.
3. Now turn the lights off. This will require coordinating with other groups and the instructor. Repeat the measurements with and without the filters. Are the two sets of measurements the same? Form a hypothesis for why or why not.
4. Now, you need to decide on the best procedure for collecting the data. Lights on or lights off? Filters or no filters? Discuss amongst your group and with the instructor or TA. You will need to measure both the first and second order lines. This is all five colors, for a total of 10 data points.

Note: When collecting the data in the steps above, be sure to estimate uncertainties on the stopping power. These come in part from the digital voltmeter, but also from the repeatability of your measurements, how well lines are centered on the slit (perhaps off-center could be better?) and so on.

Analysis

Classical vs. Quantum model of light

1. Describe the effect that passing different amounts of the same colored light through the Variable Transmission Filter has on the stopping potential and thus the maximum energy of the photoelectrons, as well as the charging time after pressing the discharge button.
2. Describe the effect that different colors of light had on the stopping potential and thus the maximum energy of the photoelectrons.
3. Defend whether this experiment supports a classical wave or a quantum model of light based on your lab results.

Read the theory of the detector operation in the Appendix, and explain why there is a slight drop in the measured stopping potential as the light intensity is reduced.

The relationship between Energy, Wavelength and Frequency

1. Use the table in Fig. 5.3 and Fig. 5.4 to find the exact frequencies and wavelengths of the spectral lines you used and plot the measured stopping potential values versus light frequency for measurements of the first and second order lines.
2. Fit your data according to $eV_0 = h\nu - \phi$, extracting values for the slope and intercept. *Note, this fitting step takes the measurement uncertainties on the stopping power and propagates them to the slope and intercept.* It's important to do this step with Igor, Matlab, or some other tool which can compute a χ^2 , minimize it with respect to the fit parameters, and then report the parameter uncertainties. From the slope, determine h using $e = 1.6 \cdot 10^{-19}$ C. Find the average value and uncertainty on the average. Does your value agree with the accepted value of $h = 6.62606957(29) \times 10^{-34}$ J-s within uncertainty?
3. From the intercepts, find the average value and uncertainty of the work function ϕ . Look up some values of work functions for typical metals. Is it likely that the detector material is a simple metal?

Appendix: Operation principle of the stopping potential detector

The schematic of the apparatus used to measure the stopping potential is shown in Fig. 5.5. Monochromatic light falls on the cathode plate of a vacuum photodiode tube that has a

low work function ϕ . Photoelectrons ejected from the cathode collect on the anode. The photodiode tube and its associated electronics have a small capacitance which becomes charged by the photoelectric current. When the potential on this capacitance reaches the stopping potential of the photoelectrons, the current decreases to zero, and the anode-to-cathode voltage stabilizes. This final voltage between the anode and cathode is therefore the stopping potential of the photoelectrons.

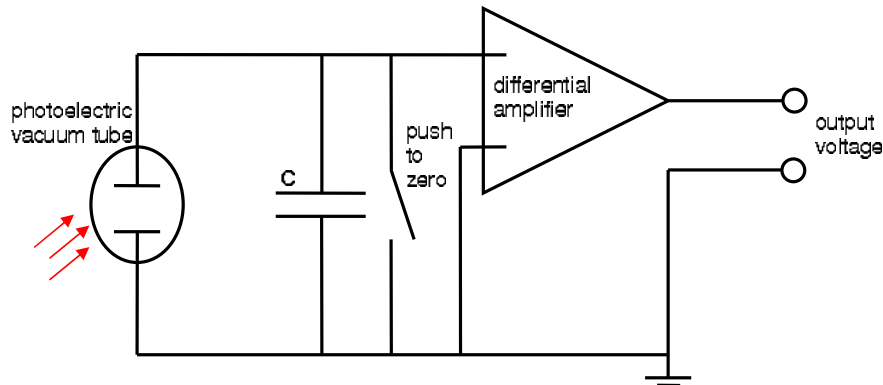


Figure 5.5: The electronic schematic diagram of the h/e apparatus.

To let you measure the stopping potential, the anode is connected to a built-in differential amplifier with an ultrahigh input impedance ($> 10^{13} \Omega$), and the output from this amplifier is connected to the output jacks on the front panel of the apparatus. This high impedance, unity gain ($V_{out}/V_{in} = 1$) amplifier lets you measure the stopping potential with a digital voltmeter.

Due to the ultra high input impedance, once the capacitor has been charged from the photodiode current, it takes a long time to discharge this potential through some leakage. Therefore a shorting switch labeled “PUSH TO Zero” enables the user to quickly bleed off the charge. While the impedance of the unity gain amplifier is very high ($10^{13} \Omega$), it is not infinite and some charge gradually leaks off. This effect can bias the measured stopping power for low intensity light sources.

Sample data tables:

Part A

Color	%Transmission	Stopping Potential	Approx. Charge Time
	100		
	80		
	60		
	40		
	20		

Part B

Color	λ (nm)	ν ($10^{14}Hz$)	Stopping Potential (V)
Yellow			
Green			
Blue			
Violet			
Ultraviolet			

You'll either want different tables for different orders, or perhaps add additional stopping power columns to the one above.

Chapter 6

Single and Double-Slit Interference, One Photon at a Time

Experiment objectives: Study wave-particle duality for photons by measuring interference patterns in the double-slit experiment using a conventional light source (laser) and a single-photon source (strongly attenuated lamp).

History

There is a rich historical background behind the experiment you are about to perform. Isaac Newton first separated white light into its colors, and, in the 1680's, hypothesized that light was composed of 'corpuscles', supposed to possess some properties of particles. This view reigned until the 1800's, when Thomas Young first performed the two-slit experiment now known by his name. Young directed light through a single aperture (slit) followed by a pair of double apertures, and then observed the result on a viewing screen (see Fig. 6.1).

Young discovered that the light appeared as light and dark fringes on the screen, a fact that seemed impossible to explain in terms of corpuscles, but was very naturally explained in terms of waves, much like the interference observed in water and sound waves. In particular, wave theory predicts that the intensity of light $I(x)$ from double slit interference should be distributed as:

$$I(x) = 4I_0 \cos^2 \left(\frac{\pi d x}{\lambda \ell} \right) \left[\frac{\sin \left(\frac{\pi a x}{\lambda \ell} \right)}{\frac{\pi a x}{\lambda \ell}} \right]^2 \quad (6.1)$$

Here, x is the position on the viewing screen, with $x = 0$ corresponding to the center of the interference pattern. The light wavelength is λ , the slits have a width of a , and they are separated by a distance d . The distance between the double slit and the viewing screen is ℓ .

Interference can also occur if there is only one slit. In this case the intensity distribution is described by Fraunhofer diffraction (see Fig. 6.7 and the derivations in the Appendix):

$$I(x) = I_0 \frac{(\sin(\frac{\pi a x}{\lambda \ell}))^2}{(\frac{\pi a x}{\lambda \ell})^2} \quad (6.2)$$

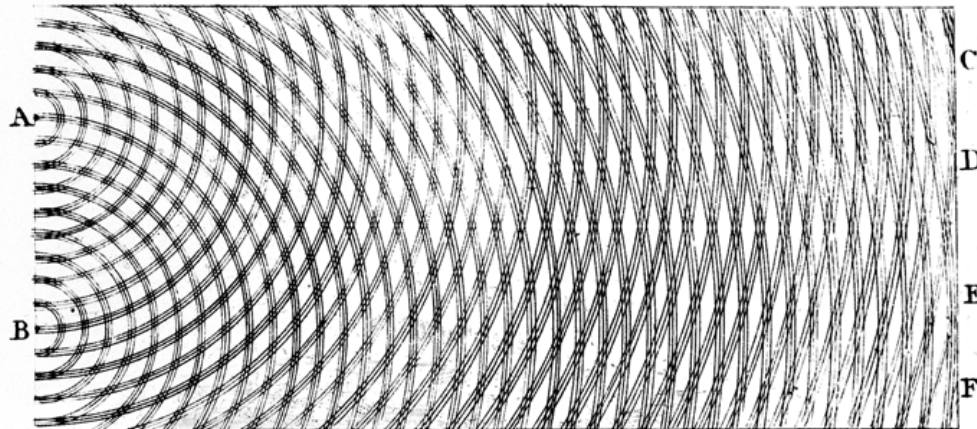
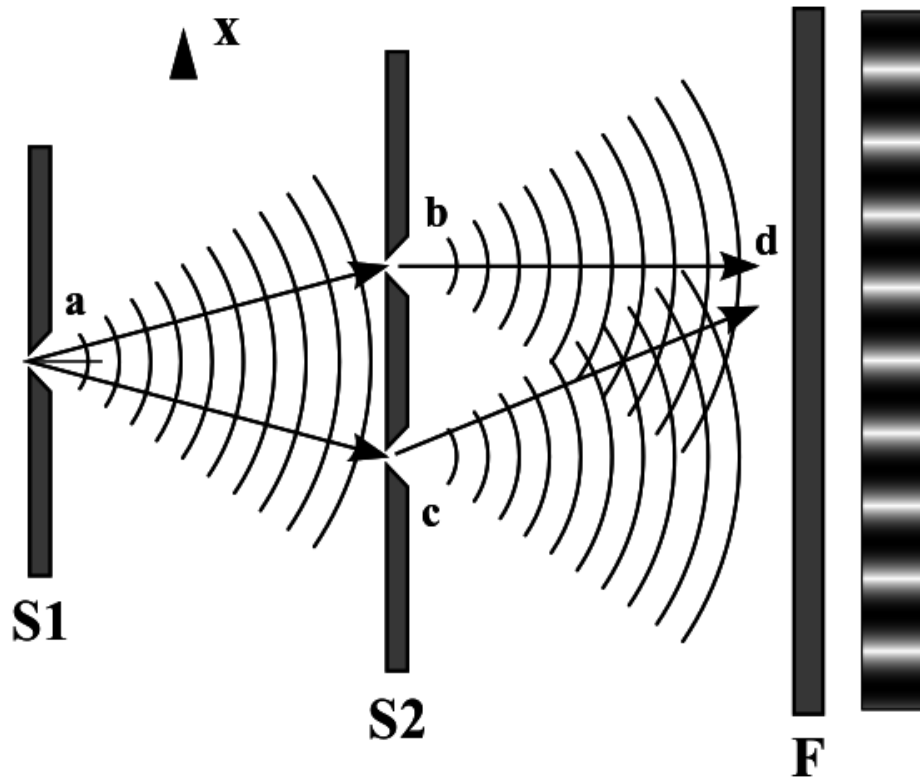


Figure 6.1: Young's double slit experiment. Top: The experimental setup. Bottom: Young's interpretation of the result as interference, analogous to the corresponding phenomenon in water waves. The figure was drawn by Young himself. *Images from Wikimedia.*

Young's experiment not only suggested that such 'light waves' existed but was also able to determine the wavelength of light. You should spend some time thinking about the latter point. How would the interference pattern change if we change λ ? Given a particular

interference pattern how might we determine λ ?

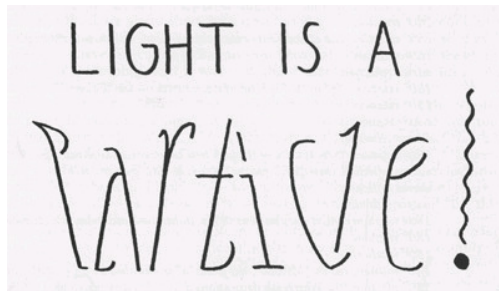


Figure 6.2: “Light is a Particle / Light is a Wave” oscillation ambigram (from *For the Love of Line and Pattern*, p. 30).

Light waves became even more acceptable with dynamical theories of light, such as Fresnel’s and Maxwell’s, in the 19th century, until it seemed that the wave theory of light was incontrovertible. And yet the discovery of the photoelectric effect, and its explanation in terms of light quanta by Einstein, threw the matter into dispute again. The explanations of blackbody radiation, of the photoelectric effect, and of the Compton effect seemed to point to the existence of ‘photons’, quanta of light that possessed definite and indivisible amounts of energy and momentum. These are very satisfactory explanations so far as they go, but they throw into question the destructive-interference explanation of Young’s experiment. Does light have a dual nature, of waves and of particles? And if experiments force us to suppose that it does, how does the light know when to behave according to each of its natures?

It is the purpose of this experimental apparatus to make the phenomenon of light interference as concrete as possible, and to give you the hands-on familiarity that will allow you to confront wave-particle duality in a precise and definite way. When you have finished, you might not fully understand the mechanism of duality – Feynman asserts that nobody really does – but you will certainly have direct experience with the actual phenomena that motivates all this discussion.

Experimental setup

Equipment needed

You will need the Teachspin “Two-slit interference” apparatus, an oscilloscope, a digital multimeter, two $\sim 1\text{m}$ RG58 coaxial cables with BNC connectors, a BNC to banana plug adapter, and a frequency counter (such as the BK Precision 1823A).

Inspecting the apparatus

Important equipment safety information

The double slit apparatus has a very sensitive and somewhat fragile sensor called a photomultiplier tube (PMT) that you will use to observe individual photons. The tube is inside the detector box at the right end of the apparatus as shown in Fig 6.3. The PMT works by being supplied a DC high voltage (500-800V) by circuitry in the same box. The PMT should never be exposed to room light, especially with the HV on. Before taking the top off of the long and narrow interference channel you need to verify that:

- A. the shutter rod is down
- B. the high voltage switch on the detector box is also off
- C. the high voltage dial on the detector box is turned all the way to zero.

As long as the top is off the rod must remain down and the HV must remain off. Finally, when the HV is on the laser/off/bulb switch at the far left end of the interference channel must be on bulb mode, or off. The laser is far too intense for the PMT. Make sure you have read this, understood, and got sign-off from the instructor before proceeding.

To inspect the inside of the apparatus, open the cover by turning the four latches that hold it closed. The details of the experimental apparatus are shown in Fig. 6.3. Take time to locate all the important components of the experiment:

- There are two distinct light sources at the left end: one a red *laser* and the other a green-filtered *light bulb*. A toggle switch on the front panel of the light source control box switches power from one source to the other.
- There are two distinct light detectors at the right-hand end of the apparatus: a *photodiode* and the aforementioned *PMT*. The photodiode is used with the much brighter laser light; it’s mounted on the bottom of the light shutter rod so that it’s in position to use when the shutter is closed (pushed down, See Fig. 6.5). The photomultiplier tube is an extremely sensitive detector able to detect individual photons (with energy of the order of 10^{-19} J); it is used with the much dimmer light-bulb source.

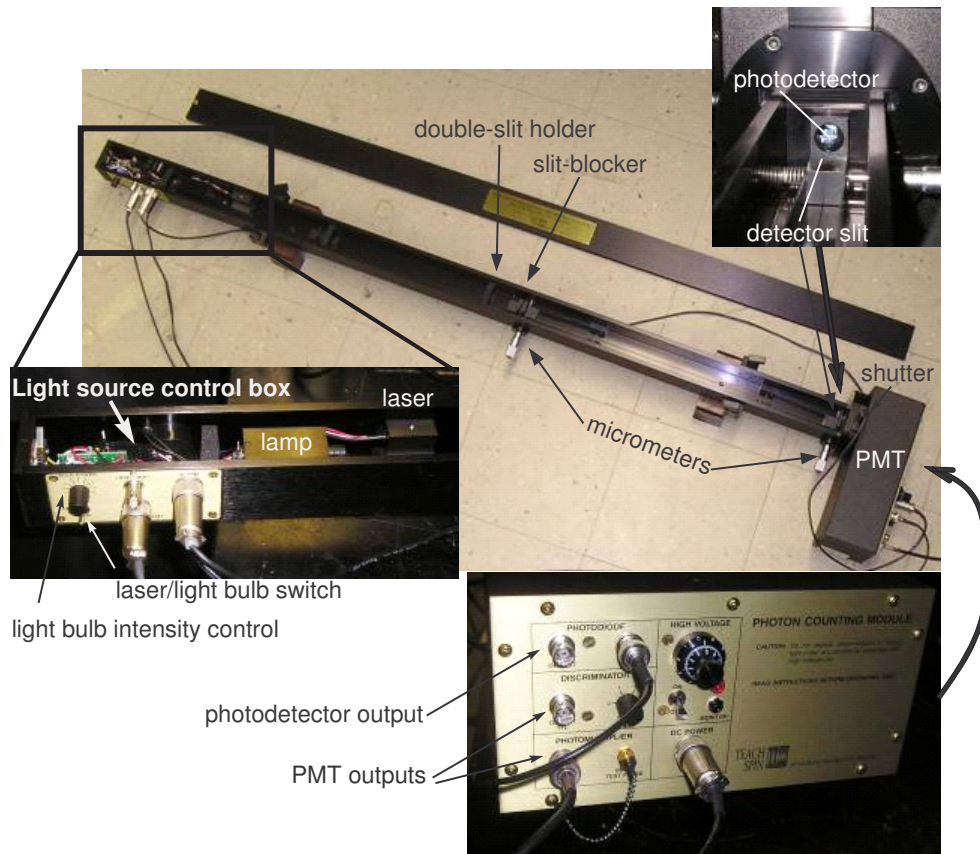


Figure 6.3: The double-slit interference apparatus.

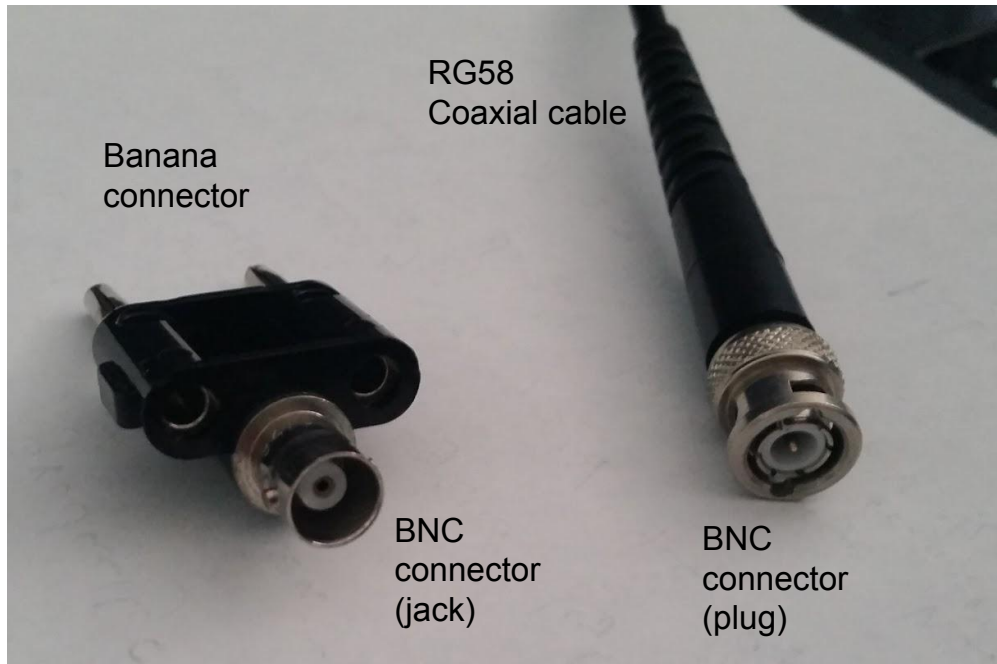


Figure 6.4: RG58 coaxial cable with a BNC connector and a BNC↔banana connector.

- There are various *slit holders* along the length of the long box. From left to right in Fig. 6.3 these are: a single slit to establish the point source of light diagrammed in Fig. 6.1; a double slit to create the interference pattern; a slit blocker just downstream of the double slit; and finally the detector slit, which sits in front of the photodetectors, allowing only a portion of the interference pattern to reach them. The distance ℓ between the double slit and detector slit is important. Be sure to measure it!
- There are two *micrometer drives*, which allow you to make mechanical adjustments to the two-slit apparatus. The right-hand dial controls the position of the detector slit. The dial near the center of the apparatus controls the position of the slit blocker. **Make sure you figure out how to read these micrometer dials!** One unit on the dial corresponds to $10 \mu\text{m}$ ¹ so turning from 100 to 200 on the dial moves the detector slit by $100 \times 10 \mu\text{m} = 1000 \mu\text{m} = 1 \text{ mm}$.

Experimental procedure

The experiment consists of three steps:

¹Yes, this is confusing! We have an unfortunate language overlap, where “micrometer” refers to both the device and the unit of distance – μm . There is not much we can do about it. If it helps, you can refer to $1 \mu\text{m}$ as a “micron”.

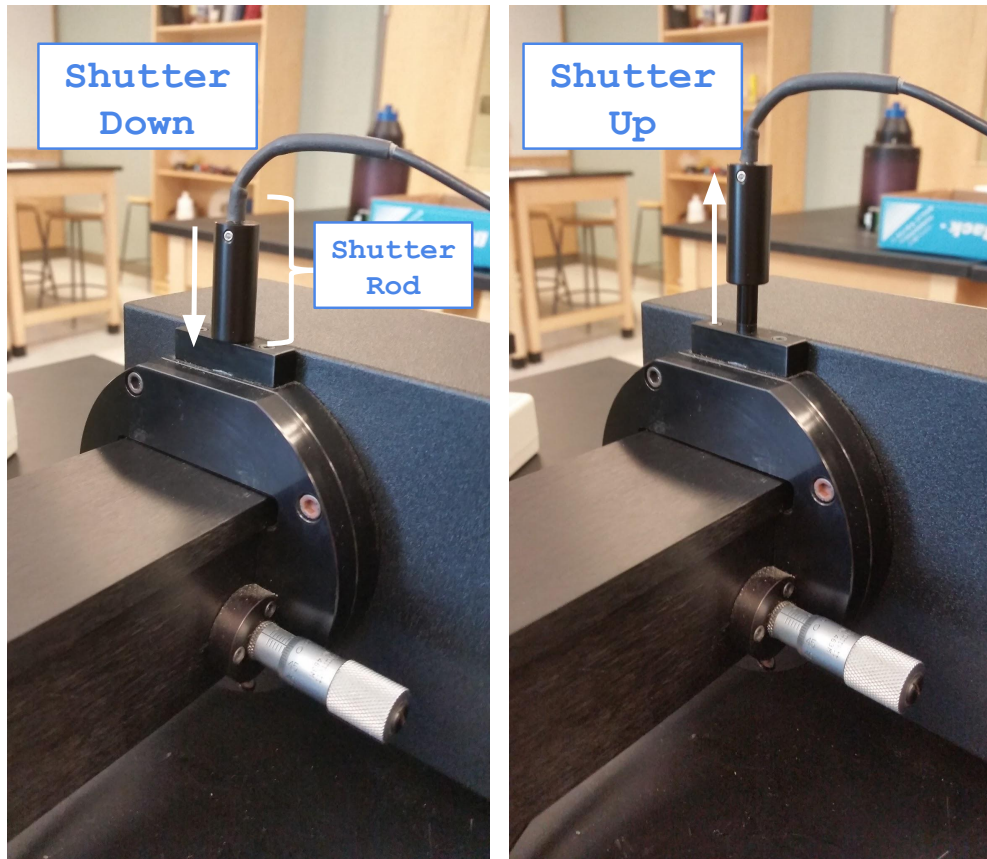


Figure 6.5: The apparatus's shutter. When down it protects the PMT and exposes the photodiode to light in the interference channel. When up, the PMT is exposed to the light. The shutter must be kept down when the box is open.

Observing interference and finding dial settings You will first identify single and double slit interference patterns directly with the top off of the apparatus. You'll do this by looking at interference fringes on a bit of paper you'll insert into the long box, to the right of the double slit in Fig. 6.3. You will record the slit blocker position corresponding to single and double slit interference in your lab book so you can switch between the two when the top is on.

Measuring interference with an intense source Using the photodiode and laser you will measure the intensity of the single- and double-slit interference patterns as a function of x , the position transverse to the laser beam. Your analysis will consist of fitting the data with equations 6.2 and 6.1. A good fit demonstrates that the wave theory of light is correct and allows you to determine the slit width and slit separation. This is a recreation of Young's original experiment.

Measuring interference with a weak source Then, using a very weak light source, you will record the intensity of the single- and double-slit interference pattern by counting the number of photons as a function of x . This measurement will introduce you to the technology we use to observe single-photons detection technology and will also demonstrate that interference occurs even if we only have one photon going through the apparatus at a time. This is the very essence of wave/particle duality!

Observing interference and finding dial settings

Important personal safety information

The apparatus has a 5-mW diode laser with an output wavelength of 670 ± 5 nm. It could potentially harm your eyes if you are not careful. Don't observe the beam directly (i.e., by staring into it) and be careful of reflections off of reflective surfaces.

For this mode of operation, you will be working with the cover of the apparatus open.

1. Switch on the laser.
2. Make a viewing screen by cutting a bit of white paper to the dimensions of the interference channel. Insert your screen just to the right of the double slit and slit blocker.
3. Adjust the position and angle of laser so that it impinges on the single slit, then the double slit, and travels all the way to the detector slit. You may need to make sure, by visual inspection, that the slit blocker is not blocking one or both of the double slits.
4. Move your screen along the beam path toward the detector slit to see the interference pattern forming. By the time your viewing card reaches the right-hand end of the apparatus, you'll see that the two overlapping ribbons of light combine to form a pattern of illumination displaying the celebrated "fringes" named after Thomas Young.

You are going to take data with the box closed and will want to be able to move the blocking slit to go from double slit mode to single slit mode without opening it. We'll also want to be able to completely block the light from both slits in order to measure the background signal from the photodiode and the PMT. Adjust the position of the blocking slit using the micrometer to find the following situations:

1. both slits are open
2. light emerges only from a single slit
3. the light from both slits is blocked.

Record the dial setting for each. Make sure that you are fully in the single and double-slit conditions as it is possible to be halfway, with the top of the pattern different than the bottom due. It is essential that you are confident enough in your ability to read, and to set, these three positions and that you are able to do so even when the box cover is closed. In your lab book describe and sketch what you see on the viewing card at the far-right end of the apparatus for each of the three settings.

Measuring interference with an intense source

In this part of the experiment you will use a photodiode to measure the intensity distribution of the interference pattern by varying the position of the detector slit. You will do this using the red laser with the shutter down.

Orienting yourself with the laser and photodiode

The apparatus should be open for this part.

The photodiode When down, the shutter positions the $\sim 1 \times 1 \text{ cm}^2$ photodiode in the path of the interference pattern. The output voltage from the photodiode is proportional to the total amount of light hitting it.

Connecting to the photodiode You need to connect a multimeter in DCV mode to the photodiode output on the detector box. To do this you will need a coaxial cable with BNC connectors on each end and a BNC \leftrightarrow banana plug converter. These items are shown in Fig. 6.4.

The detector slit The detector slit masks most of the photodiode, only allowing light through in a narrow window range along the horizontal (x) axis. By adjusting the micrometer screw of the detector slit, you can move the slit over the interference pattern, eventually mapping out its intensity distribution quantitatively.

Getting in position For now, ensure that the detector slit is located somewhere near the edge middle of the two-slit interference pattern, and have the slit-blocker set to give you double-slit interference.

Quick scan of the pattern Now, turn the micrometer dial to move the detector slit. Observe how the voltage output of the photodiode changes as you translate the slit across light and dark fringes. To take good data you will do the same thing with the box closed, stopping at even intervals to record the photodiode output.

Taking photodiode data

Now, close the apparatus. Be gentle when putting the top on! **Note: you will have to follow the procedure below twice, once for double slit, and then again for single slit.**

Record the pedestal Turn off the laser and observe that the photodiode produces an output even without light. This is the “zero offset” or “pedestal” reading. Record it in your lab book, along with an uncertainty. You will need this for your analysis.

Take a datapoint Now you are ready to take data. Move the detector slit to one end of the interference pattern. Collect one datapoint by recording the detector slit position (by reading the micrometer) and the voltage from the photodiode. You will be taking a lot of datapoints and it is wise to plot them as you collect them. Therefore you should record the readings in a spreadsheet and do not need to put them in your lab book.

Uncertainty – Part 1 Estimate your uncertainty from the micrometer and multi-meter precision and stability. Record your uncertainty and any comments you have about it in your lab book. We will come back to this later.

Collect more points Move the detector slit by some small interval, collecting additional datapoints. You want to map out the interference pattern with sufficient details that you can easily resolve the light and dark fringes. For double-slit interference, taking data every $100\ \mu\text{m}$ should be OK. The single slit pattern is less detailed and you need only take data every $200\ \mu\text{m}$.

Plot the data Plot your data as you take it to verify that you are seeing an interference pattern. Remake the plot every 20 steps or so. (This is easiest to do as a scatterplot in a spreadsheet).

The goal The goal is to take about 70 points in double slit mode and 35 in single slit mode. This should allow you to see multiple maxima and minima in double slit, and the central maximum in single slit mode, along with (hopefully) a secondary maximum on at least one side. **Note: before moving from double to single-slit mode you should estimate your uncertainties by following the procedure described below.**

Uncertainties

Are your uncertainties really just due to the measurement precision of the micrometer dial and DVM? It is wise to test this by repeating some measurements in both single and double slit modes.. To do so:

Back to the beginning Move the detector slit all the way back to the starting point. Go a little past the point as you do so, then come back to it going in the direction that you took all the data. Record the old and new reading of that point in your lab book.

Repeat points Now, translate the detector slit and remeasure 8 representative points in your dataset. Recording the old and new values.

And again Do the same thing again, so that you have three repeated readings at each of the 8 points.

Uncertainty – Part 2 Use the standard deviation in each of the 8 measurements to estimate your uncertainty due to repeatability. Is it larger than that due to the precision? Talk this over with the instructor and how to generalize what you found to all the points.

Measuring interference with a weak source

Orienting yourself with the bulb and PMT

Now you need to change the apparatus to use the light bulb and PMT. Keep the shutter down for now.

Open up Open the cover and slide the laser source to the side (do not remove the laser from the stand).

Bulb settings Now set the 3-position toggle switch to the “bulb” position and dial its intensity to 8. As long as you haven’t touched anything else the apparatus is still aligned and light from the bulb will form interference patterns that you can measure with the PMT.

Double slit mode Use the dial to move the slit blocker into double slit mode.

Close it up Now close and latch the cover.

High voltage and PMT pulses Have an instructor show you how to use the DVM to measure the voltage supplied to the PMT and how to observe the PMT output on an oscilloscope. With the instructor, gradually turn on the HV to get up to a setting of around 700 V. You should see pulses on the oscilloscope screen.

Counting Pulses The instructor will also show you how to look at the discriminator output on the scope and how to count pulses with a frequency counter. The counter should be set to count for 1 second. *Pause to document the setup procedure in your lab book. Sketch connections, record the HV rate, etc. You should document this well enough to repeat it in a year.*

Unstable Counter! You’ve noticed that the counting rate isn’t stable. This is because you are observing a random process. So, how do we assign an uncertainty? Begin by recording 10 different readings N_i from the frequency counter in your lab notebook.

Counting uncertainty Compute the mean of the readings and their standard deviation and record those too. Compare the standard deviation to $\sqrt{N_i}$ for a couple of the points. Are the values similar? It turns out that for a random counting process, if you count N_i counts then the associated uncertainty is $\delta N_i = \sqrt{N_i}$. This is what you will use in your analysis².

The dark rate By the way, you have been observing counts even without light shining on the PMT, since the shutter is down. This is the so-called “dark rate” of the PMT. It is similar to the “zero offset” or “pedestal” that you saw for the photodiode. You should record it in your lab book. Ask your instructor what causes it and to explain how the PMT works. **Then, finally, go ahead and raise the shutter before moving on to the next section.**

Taking PMT data

The data taking must be done with the box closed and high voltage on. If for any reason you have to open the box you need to make sure the high voltage is off and the shutter is down.

You are now going to measure single and double slit interference just like you did with the laser and photodiode. In particular you should take about the same number of points with the same step size. Now, however you will be using the PMT counting rate as a measure of the light intensity, rather than the photodiode output, and you are observing a very dim light source. In fact the source is so dim that you can convince yourself with a “back of the envelope” computation that there is usually only one photon in the region downstream of the double slits at any given time. That calculation is described in the data analysis section and it should appear in your report. The ramification is that the interference pattern must be due to photons interfering with themselves! This experimentally establishes wave-particle duality.

You don’t need to repeat datapoints to establish an uncertainty. The dominant contribution is just due to the statistical noise in the counting rate ($\delta N_i = \sqrt{N_i}$). In Matlab, if you have your N_i in an array `N` then the δN_i can just be computed as `dN=sqrt(N)`, or similar in Python `dN=np.sqrt(N)`. You can then plot the data and uncertainties with the `errorbar` function in either Matlab or Python

You should be careful not to wait to record “round numbers” or other aesthetically pleasing readings from the counter. The best way of doing this is to establish a routine. For example, turn the micrometer dial and then look at the counter. Record the second reading that you see since the first might be corrupted by the move. You need only record the rate as an integer (i.e., you can neglect any numbers after the decimal place).

²So, if you count $N = 9$ counts in some period of time the uncertainty is apparently $\delta N = \sqrt{9} = 3$. But, what if you are sure it was 9 counts, not 8, not 10, etc? You are really positive you didn’t miss any. What then? You should discuss this with your instructor.

Qualitative observations

In both the one and two slit cases make some qualitative observations. Do the patterns appear to agree with Equations. [6.1-6.2](#)? What would happen if we increased or decreased d , a , λ and ℓ . Record your thoughts in your labbook, and use these insights in setting the initial parameters when fitting your data.

Your instructor may have told you to do a “brief writeup” of this experiment. If so, please see the instructions starting on the next page (page [62](#)).

Instructions for a brief writeup

In this lab you have collected 4 independent datasets (red laser and green bulb with single or double slit) and the analysis will require fits to these data. Therefore, a formal lab report with abstract, introduction, conclusion, etc. is **not** required. For this brief writeup: you only need to include the tables and figures requested below, as well as answering the questions listed in this section. **Please restate each one so it's easy for us to follow along.** Refer to the relevant section earlier in the manual or later in Appendices for additional discussion of the analysis procedure. This is just a bullet point summary of the deliverables.

Data Tables and Uncertainties

- Tabulate your data for each of the four interference patterns and include them (with labels and captions).
- Describe how uncertainties on position, photodiode voltage, and photon counting rate were determined, and report your estimated uncertainty values.

Data Analysis and Fitting

For each of the four interference patterns, address the following points:

1. Plot the measurements for the photodiode voltage or photon counting rate (y-axis) vs position (x-axis) with uncertainties and fit to Eqs. (6.1) and (6.2) for Double and Single Slit, respectively. Include a figure with the plot of your data and the resulting fit in your report (with labels and captions).
2. Is it a good fit: in terms of the observed χ^2 and number of degrees of freedom?
3. What are the best fit parameters and uncertainties?
4. Is B consistent with what you expect from the background (*i.e.* pedestal or dark rate) measurements you collected during lab? Comment on any discrepancies.

Finally, quantitatively compare your measurements of a and d between the four datasets. Are they consistent?

Did we really see single photon interference?

To conclude that photons interfere with themselves, rather than with other nearby photons, we have to be sure that the rate of photons is low enough that only one of them is in the vicinity of the double slit at any given time. You need to work through the steps in the Appendix “Computing the photon rate” below, writing numbers in your lab book. In this brief writeup summarize the results in a paragraph or two, including the numbers you calculate at each stage. Write a nice bit of narrative text, not just a bunch of bullets with numbers.

Appendix: Computing the photon rate

This is a so-called back of the envelope calculation.

1. Begin by computing the amount of electrical power that is converted to light by the light bulb. Assume we operate the bulb at 6 VDC and it draws 0.2 A of current. From this you can get the power drawn by the bulb.
2. Most of the power goes into heating the bulb. Assume 5% of it is converted to light evenly distributed from 500-1500 nm.
3. The bulb has a green filter that blocks the light outside the range 541-551 nm. From this you can get the amount of power radiated as green light.
4. You can convert the power (energy/time) into a rate of photons using the fact that $E = \frac{hc}{\lambda}$. From this you can find the rate of green photons emitted from the bulb+filter.
5. The photons emitted by the bulb get diluted by the apertures in the interference channel. The first is a single slit about $R = 10$ cm from the bulb with a width of 0.1 mm and a height of 1 cm. Assuming that the bulb radiates evenly onto a spherical surface of radius R , compute the fraction of the light that would pass through the slit.
6. After the single slit the beam is diffracted into a single slit interference pattern and impinges on the double slit. If you had the apparatus in front of you, you could estimate the area that the interference pattern covers. A good guess is 1 cm^2 . Assume this and that the double slits have an area twice that of the single slit. You can compute the rate of photons passing through the double slits.
7. We know that a photon travels at a speed $c = 3 \times 10^8$ m/sec. Divide that by the number of photons per second that pass through the double slits. This gives you the typical space between photons in meters/photon.
8. Invert the number from the previous bullet point and multiply by the 0.5 m between the double slit and detector slit to determine how many photons are in the setup on average. Is it a number much less than one? If so, there is a good reason to believe that the photons must be interfering with themselves to form the light and dark fringes you see.

Appendix: Fitting the interference data

You need to fit your data with Eqs. (6.1) and (6.2). A detailed example is provided in “Help on the double slit analysis” in the Matlab folder on Blackboard or “Fitting Help: Single and Double Slit” in the Python folder on Blackboard. *Please utilize this, it will save you a lot of time!*

A few fitting tips and notes:

- Make sure that units of all your measured values are self-consistent - the program will go crazy if you mix meters and micrometers together!
- Try to plot your function using estimated values of the parameters before doing a fit with it. This catches many silly errors.
- The fitting function has a term to account for the overall normalization and also for the fact that the maximum is not set at $x = 0$. In other words, we substitute $x \rightarrow x - x_0$ when coding the fit function. Estimate both of these (for the normalization, look at what happens when $x - x_0 = 0$) but include them as free parameters in the fit.
- The fitting function has a term to account for the “pedestal” (for the photodiode) or “dark rate” (for the PMT) you observed when the lights were off. The estimated value should be changed to agree with what you observed.
- Just a note: fitting functions should include the minimal number of parameters. If I have parameters a and b but they always appear as ab in my function, then I am much better off including a term $c = ab$ when doing the fit. Otherwise the fit is underconstrained. Adjusting a up has the same effect as adjusting b down so it’s impossible to converge on unique values for a and b . Fitting algorithms really dislike this situation.
- If the program has trouble fitting all the parameters, first hold the values of the parameters you know fairly well (such as the light wavelength, maximum peak intensity, background, etc.). Once you determine the approximate values for all other parameters, you can release the fixed ones, and let the program adjust everything to make the fit better.

Appendix: Two-Slit interference with atoms³

According to quantum mechanics, the wave-particle duality must be applied not only to light, but to any “real” particles as well. That means that under the right circumstance, atoms should behave as waves with wavelength $\lambda_{\text{atom}} = h/\sqrt{2mE} = h/p$ (often called de Broglie wavelength), where h is Planck’s constant, m is the mass of the particle, and E and p are respectively the kinetic energy and the momentum of the particle. In general, wave effects with “massive” particles are much harder to observe compare to massless photons, since their wavelengths are much shorter. Nevertheless, it is possible, especially now when scientists have mastered the tools to produce ultra-cold atomic samples at nanoKelvin temperatures. As the energy of a cooled atom decreases, its de Broglie wavelength becomes larger, and the atom behaves more and more like waves. For example, in several experiments, researchers used a Bose-Einstein condensate (BEC) – the atomic equivalent of a laser – to demonstrate the atomic equivalent of the Young’s double-slit experiment. As shown in Fig. 6.6(a), an original BEC sits in single-well trapping potential, which is slowly deformed into a double-well trapping potential thus producing two phase-coherent atom wave sources. When the trapping potential is turned off, the two BECs expand and interfere where they overlap, just as in the original Young’s double-slit experiment.

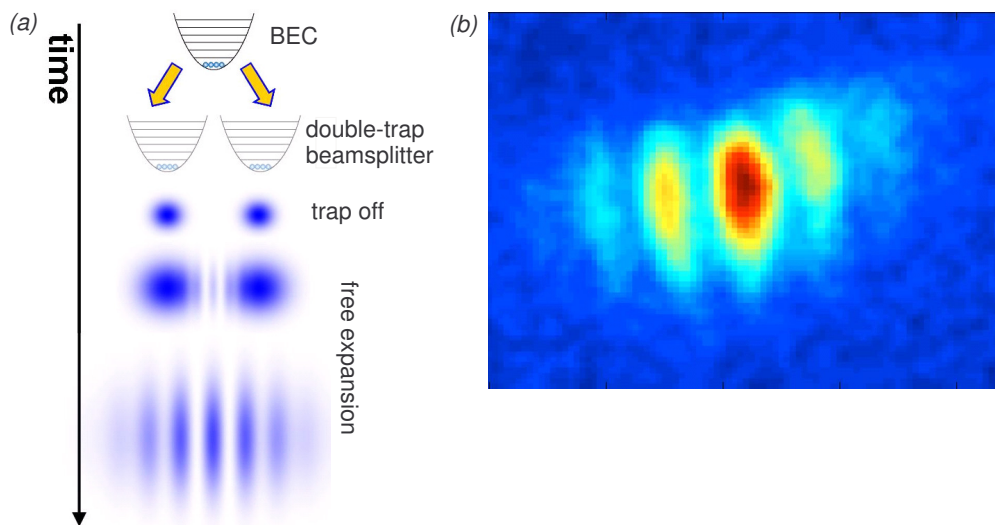


Figure 6.6: Atom interferometry version of Young’s double-slit experiment: (a) schematic and (b) experimentally measured interference pattern in an ⁸⁷Rb Bose-Einstein condensate.

Fig. 6.6(b) shows the resulting interference pattern for a ⁸⁷Rb BEC. Atom interferometry is an area of active research, since atoms hold promise to significantly improve interferometric resolution due their much shorter de Broglie wavelength compared to optical photons. In fact, the present most accurate measurements of accelerations, rotations, and gravity gradients are based on atomic interference.

³Special thanks to Prof. Seth Aubin for providing the materials for this section

Appendix: Fraunhofer Diffraction at a Single Slit and Two-Slit interference

Diffraction at a Single Slit We will use a *Fraunhofer* diffraction model to calculate the intensity distribution resulting from light passing a single-slit of width a , as shown in Fig. 6.7(a). We will assume that the screen is far away from the slit, so that the light beams passed through different parts of the slit are nearly parallel. To calculate the total intensity on the

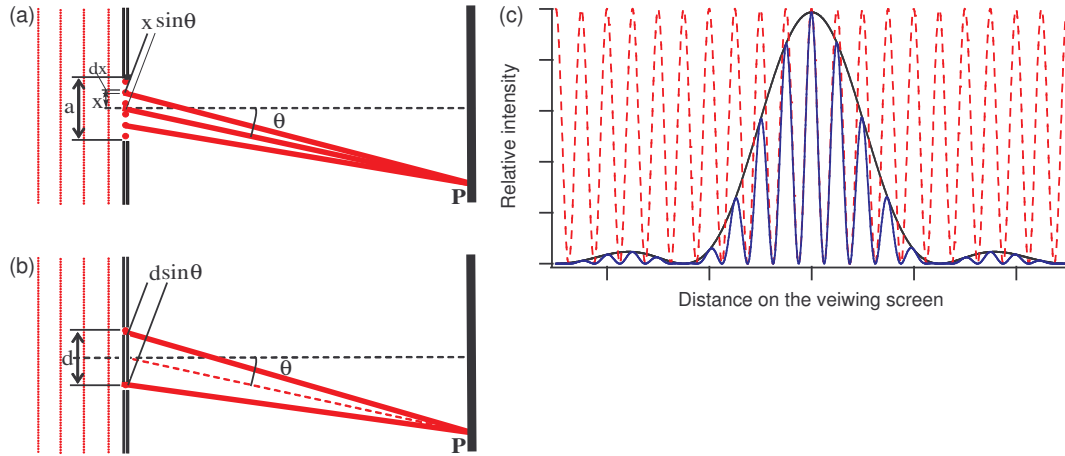


Figure 6.7: (a) Single slit diffraction pattern formation. (b) Two-slit interference pattern formation. (c) Examples of the intensity distributions on a viewing screen after passing one slit (black), two infinitely small slits (red), two slits of finite width (blue).

screen we need to sum the contributions from different parts of the slit, taking into account phase difference acquired by light waves that traveled different distance to the screen. If this phase difference is large enough we will see an interference pattern. Let's break the total height of the slit by very large number of point-like radiators with length dx each and positioned at the height x above the center of the slit (see Fig. 6.7(a)). Since it is more convenient to work with complex numbers, we will assume that the original incident wave is a real part of $E(z, t) = E_0 e^{ikz - i2\pi\nu t}$, where $k = 2\pi/\lambda$ is the wave number. Then the amplitude of each point radiator on a slit is [a real part of] $dE(z, t) = E_0 e^{ikz - i2\pi\nu t} dx$. A beam emitted by a radiator at the height x above the center of the slit must travel an extra distance $x \sin \theta$ to reach the plane of the screen, acquiring an additional phase factor. Then we may write a contribution at the point P from a point radiator dx as the real part of:

$$dE_P(z, t, x) = E_0 e^{ikz - i2\pi\nu t} e^{ikx \sin \theta} dx. \quad (6.3)$$

To find the overall amplitude at that point we need to add up the contributions from all point sources along the slit:

$$E_P = \int_{-a/2}^{a/2} dE(z, t) = E_0 e^{ikz - i2\pi\nu t} \int_{-a/2}^{a/2} e^{ikx \sin \theta} dx = A_P \times E_0 e^{ikz - i2\pi\nu t}. \quad (6.4)$$

Here A_P is the relative amplitude of the electromagnetic field at the point P :

$$A_P = \frac{1}{ik \sin \theta} \cdot (e^{ik\frac{a}{2} \sin \theta} - e^{-ik\frac{a}{2} \sin \theta}) \propto \frac{\sin(\frac{\pi D}{\lambda} \sin \theta)}{\frac{\pi D}{\lambda} \sin \theta} \quad (6.5)$$

The intensity is proportional to the square of the amplitude and thus

$$I_P \propto \frac{(\sin(\frac{\pi a}{\lambda} \sin \theta))^2}{(\frac{\pi a}{\lambda} \sin \theta)^2} \quad (6.6)$$

The minima of the intensity (“dark fringes”) occur at the zeros of the argument of the sin function: $\frac{\pi D}{\lambda} \sin \theta = m\pi$, while the maxima (“bright fringes”) almost exactly match $\frac{\pi D}{\lambda} \sin \theta = (m + \frac{1}{2})\pi$ for $m = 0, \pm 1, \pm 2, \dots$.

Let us now consider the case of the interference pattern from two identical slits separated by the distance d , as shown in Fig. 6.7(b). We will assume that the size of the slits is much smaller than the distance between them, so that the effect of Fraunhofer diffraction on each individual slit is negligible. Then going through the similar steps the resulting intensity distribution on the screen is given by familiar Young formula:

$$I(\theta) = |E_0 e^{ikd/2 \sin \theta} + E_0 e^{-ikd/2 \sin \theta}|^2 = 4I_0 \cos^2 \left(\frac{\pi d}{\lambda} \sin \theta \right), \quad (6.7)$$

where $k = 2\pi/\lambda$, $I_0 = |E_0|^2$, and the angle θ is measured with respect to the normal to the plane containing the slits.

If we now include the Fraunhofer diffraction on each slit as we did before, we arrive to the total intensity distribution for two-slit interference pattern:

$$I(\theta) \propto \cos^2 \left(\frac{\pi d}{\lambda} \sin \theta \right) \left[\frac{\sin(\frac{\pi a}{\lambda} \sin \theta)}{\frac{\pi a}{\lambda} \sin \theta} \right]^2. \quad (6.8)$$

The examples of the light intensity distributions for all three situations are shown in Fig. 6.7(c). Note that the intensity distributions derived here are functions of the angle θ between the normal to the plane containing the slits and the direction to the point on the screen. To connect these equations to Eqs. (6.2) and (6.1) we assume that $\sin \theta \simeq \tan \theta = x/\ell$ where x is the distance to the point P on the screen, and ℓ is the distance from the two slit plane to the screen.

Chapter 7

Faraday Rotation

Experiment objectives: Observe the *Faraday Effect*, the rotation of a light wave's polarization vector in a material with a magnetic field directed along the wave's direction. Determine the relationship between the magnetic field and the rotation by measuring the so-called *Verdet constant* of the material. Become acquainted with some new tools: an oscilloscope, a function generator and an amplifier, and a new technique: phase-locking.

Introduction

The term polarization refers to the direction of the electrical field in a light wave. Generally, light is not polarized when created (e.g., by atomic deexcitations) but can be made so by passing it through a medium that transmits electric fields oriented in one direction and absorbs all others. Imagine we create a beam of light traveling in the z direction. We then polarize it in the x direction ($\mathbf{E} = \hat{\mathbf{x}}E_0 \cos(kz - \omega t)$) by passing it through a polarizer and then pass it through a second polarizer, with a transmission axis oriented at an angle θ with respect to the x axis. If we detect the light beam after the second polarizer, the intensity is

$$I = I_0 \cos^2 \theta \quad (7.1)$$

In 1845, Michael Faraday discovered that he could create polarized light and then rotate the direction of the polarization using a magnetic field. The field, created in our laboratory by a solenoid, points in the direction of the light beam. The Faraday effect is caused by a combination of factors:

1. We can describe the polarization vector in terms of right- and left-handed components. In this description, the electric field of the right-handed component rotates clockwise around the z axis as the wave travels. The left-handed component rotates counter-clockwise.
2. At the particle level, the right and left hand components correspond to photons with angular momentum $\hbar = h/2\pi$ directed parallel (right) or anti-parallel (left) to the z axis.

3. When an atom is placed in a magnetic field, single atomic energy levels may divide into multiple levels, each with slightly different energies. This is called the *Zeeman effect* and qualitatively it occurs because the moving atomic electrons may be thought of as a current \mathbf{J} which interacts with the external magnetic field \mathbf{B} .
4. The atomic current \mathbf{J} contains components with different angular momenta, and those components interact differently with right and left handed photons.
5. The macroscopic effect is that a material in a magnetic field can have a different index of refraction n for left and right handed light. Since the phase velocity of light c' depends on n as $c' = c/n$, the relative phase between the left and right handed components changes as the light travels a distance L through the material.
6. That phase change causes a rotation in the polarization vector.

The polarization vector rotates in proportion to the length of the material, the magnitude of the magnetic field, and a material dependent constant C_V called the *Verdet constant*:

$$\phi = C_V B L \quad (7.2)$$

Typically C_V depends on the wavelength of the light and has a value of a few rad/T · m. For the solenoid we'll use in this lab, the field at the center is $B = 11.1 \text{ mT/A}$, and our material, a special sort of glass, is 10 cm long. For a current of 0.1 A, we expect a rotation of a few $\times 10^{-4}$ radians. This is a pretty small angle and it will require a special technique to detect.

We are going to take the polarized laser light and direct it through the glass rod, which is inserted into the center of the solenoid. The beam will then pass through a second polarizer with the transmission axis at an angle θ with respect to the initial polarization of the laser. The intensity of the transmitted light will then depend on the sum of the angle θ and the additional rotation ϕ caused by the magnetic field:

$$I = I_0 \cos^2(\theta + \phi) \quad (7.3)$$

$$= I_0 \frac{1 + \cos(2\theta + 2\phi)}{2} \quad (7.4)$$

$$= \frac{I_0}{2} + \frac{I_0}{2} [\cos 2\theta \cos 2\phi - \sin 2\theta \sin 2\phi] \quad (7.5)$$

$$\approx I_0 \left[\frac{1 + \cos 2\theta}{2} - \phi \sin 2\theta \right] \quad (\phi^2 \ll 1) \quad (7.6)$$

$$= I_0 \left[\frac{1 + \cos 2\theta}{2} - C_V L B \sin 2\theta \right] \quad (7.7)$$

We'll see the Faraday effect by observing changes in the intensity of light as we vary the magnetic field. But, there is a problem. The term involving ϕ is much smaller than the other terms and it can easily be buried in noise from a photo-detector. We'll use an experimental technique, called phase-locking, to get around the problem. We will then relate the changes

in the intensity to changes in the angle ϕ using a special calibration dataset. Since we know the corresponding changes in B we can use Eq. 7.2 to extract C_V .

The phase-locking technique works in the following way. We'll vary the magnetic field periodically with time as a sine wave, and then observe the signal from the photodiode as a function of time. The signal will look like a large constant with a small sine wave "wobble" on it, along with some random noise with a similar magnitude to the wobble. However, we can subtract off the non time-varying portion of the signal, using a high pass filter. Then, since we know the period and phase of the magnetic field, we can time our observations to be exactly in sync with the magnetic field, and use this to average out the noise. What's left over is the wobble in I , essentially the $C_V L B \sin 2\theta$ term. Knowing B, L and θ , we can determine C_V .

Experimental Setup

The setup consists of:

- A solenoid to produce the magnetic field.
- A glass rod inside the solenoid. You'll measure C_V of this material.
- A polarized HeNe laser, to shine light through the field.
- A second polarizer, the "analyzer", to define the angle θ .
- A photodiode to detect the laser light after the coil and the analyzer.
- A function generator and amplifier to supply current to the solenoid.
- A digital multimeter (DMM) to measure the solenoid current.
- A digital oscilloscope to read out the photodiode.

The experimental setup is shown in Fig. 7.1.

Experimental Procedure

Preliminaries

Laser & photodiode setup The amplifier includes the laser power supply on the back. Plug the laser in, **being careful to match colors between the cable and the power supply's connectors**. Align the laser so it travels down the center of the solenoid, through the glass rod, and into the center of the photodiode. Set the photodiode load resistor to $1\text{ k}\Omega$. Plug the photodiode into a DMM and measure DC voltage.

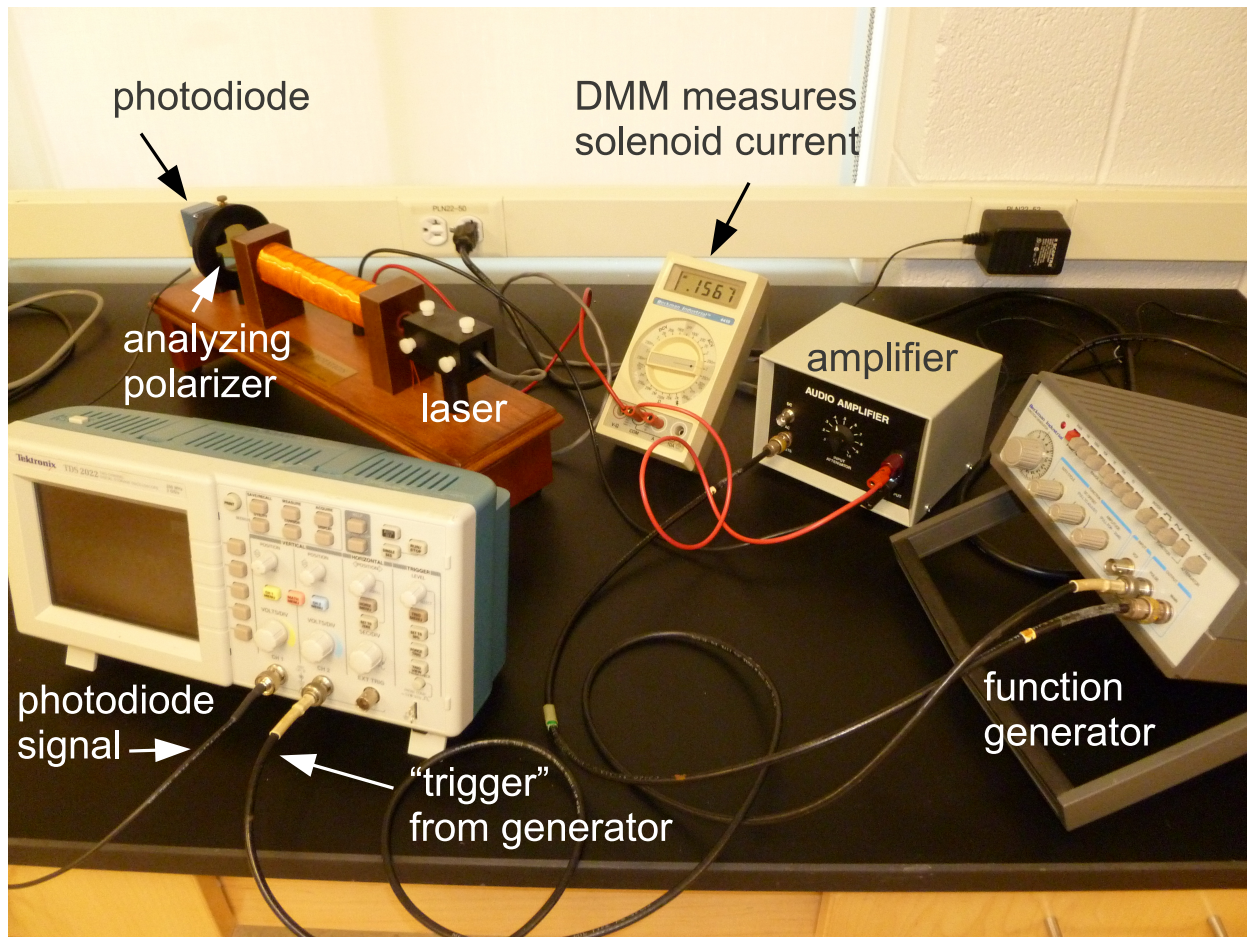


Figure 7.1: The experimental setup.

Calibration: intensity vs. θ We want to understand how the angle between the polarization vector of the laser light and the polarizer direction affects the intensity. Vary the angle of the analyzing polarizer and use a white screen (e.g., piece of paper) to observe how the intensity of the transmitted light changes. Find the angles that give you maximum and minimum transmission. Then, use the DMM to measure the photodiode output as a function of θ , going between the maximum and minimum in 5° steps. Tabulate this data. Graph it. What functional form should it have? Does it?

Function Generator Setup Plug the function generator output and its trigger (a.k.a. pulse) output into different channels on the scope. Trigger the scope on the trigger/pulse output from the function generator and look at the function generator signal. Modify the function generator to provide a 200 Hz sine wave with an amplitude of about 1 V. There is no more need to touch dials on the function generator.

Amplifier setup Now, plug the output of the function generator into the amplifier. Use a coaxial↔double-prong connector to feed the amplifier output into the scope, still

triggering on the generator. Vary the amplifier dial setting and observe the change in the output. At some point, the output will become clipped, as the amplifier reaches its power output. Record this dial setting, and the amplitude of the sine wave, just as the clipping sets in. This is the maximum useful output from the amplifier. Turn the dial all the way off, and hook the amplifier up to the solenoid, with the DMM in series to measure AC current. Go back to the maximum setting and measure the current flowing through the solenoid. This is the maximum useful current. You now have a time varying magnetic field in the solenoid and you can control its magnitude with the amplifier.

Measuring Faraday Rotation

Choice of θ You need to pick an angle θ , which may seem arbitrary. There is a best choice. Examine Eq. 7.7. Pick θ and be sure to tighten the thumbscrew.

Faraday rotation Plug the photodiode output into the scope, set the scope so its channel is DC coupled, and make sure that the “probe” setting is at 1x. Turn the amplifier dial about halfway to the maximum setting you found. Observe the photodiode trace on the scope, perhaps changing the volts/div setting so you can see the trace more clearly. What is the voltage? Record it. The changing magnetic field should be causing a change in the polarization angle of the laser light, which should cause a sinusoidal time dependence to the photodiode signal, referred to as the “wobble.” Can you see any wobble?

AC coupling The wobble is riding atop a large constant (DC) signal. The scope can remove the DC signal by “AC coupling” the photodiode channel. This essentially directs the scope input through a high pass filter. Do this, and then set the photodiode channel to the 2 mV setting. You should now see a wobble. Vary the amplifier dial’s setting and notice how the amplitude of the wobble changes. You are seeing the Faraday effect.

Remove the noise The signal is noisy, but now we’ll really benefit from knowing the waveform that the function generator is producing. Because we trigger the scope on the function generator, the maxima and minima will, neglecting random noise, occur at the same point on the scope screen (and in its memory bank). The scope has a feature that allows you to average multiple triggers. Doing this mitigates the noise, since at each point on the trace we are taking a mean, and the uncertainty in a mean decreases as we increase the number of measurements N as $1/\sqrt{N}$. Turn on the averaging feature by going to the “Acquire” menu. Observe how the averaged trace becomes more stable as you increase the number of traces being averaged. The larger the better, but ~ 100 traces should be enough.

Take measurements You should now systematically measure the amplitude of the wobble as a function of the current in the solenoid. There should be a linear relationship, which can be fit to extract C_V . Take about 10 measurements, evenly separated between

the smallest current for which there is a measurable wobble, and the maximum you found earlier. In each case, you want to start acquisition (Run/Stop on the scope), let the averaged signal converge onto a nice sine wave, stop acquisition and measure the amplitude of the signal using the scope's cursors. One measurement is shown in Fig. 7.2. Record the negative and positive amplitudes V_{low} and V_{high} ($\pm 640 \mu\text{V}$ in Fig. 7.2) and the peak to peak voltage ΔV (1.28 mV), along with the current in the coil - I_{coil} . Estimate the uncertainty in your measurements.

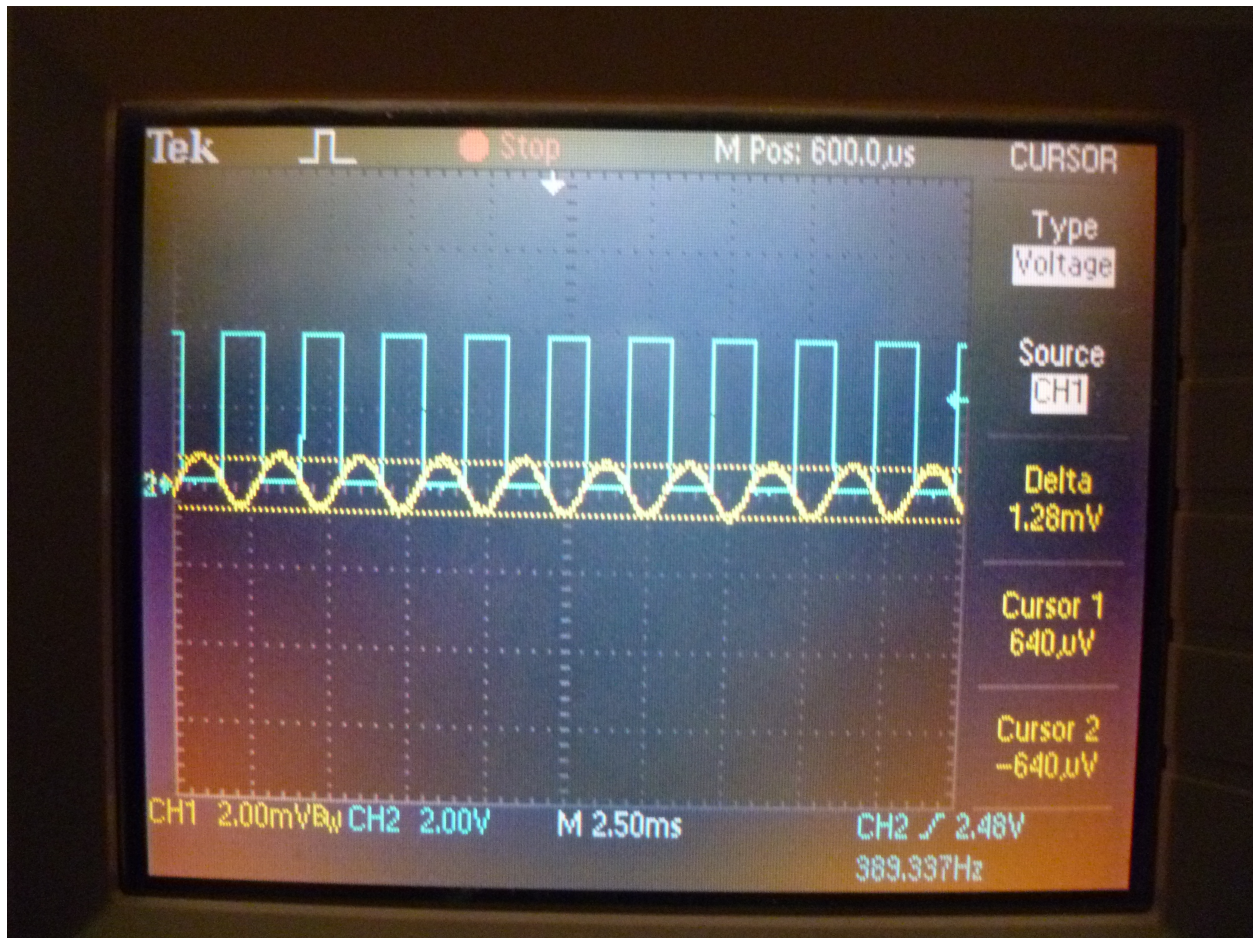


Figure 7.2: An example scope trace. The yellow curve is the output of the photodiode, AC coupled and averaged over 128 traces. The blue curve is the trigger output from the function generator and is being used to trigger the scope readout so that it's in phase with the changing magnetic field. The maximum and minimum amplitudes of the photodiode signal are measured with the scope's cursors.

Data Analysis

A variation in the angle ϕ is related to a variation in the magnetic field B according to:

$$\Delta\phi = C_V L \Delta B \quad (7.8)$$

We want to use this equation to extract C_V . Both ϕ and B vary with time as sine waves. We'll take $\Delta\phi$ and ΔB to be the amplitude of those waves (half the peak to peak). We didn't directly measure either quantity, but we can compute them. For ΔB it's easy: $\Delta B = 11.1 \text{ mT/A} \times \sqrt{2} I_{\text{coil}}$, where the $\sqrt{2}$ accounts for the fact that DMMs measure the root-mean-squared (RMS) value of an AC signal, not the peak value.

Figuring out $\Delta\phi$ is a little more difficult. We need to use our calibration dataset, which relates the photodiode signal to the angle between the laser polarization and the analyzer's orientation. From that dataset, you can estimate $dV/d\theta$ at the value of θ you are using in your experiment. Of course a change in the polarization of the laser, $\Delta\phi$ is equivalent to holding the laser polarization constant and changing the angle of the analyzer by $-\Delta\theta$. So, we can calculate:

$$\Delta\phi = \left[\frac{dV}{d\theta} \right]^{-1} \frac{\Delta V}{2} \quad (7.9)$$

The factor of two is because we defined ΔV as the peak to peak voltage.

Compute C_V for a few points, along with the uncertainty. Complete the analysis by fitting $\Delta\phi$ vs. ΔB to a straight line and then use Eq. 7.8 to extract C_V and its uncertainty. Do you see a nice linear relationship? What should the intercept be? Is it what you expect? Are there any outlier points, particularly at the ends of your curve? You could do the same analysis using V_{low} or V_{high} ... does doing so yield the same answer for C_V ?

Chapter 8

Atomic Spectroscopy

Experiment objectives: test a diffraction grating-based spectrometer, study the energy spectrum of atomic hydrogen (H) and a hydrogen-like atomic sodium (Na), determine values of quantum defects of low angular momentum states of Na, and measure fine splitting using Na yellow doublet.

History

The observation of discrete lines in the emission spectra of atomic gases gives insight into the quantum nature of atoms. Classical electrodynamics cannot explain the existence of these discrete lines, whose energies (or wavelengths) are given by characteristic values for specific atoms. These emission lines are so fundamental that they are used to identify atomic elements in objects, such as in identifying the constituents of stars in our universe. When Niels Bohr postulated that electrons can exist only in orbits of discrete energies, the explanation for the discrete atomic lines became clear. In this laboratory you will measure the wavelengths of the discrete emission lines from two elements - hydrogen and sodium - to determine the energy levels in the hydrogen-like atoms.

Hydrogen Spectrum

The hydrogen atom is the simplest atom: it consists of a single proton and a single electron. Since it is so simple, it is possible to calculate the energy spectrum of the electron bound states - in fact you will do that in your Quantum Mechanics course. From these calculations it follows that the electron energy is quantized, and these energies are given by the expression:

$$E_n = -\frac{2\pi^2 m_e e^4}{(4\pi\epsilon_0)^2 h^2 n^2} = -hcR_y \frac{1}{n^2} \quad (8.1)$$

where n is the **principal quantum number** of the atomic level, and

$$R_y = \frac{2\pi m_e e^4}{(4\pi\epsilon_0)^2 ch^3} \quad (8.2)$$

is a fundamental physical constant called the **Rydberg constant** (here m_e is the electron mass). Numerically, $R_y = 1.0974 \times 10^5 \text{cm}^{-1}$ and $hcR_y = 13.605 \text{eV}$.

Because of the allowed energies of an electron in a hydrogen atom, the electron can change its state only by making a transition (“jump”) from one state of energy E_1 to another state of lower energy E_2 by emitting a photon of energy $h\nu = E_1 - E_2$ that carries away the excess energy. Thus, by exciting atoms into high-energy states using a discharge and then measuring the frequencies of emission one can figure out the energy separation between various energy levels. Since it is more convenient to use a wavelength of light λ rather than its frequency ν , and using the standard connection between the wavelength and the frequency $h\nu = hc/\lambda$, we can write the wavelength of a photon emitted by electron jumping between the initial state n_1 and the final state n_2 as follows:

$$\frac{1}{\lambda_{12}} = \frac{2\pi^2 m_e e^4}{(4\pi\epsilon_0)^2 c h^3} \left[\frac{1}{n_2^2} - \frac{1}{n_1^2} \right] = R_y \left[\frac{1}{n_2^2} - \frac{1}{n_1^2} \right] \quad (8.3)$$

Fig. 8.1 shows the energy levels of hydrogen, and indicates a large number of observed transitions, grouped into series with the common electron final state. Emitted photon frequencies span the spectrum from the UV (UltraViolet) to the IR (InfraRed). Among all the series, only the Balmer series, corresponding to $n_2 = 2$, has its transitions in the visible part of the spectrum. Moreover, from all the possible transitions, we will be able to observe and measure only the following four lines: $n_1 = 6 \rightarrow 2$, $5 \rightarrow 2$, $4 \rightarrow 2$, and $3 \rightarrow 2$.

Sodium spectrum

Sodium (Na) belongs to the chemical group of *alkali metals* [together with lithium (Li), potassium (K), rubidium (Rb), cesium (Cs) and Francium (Fr)]. All these elements consist of a closed electron shell with one extra unbound electron. Not surprisingly, the energy level structure for this free electron is very similar to that of hydrogen. For example, a Na atom has 11 electrons, and its electronic configuration is $1s^2 2s^2 2p^6 3s$. Ten closed-shell electrons effectively screen the nuclear charge number ($Z = 11$) to an effective charge $Z^* \approx 1$, so that the $3s$ valence electron experiences the electric field potential similar to that of a hydrogen atom, given by the Eq. 8.1.

However, there is an important variation of the energy spectrum of alkali metals, which is related to the electron angular momentum l . In hydrogen the energy levels with same principle quantum number n but different electron angular momentum $l = 0, 1, \dots, (n - 1)$ are degenerate. For Na and others the levels with different values of l are shifted with respect to each other. This is mainly due to the interaction of the unpaired electrons with the electrons of the closed shells. For example, the orbits of the electron with large angular momentum values l are far above the closed shell, and thus their energies are basically the same as for the hydrogen atom. An electron with smaller l spends more time closer to the nucleus, and “feels” stronger bounding electrostatic potential. As a result the corresponding energy levels are pulled down compared to those of hydrogen, and the states with the same principle number n but different angular momenta l are split (*i.e.* have different energies).

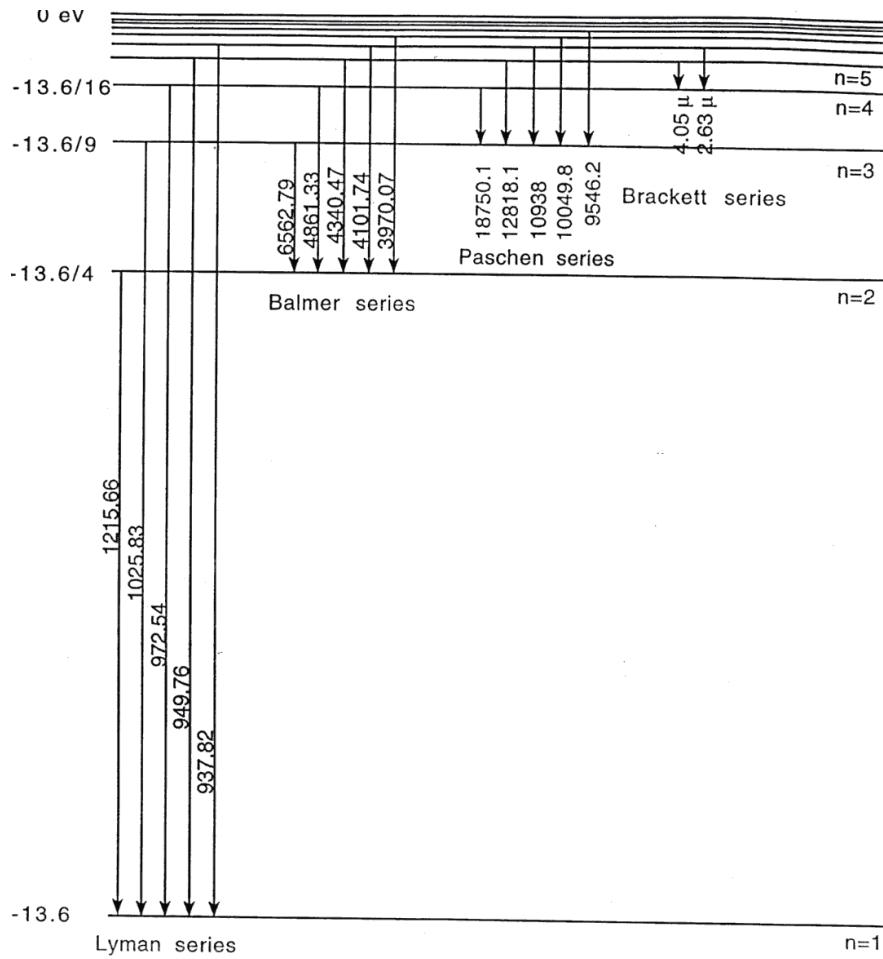


Figure 8.1: Spectrum of Hydrogen. The numbers on the left show the energies of the hydrogen levels with different principle quantum numbers n in eV. The wavelength of the emitted photon in Å is shown next to each electron transition.

To take into account the modification of the atomic spectra while still using the same basic equations used for hydrogen, it is convenient to introduce a small correction Δ_l to the principle quantum number n to take into account the level shifts. This correction is often called a *quantum defect*, and its value depends on the angular momentum l , and does not vary much between states with different principle quantum numbers n but same l ¹:

$$E_{nl} = -\frac{hcR_y}{(n - \Delta_l)^2} \quad (8.4)$$

In particular, the energies of two states with the lowest angular momentum (s and p) are noticeably affected by the more complicated electron structure of Na, while the energy levels of the states with the higher values of angular momentum (d , f) are identical to the hydrogen energy spectrum.

The spectrum of Na is shown in Fig. 8.2. One can immediately see that there are many more optical transitions because of the lifted degeneracy of energy states with different angular momenta. However, not all electronic transitions are allowed: since the angular momentum of a photon is 1, then the electron angular momentum cannot change by more than one while emitting one spontaneous photon. Thus, it is important to remember the following *selection rule* for atomic transitions:

$$\Delta l = \pm 1. \quad (8.5)$$

According to that rule, only transitions between two “adjacent” series are possible: for example $p \rightarrow s$ or $d \rightarrow p$ are allowed, while $s \rightarrow s$ or $s \rightarrow d$ are forbidden. The strongest allowed optical transitions are shown in Fig. 8.3. Note that each level for given n and l is split into two because of the *fine structure splitting*. This splitting is due to the effect of electron *spin* and its coupling with the angular momentum. Proper treatment of spin requires knowledge of quantum electrodynamics and solving the Dirac equation; for now spin can be treated as an additional quantum number associated with any particle. The spin of electron is $1/2$, and it may be oriented either along or against the non-zero electron’s angular momentum. Because of the weak coupling between the angular momentum and spin, these two possible orientations result in small differences in energy for corresponding electron states.

¹The accepted notation for different electron angular momentum states is historical, and originates from the days when the proper quantum mechanical description for atomic spectra had not been developed yet. Back then spectroscopists had categorized atomic spectral lines corresponding to their appearance: for example any spectral lines from electron transitions from s -orbital ($l = 0$) always appeared **S**harp on a photographic film, while those with initial electron states of d -orbital ($l = 2$) always appeared **D**iffuse. Also spectral lines of **P**inciple series (initial state is p -orbital, $l = 1$) reproduced the hydrogen spectrum most accurately (even though at shifted frequencies), while the **F**undamental (initial electron state is f -orbital, $l = 3$) series matched the absolute energies of the hydrogen most precisely. The orbitals with higher values of the angular momentum are denoted in an alphabetic order (g , h , *etc.*)

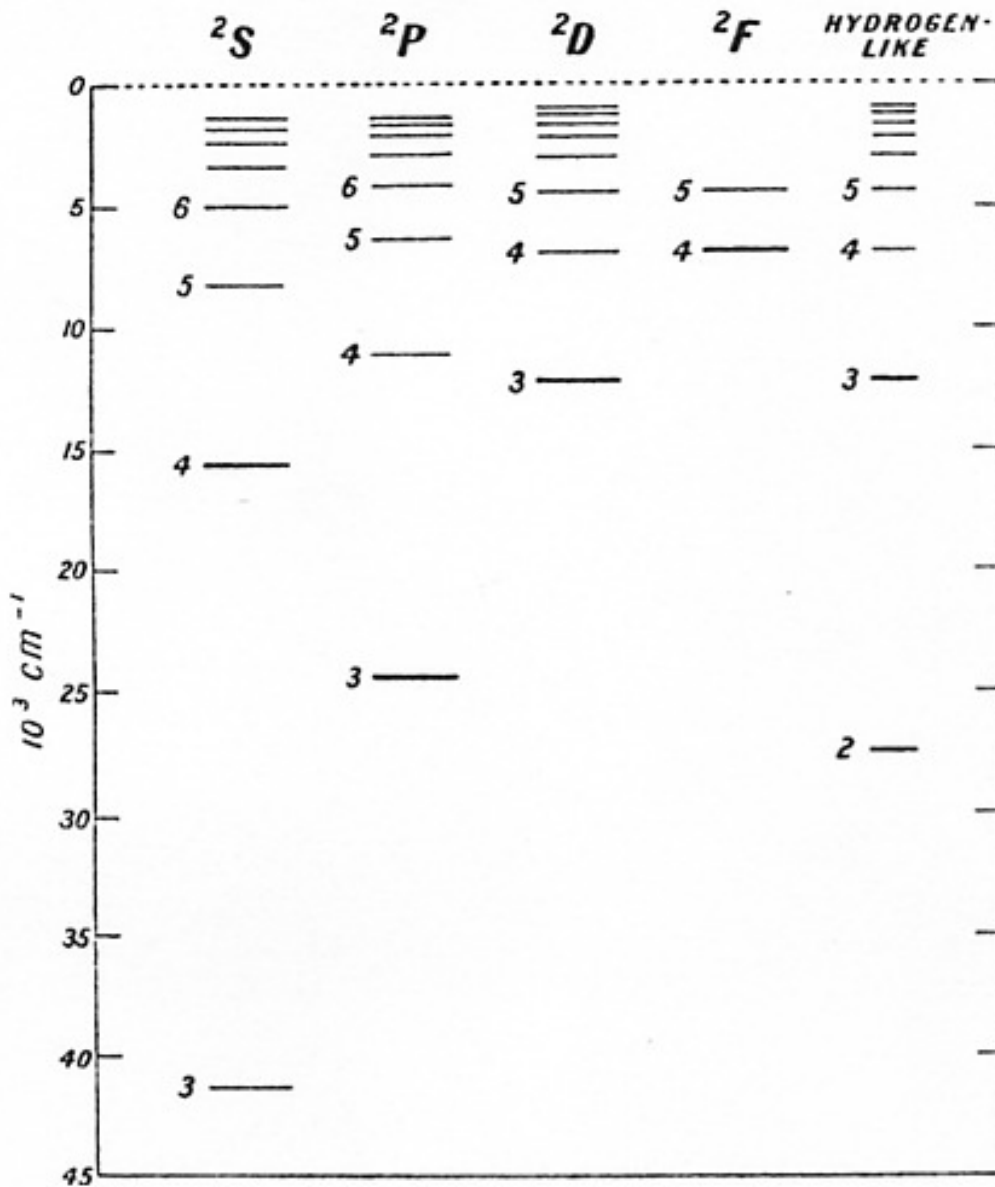


Figure 8.2: Energy spectrum of Na. The energy states of H are shown in far right for comparison.

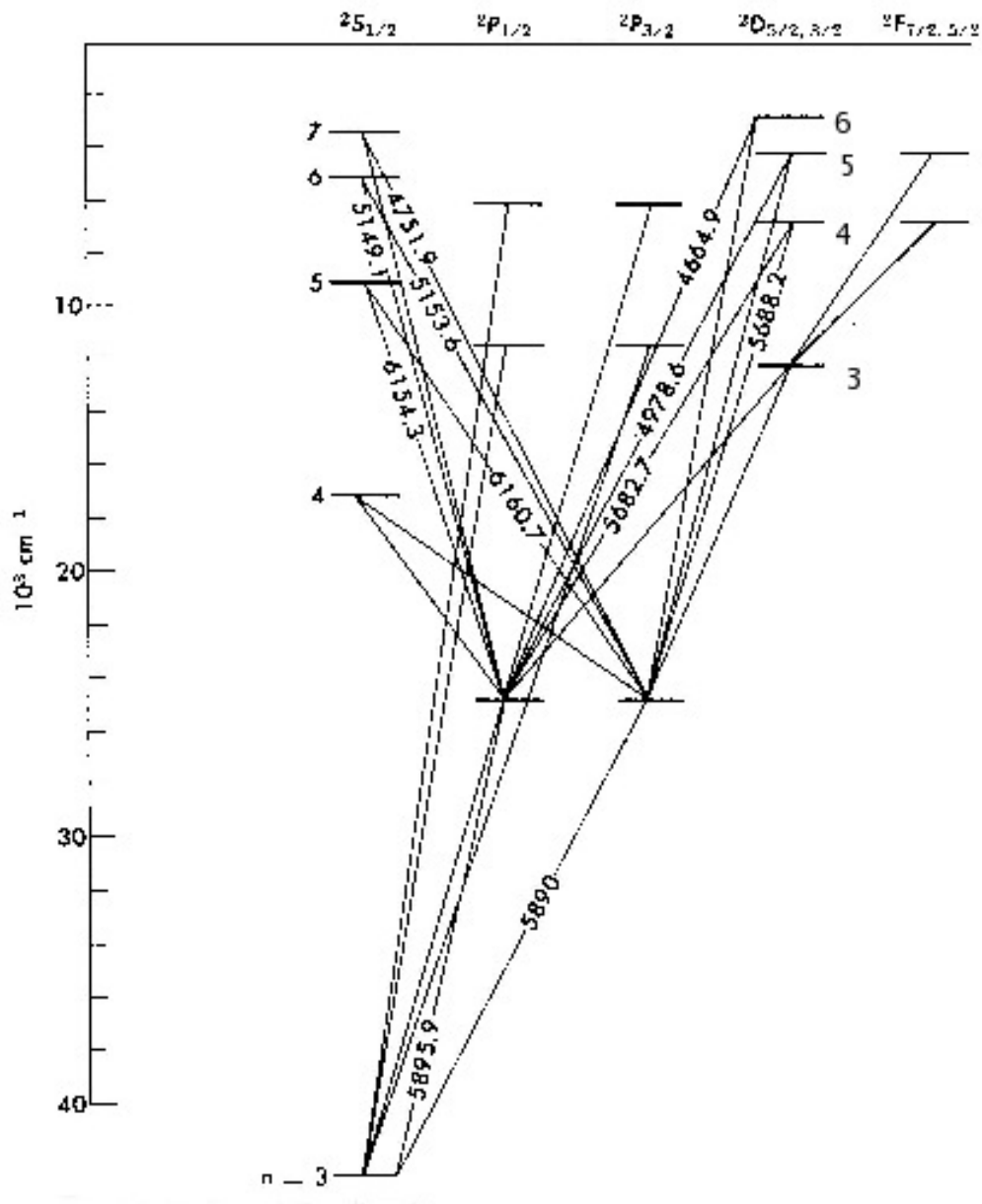


Figure 8.3: Transitions for Na. The wavelengths of selected transitions are shown in \AA . Note that p state is now shown in two columns, one referred to as $P_{1/2}$ and the other as $P_{3/2}$. The small difference between their energy levels is the “fine structure”.

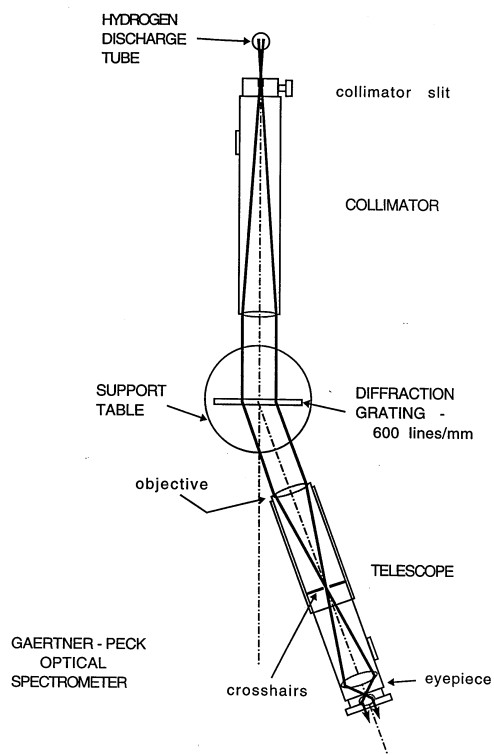


Figure 8.4: Gaertner-Peck Spectrometer

Experimental setup

Equipment needed: Gaertner-Peck optical spectrometer, hydrogen discharge lamp, sodium discharge lamp.

Fig. 8.4 gives a top view of the Gaertner-Peck optical spectrometer used in this lab. The spectrometer consists of two tubes. One tube holds a vertical collimator slit of variable width, and should be directed toward a discharge lamp. The light from the discharge lamp passes through the collimator slit and then gets dispersed on the diffraction grating mounted in the mounting pedestal in the middle of the apparatus. The other tube holds the telescope that allows you to magnify the image of the collimator slit to get a more accurate reading on the rotating table. This tube can be rotated around the central point, so that you will be able to align and measure the wavelength of all visible spectral lines of the lamp in all orders of diffraction grating.

It is likely that you will find a spectrometer at nearly aligned condition at the beginning on the lab. Nevertheless, take time making sure that all elements are in order to insure good quality of your data.

Telescope Alignment: Start by adjusting the telescope eyepiece in or out to bring the crosshairs into sharp focus. Next aim the telescope out the window to view a distant object such as leaves in a tree. If the distant object is not in focus, you will have to adjust the position of the telescope tube - ask your instructor for directions.

Collimator Conditions: Swing the telescope to view the collimator which is accepting light from the hydrogen discharge tube through a vertical slit of variable width. The slit opening should be set to about 5-10 times the crosshair width to permit sufficient light to see the faint violet line and to be able to see the crosshairs. If the bright column of light is not in sharp focus, you should align the position of the telescope tube again (with the help of the instructor).

Diffraction Grating Conditions: In this experiment you will be using a diffraction grating that has 600 lines per mm. A brief summary of diffraction grating operation is given in the Appendix of this manual. If the grating is not already in place, put it back on the baseclamp and fix it there. The table plate that holds the grating can be rotated, so try to orient the grating surface to be maximally perpendicular to the collimator axis. However, the accurate measurement of angles does not require the perfect grating alignment. Instead, for each spectral line in each diffraction order you will be measuring the angles on the left (θ_l) and on the right (θ_r), and use both of the measurements to figure out the optical wavelength using the following equation:

$$m\lambda = \frac{d}{2}(\sin \theta_r + \sin \theta_l), \quad (8.6)$$

where m is the diffraction order, and d is the distance between the lines in the grating.

Then, with the hydrogen tube ON and in place at the collimator slit, swing the rotating telescope slowly through 90 degrees both on the left & right sides of the forward direction. You should observe diffraction maxima for most spectral wavelength, λ , in 1st and 2nd order. If these lines seem to climb uphill or drop downhill the grating has to be adjusted in its baseclamp to bring them all to the same elevation.

Also, turn on the sodium lamp as soon as you arrive, since it requires 10-15 minutes to warm up and show clear Na spectrum.

Experimental studies of Hydrogen Balmer series

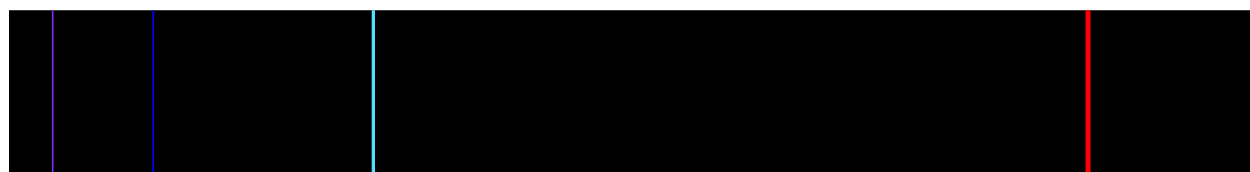


Figure 8.5: Hydrogen visible spectrum

Swing the rotating telescope slowly and determine which spectral lines from Balmer series you observe. You should be able to see three bright lines - Blue, Green and Red - in the first ($m=1$) and second ($m=2$) diffraction orders on both left & right sides.

One more line of the Balmer series is in the visible range - Violet, but its intensity is much lower than for the other three line. However, you will be able to find it in the first order if you look carefully with the collimator slit open a little wider. You should see spectrum resembling one shown in Fig. 8.5.

After locating all the lines, measure the angles at which each line occurs. The spectrometer reading for each of the first order lines should be measured at least *twice* by *different* lab partners to avoid systematic errors. For the second order lines, you only need one measurement for your group. **Don't forget:** for every line you need to measure the angles to the right and to the left!

You should be able to determine the angle with accuracy of 1 minute, but you should know how to read angles with high precision in the spectrometer: first use the bottom scale to get the rough angle reading within a half of the degree. Then use the upper scale for more accurate reading in minutes. To get this reading find a tick mark of the upper scale that aligns perfectly with some tick mark of the bottom scale - this is your minute reading. Total angle is the sum of two readings.

To measure the frequency precisely, center the crosshairs on the line as accurately as possible. Choose the width of lines by turning the collimator slit adjustment screw. If the slit is too wide, it is hard to judge the center of the line accurately; if the slit is too narrow, then not enough light is available to see the crosshairs. For Violet the intensity is noticeably less than for the other three lines, so you may not see it. However, even if you find it, a little assistance is usually required in order to locate the crosshairs at this line. We suggest that a low intensity flashlight be aimed toward the Telescope input, and switched ON and OFF repeatedly to reveal the location of the vertical crosshair relative to the faint Violet line.

Data analysis for Hydrogen Data

Calculate the wavelength λ for each line observed in all orders, calculate the average wavelength value and uncertainty for each line, and then identify the initial and final electron states principle numbers (n_1 and n_2) for each line using Fig. 8.1.

Make a plot of $1/\lambda$ vs $1/n_1^2$ where n_1 = the principal quantum number of the electron's initial state. Put all λ values you measure above on this plot. Your data points should form a straight line. From Equation (8.3) determine the physical meaning of both slope and intercept, and compare the data from the fit to the expected values for each of them. The slope should be the Rydberg constant for hydrogen, R_y . The intercept is $R_y/(n_2)^2$. From this, determine the value for the principal quantum number n_2 . Compare to the accepted value in the Balmer series.

Experimental studies of Sodium Spectrum

Switch to Sodium lamp and make sure the lamp warms up for approximately 5-10 minutes before starting the measurements.

Double check that you see a sharp spectrum in the spectrometer (adjust the width of the collimator slit if necessary). In the beginning it will be very useful for each lab partner to quickly scan the spectrometer telescope through all first-order lines, and then discuss which lines you see correspond with which transitions in Table 8.1 and Fig. 8.3. Keep in mind that the color names are symbolic rather than descriptive!

Color	Line ₁ (Å)	Line ₂ (Å)
Red	6154.3	6160.7
Yellow	5890.0	5895.9
Green	5682.7	5688.2
	5149.1	5153.6
	4978.6	4982.9
Blue	4748.0	4751.9
	4664.9	4668.6
Blue-Violet	4494.3	4497.7

Table 8.1: Wavelength of the visible sodium lines.

After that, carefully measure the left and right angles for as many spectral lines in the first and second orders as possible. The spectrometer reading for each line should be measured at least *once* by both lab partners to avoid systematic errors.

Determine the wavelengths of all measured Na spectral lines using Eq. 8.6. Compare these measured mean wavelengths to the accepted values given in Fig. 8.3 and in Tab. 8.1. Identify at least seven of the lines with a particular transition. The eighth line in Tab. 8.1 has a wavelength $\lambda = 4494.3\text{\AA}$. It corresponds to the $8d \rightarrow 3p$ transition but isn't shown in Fig. 8.3.

Line₁ and Line₂ correspond to transitions to two fine-splitting $3p$ states $P_{1/2}$ and $P_{3/2}$. These two transition frequencies are very close to each other, and to resolve them with the spectrometer the width of the slit should be very narrow. However, you may not be able to see some weaker lines then. In this case you should open the slit wider to let more light in when searching for a line. If you can see a spectral line but cannot resolve the doublet, record the reading for the center of the spectrometer line, and use the average of two wavelengths given above.

Measurements of the fine structure splitting. Once you measured all visible spectral lines, go back to some bright line (yellow should work well), and close the collimator slit such that you can barely see any light going through. In this case you should be able to see the splitting of the line because of the *fine structure splitting* of states $P_{1/2}$ and $P_{3/2}$. For the Na D doublet the splitting between two lines $\Delta\lambda = \lambda_{3/2} - \lambda_{1/2}$.

Measure the splitting between two lines in the first and the second orders. Which one works better? Discuss this issue in your lab report. Compare to the accepted value: $\Delta\lambda = 5.9\text{ \AA}$. Compare this approach to the use of the Fabry-Perot interferometer.

Data analysis for Sodium

Calculation of a quantum defect for $n = 3, p$ state.

Once you identified all of your measured spectral lines, choose only those that correspond to optical transitions from any d to $n = 3, p$ states. Since the energy states of d series follows

the hydrogen spectra almost exactly, the wavelength of emitted light λ is given by:

$$\frac{hc}{\lambda} = E_{nd} - E_{3p} = -\frac{hcR_y}{n_d^2} + \frac{hcR_y}{(3 - \Delta_p)^2}, \quad (8.7)$$

or

$$\frac{1}{\lambda} = \frac{R_y}{(3 - \Delta_p)^2} - \frac{R_y}{n_d^2}, \quad (8.8)$$

where n_d is the principle number of the initial d state. Verify this expression by plotting $1/\lambda$ versus $1/n_d^2$ for the $n_d = 4, 5$, and 6 . From the slope of this curve determine the value of the Rydberg constant R_y . The value of the intercept in this case is $\frac{R_y}{(3 - \Delta_p)^2}$, so use it to find the quantum defect Δ_p .

Compare the results of your calculations for the quantum defect with the accepted value $\Delta_p = 0.86$.

Appendix: Operation of a diffraction grating-based optical spectrometer

A diffraction grating is a common optical element, which consists of a pattern with many equidistant slits or grooves. Interference of multiple beams passing through the slits (or reflecting off the grooves) produces sharp intensity maxima in the output intensity distribution, which can be used to separate different spectral components on the incoming light. In this sense the name “diffraction grating” is somewhat misleading, since we are used to talking about diffraction with regard to the modification of light intensity distribution to the finite size of a single aperture.

To describe the properties of a light wave after passing through the grating, let us first consider the case of 2 identical slits separated by the distance h , as shown in Fig. 8.6a. We will assume that the size of the slits is much smaller than the distance between them, so that the effect of Fraunhofer diffraction on each individual slit is negligible. Then the resulting intensity distribution on the screen is given by the familiar Young formula:

$$I(\theta) = |E_0 + E_0 e^{ikh \sin \theta}|^2 = 4I_0 \cos^2 \left(\frac{\pi h}{\lambda} \sin \theta \right), \quad (8.9)$$

where $k = 2\pi/\lambda$, $I_0 = |E_0|^2$, and the angle θ is measured with respect to the normal to the plane containing the slits.

Interference on N equidistant slits illuminated by a plane wave (Fig. 8.6b) produces much sharper maxima. To find light intensity on a screen, the contributions from all N slits must be summarized taking into account their acquired phase difference, so that the optical field intensity distribution becomes:

$$I(\theta) = |E_0 + E_0 e^{ikh \sin \theta} + E_0 e^{2ikh \sin \theta} + \dots + E_0 e^{(N-1)ikh \sin \theta}|^2 = I_0 \left[\frac{\sin \left(N \frac{\pi h}{\lambda} \sin \theta \right)}{\sin \left(\frac{\pi h}{\lambda} \sin \theta \right)} \right]^2. \quad (8.10)$$

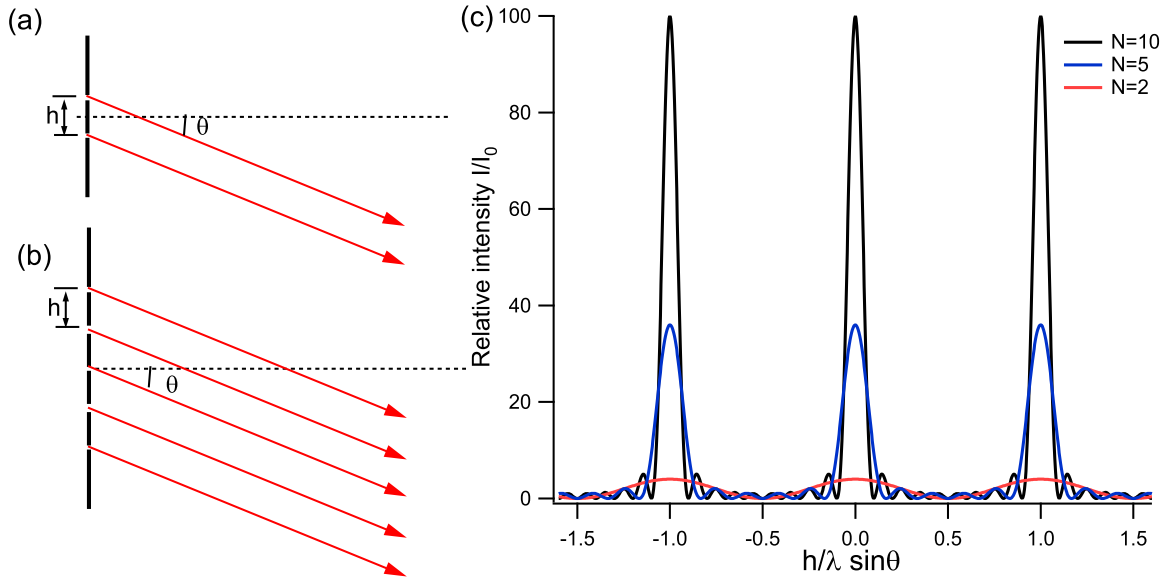


Figure 8.6: Intensity Pattern for Fraunhofer Diffraction

Here we again neglect the diffraction from each individual slit, assuming that the size of the slit is much smaller than the separation h between the slits.

The intensity distributions from a diffraction grating with illuminated $N = 2, 5$ and 10 slits are shown in Fig. 8.6c. The tallest (*principle*) maxima occur when the denominator of Eq.(8.10) becomes zero: $h \sin \theta = \pm m \lambda$ where $m = 1, 2, 3, \dots$ is the diffraction order. The heights of the principle maxima are $I_{\max} = N^2 I_0$, and their widths are $\Delta \theta = 2\lambda / (Nh)$. Notice that the more slits are illuminated, the narrower diffraction peaks are, and the better the resolution of the system is:

$$\frac{\Delta \lambda}{\lambda} = \frac{\Delta \theta}{\theta} \simeq \frac{1}{Nm} \quad (8.11)$$

For that reason in any spectroscopic equipment a light beam is usually expanded to cover the maximum surface of a diffraction grating.

Diffraction Grating Equation when the Incident Rays are not Normal

Up to now, we assumed that the incident optical wavefront is normal to the pane of a grating. Let's now consider more general case when the angle of incidence θ_i of the incoming wave is different from the normal to the grating, as shown in Fig. 8.7(a). Rather than calculating the whole intensity distribution, we will determine the positions of principle maxima. The path length difference between two rays 1 and 2 passing through the consequential slits is $a + b$, where:

$$a = h \sin \theta_i; \quad b = h \sin \theta_R \quad (8.12)$$

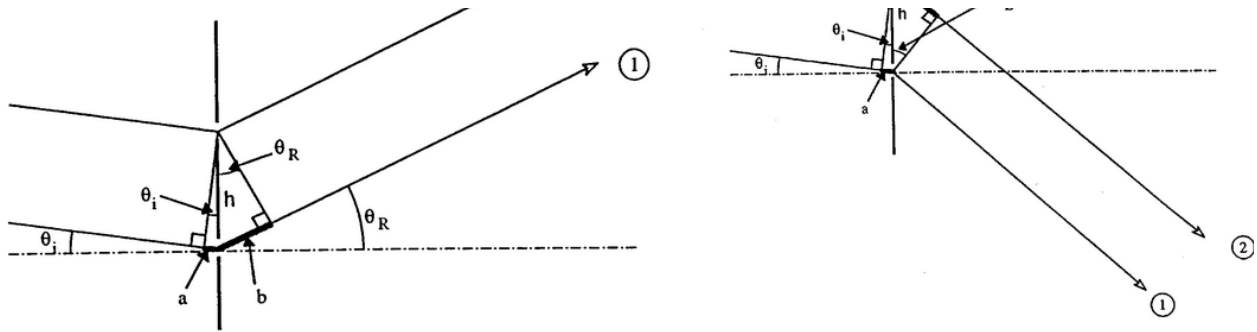


Figure 8.7: Diagram of the light beams diffracted to the right (a) and to the left (b).

Constructive interference occurs for order m when $a + b = m\lambda$, or:

$$h \sin \theta_i + h \sin \theta_R = m\lambda \quad (8.13)$$

Now consider the case shown in Fig. 8.7(b). The path length between two beams is now $b - a$ where $b = h \sin \theta_L$. Combining both cases we have:

$$h \sin \theta_L - h \sin \theta_i = m\lambda \quad (8.14)$$

$$h \sin \theta_R + h \sin \theta_i = m\lambda$$

Adding these equations and dividing the result by 2 yields the following expression connecting the right and left diffraction angles:

$$m\lambda = \frac{h}{2} (\sin \theta_L + \sin \theta_R) \quad (8.15)$$

Chapter 9

Superconductivity

Experiment objectives: study the behavior of a high temperature superconducting material Yttrium-Barium-Copper-Oxide (YBCO, $\text{YBa}_2\text{Cu}_3\text{O}_7$) in magnetic field, measure the critical temperature for a phase transition in a superconductor.

History

Solids can be roughly divided into four classes, according to the way they conduct electricity. They are: Metals, Semiconductors, Insulators and Superconductors. The behavior of these types of materials is explained by quantum mechanics. Basically, when atoms form a solid, the atomic levels of the electrons combine to form bands. That is, over a finite range of energies there are states available to electrons. Since only one electron can occupy a given state, as described by the **Pauli Exclusion Principle**, electrons will fill these states up to some maximum: the Fermi Energy, E_f . A solid is a metal if it has an energy band that is not full. The electrons are then free to move about, making a metal a good conductor of electricity. If the solid has a band that is completely full, with an energy gap to the next band, that solid will not conduct electricity very well, making it an insulator. A semiconductor is between a metal and insulator: while it has a full band (the valence band), the next band (the conduction band) is close enough in energy, and so the electrons can easily reach it. Superconductors are in a class by themselves. They can be metals or insulators at room temperature. Below a certain temperature, called the critical temperature, the electrons “pair” together (in Cooper pairs) and travel through the solid without resistance. Current in a superconductor below the critical temperature will travel indefinitely without dissipation.

Superconductivity was discovered in 1911 by H. Onnes. He discovered that simple metals (Pb, Nb) superconduct when placed in liquid helium (4 K). This was an important discovery, but the real excitement came in 1986 when Swiss scientists discovered that certain ceramics would superconduct at 35 K. Several groups later discovered materials that would superconduct at temperatures up to 125 K. These materials are called high temperature superconductors (HTS). Their discovery was a breakthrough, because it meant that these superconductors will work in liquid nitrogen (at 77 K), which is relatively cheap and

abundant.

A fascinating fact about superconductors: they will carry a current nearly indefinitely, without resistance. Superconductors have a critical temperature, above which they lose their superconducting properties.

Another striking demonstration of superconductivity is the **Meissner effect**. Magnetic fields cannot penetrate superconducting surfaces; instead a superconductor attempts to expel all magnetic field lines. It is fairly simple to intuitively understand the Meissner effect if you imagine a perfect conductor of electricity. If a superconductor placed in a magnetic field, Faraday's Law says an induced current that opposes the field would be set up. But unlike in an ordinary metal, this induced current does not dissipate in a perfect conductor. So, this induced current would always be present to produce a field which opposes the external field. In addition, microscopic dipole moments are induced in the superconductor that oppose the applied field. This induced field repels the source of the applied magnetic field, and will consequently repel the magnet associated with this field. Thus, a superconductor will levitate a magnet placed upon it (this is known as magnetic levitation).

Safety

- Wear glasses when pouring liquid nitrogen. Do not get it on your skin or in your eyes!
- Do not touch anything that has been immersed in liquid nitrogen until the item warms up to the room temperature. Use the provided tweezers to remove and place items in the liquid nitrogen.
- Do not touch the superconductor: it contains poisonous materials!
- Beware of the current leads: they are carrying a lethal current!

Experimental procedure

Equipment needed: YBCO disk, tweezers, thermal insulating container, small magnet.

Magnetic Levitation (the Meissner effect)

1. Place one of the small magnets (provided) on top of the superconducting disk at room temperature. Record the behavior of the magnet.
2. Using the tweezers, place the superconducting disk in the thermal insulating container. Attach the thermocouple leads (see diagram) to a multimeter reading on the mV scale. Slowly pour liquid nitrogen over the disk, filling the container as much as you can. The nitrogen will boil, and then settle down. When the multimeter reads about 6.4 mV, you are at liquid nitrogen temperature (77 K).

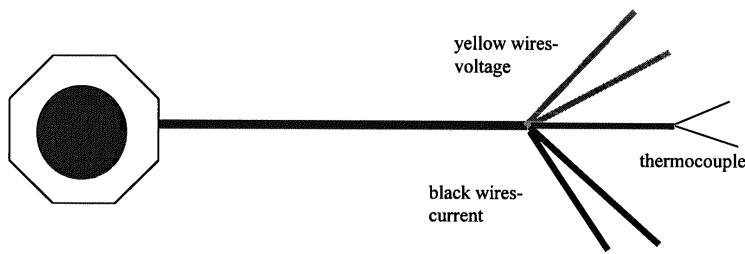


Figure 9.1: The superconducting disk with leads.

3. After the disk is completely covered by the liquid nitrogen, use the tweezers to pick up the provided magnet and attempt to balance it on top of the superconductor disk. Record what you observe.
4. Try demonstrating a *frictionless magnetic bearing*: if you carefully set the magnet rotating, you will observe that the magnet continues to rotate for a long time. Also, try moving the magnet across the superconductor. Do you feel any resistance? If you feel resistance, why is this?
5. Using tweezers, take the disk (with the magnet on it) out of the nitrogen and place it beside the container, allowing it to warm. Watch the thermocouple reading carefully, and take a reading when the magnet fails to levitate any longer. This is a rough estimate of the critical temperature. Make sure you record it!
6. Repeat the experiment by starting with the magnet on top of the superconductor disk and observe if the magnet starts levitating when the disk's temperature falls below critical.

Measuring resistance and critical temperature

We will measure the resistance by a **four probe method**, as a function of temperature. Using four probes (two for current and two for voltage) eliminates the contribution of resistance due to the contacts, and is good to use for samples with small resistances. Connect a voltmeter (with 0.01 mV resolution) to the yellow wires. Connect a current source through an ammeter to the **black** wires. Place a current of about 0.2 Amps (200 mA) through the black leads. Note: **DO NOT EXCEED 0.5 AMP!!!!** At room temperature, you should be reading a non-zero voltage reading.

1. With the voltage, current and thermocouple leads attached, carefully place the disk in the container. Pour liquid nitrogen into the container. Wait until the temperature reaches 77 K.

2. With tweezers, take the disk out of nitrogen and place it outside of the container. **Start quickly recording the current, voltage and thermocouple readings as the disk warms up.** When superconducting, the disk should have $V=0$ ($R=0$). At a critical temperature, you will see a voltage (resistance) appear.
3. Repeat this measurement several times to acquire a significant number of data points near the critical temperature (6.4-4.5 mV). Make a plot of resistance versus temperature, and make an estimate of the critical temperature based on this plot.

Conversion from mV to Kelvin

°K	0	1	2	3	4	5	6	7	8	9	10	°K
60	7.60	7.53	7.46	7.40	7.33	7.26	7.19	7.12	7.05	6.99	6.92	60
70	6.92	6.85	6.78	6.71	6.64	6.56	6.49	6.42	6.37	6.33	6.29	70
80	6.29	6.25	6.21	6.17	6.13	6.09	6.05	6.01	5.97	5.93	5.90	80
90	5.90	5.86	5.83	5.79	5.75	5.72	5.68	5.64	5.60	5.56	5.52	90
100	5.52	5.48	5.44	5.41	5.37	5.34	5.30	5.27	5.23	5.20	5.16	100
110	5.16	5.13	5.09	5.06	5.02	4.99	4.95	4.91	4.88	4.84	4.81	110
120	4.81	4.77	4.74	4.70	4.67	4.63	4.60	4.56	4.53	4.49	4.46	120
130	4.46	4.42	4.39	4.35	4.32	4.28	4.25	4.21	4.18	4.14	4.11	130
140	4.11	4.07	4.04	4.00	3.97	3.93	3.90	3.86	3.83	3.79	3.76	140
150	3.76	3.73	3.69	3.66	3.63	3.60	3.56	3.53	3.50	3.47	3.43	150
160	3.43	3.40	3.37	3.34	3.30	3.27	3.24	3.21	3.18	3.15	3.12	160
170	3.12	3.09	3.06	3.03	3.00	2.97	2.94	2.91	2.88	2.85	2.82	170
180	2.82	2.79	2.76	2.73	2.70	2.67	2.64	2.61	2.58	2.53	2.52	180
190	2.52	2.49	2.46	2.43	2.40	2.37	2.34	2.31	2.29	2.26	2.23	190
200	2.23	2.20	2.17	2.14	2.11	2.08	2.05	2.02	1.99	1.96	1.93	200
210	1.93	1.90	1.87	1.84	1.81	1.78	1.75	1.72	1.69	1.66	1.64	210
220	1.64	1.61	1.59	1.56	1.54	1.51	1.49	1.46	1.44	1.41	1.39	220
230	1.39	1.36	1.34	1.31	1.29	1.26	1.24	1.21	1.19	1.16	1.14	230
240	1.14	1.11	1.09	1.07	1.04	1.02	0.99	0.97	0.94	0.92	0.89	240
250	0.89	0.87	0.84	0.82	0.79	0.77	0.74	0.72	0.69	0.67	0.65	250
260	0.65	0.62	0.60	0.58	0.55	0.53	0.50	0.48	0.45	0.42	0.40	260
270	0.40	0.38	0.36	0.34	0.32	0.30	0.28	0.26	0.24	0.22	0.20	270
280	0.20	0.18	0.16	0.14	0.12	0.10	0.08	0.06	0.04	0.02	0.00	280
290	0.00	-0.02	-0.04	-0.06	-0.08	-0.10	-0.12	-0.14	0.16	-0.18	-0.20	290
300	-0.20	-0.22	-0.24	-0.26	-0.28	-0.30	-0.32	-0.34	-0.36	-0.38	-0.40	300

Instructions for a brief writeup

The abstract, introduction and conclusion are not needed. Do the following analysis steps and/or answer the following questions. Make it clear which question you are addressing. For example:

Thermocouple

Blah, blah, blah, blah...

Address the following points

Diagram the experimental setup, including the current source, the superconductor and its wires, and the DMMs.

Thermocouple Explain in your own words what a thermocouple is and how it works (i.e., the physics). No more than a paragraph is needed.

Resistance In the Meissner effect demonstration, when moving the magnets around, did you feel resistance? Why or why not? Explain in terms of physics concepts and principles that you learned in PHYS 102.

Tabulate your data, including columns for voltage, current, resistance, thermocouple voltage, and temperature.

Uncertainties Estimate them and state them.

Temperature Explain how you got the temperature from the thermocouple voltage. What was your procedure? No more than a couple of sentences are needed.

Plot the resistance (y-axis) vs temperature (x-axis). Based on the plot, what is the critical temperature for YBCO?

Fit the R vs T plot. What function? You tell me. Based on the fitted function, can you estimate the critical temperature? Parameterizing a dataset and then using it to characterize the data, perhaps with a single number, is a common laboratory task. Get used to it!