Chapter 1

Electron Diffraction

Experiment objectives: observe diffraction of the beam of electrons on a graphitized carbon target and calculate the intra-atomic spacings in the graphite.

History

A primary tenet of quantum mechanics is the wavelike properties of matter. In 1924, graduate student Louis de Broglie suggested in his dissertation that since light has both particle-like **and** wave-like properties, perhaps all matter might also have wave-like properties. He postulated that the wavelength of objects was given by $\lambda = h/p$, where h is Planck's constant and $p = mv$ is the momentum. *This was quite a revolutionary idea*, since there was no evidence at the time that matter behaved like waves. In 1927, however, Clinton Davisson and Lester Germer discovered experimental proof of the wave-like properties of matter — particularly electrons. This discovery was quite by mistake: while studying electron reflection from a nickel target, they inadvertently crystallized their target, while heating it, and discovered that the scattered electron intensity as a function of scattering angle showed maxima and minima. That is, electrons were "diffracting" from the crystal planes much like light diffracts from a grating, leading to constructive and destructive interference. Not only was this discovery important for the foundation of quantum mechanics (Davisson and Germer won the Nobel Prize for their discovery), but electron diffraction is an extremely important tool used to study new materials. In this lab you will study electron diffraction from a graphite target, measuring the spacing between the carbon atoms.

Theory

Consider planes of atoms in a **crystal** as shown in Fig. 1.1 separated by distance d. Electron "waves" reflect from each of these planes. Since the electron is wave-like, the combination of the reflections from each interface produces to an interference pattern. This is completely analogous to light interference, arising, for example, from different path lengths in the Fabry-Perot or Michelson interferometers. The de Broglie wavelength for the electron is given by
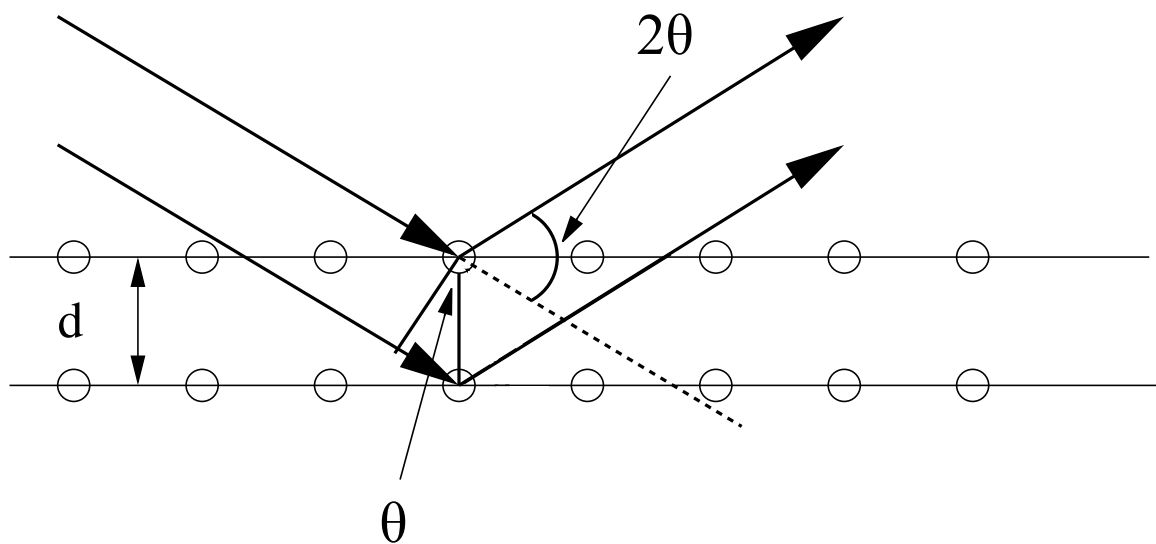
Figure 1.1: Electron Diffraction from atomic layers in a crystal.

$\lambda = h/p$, where p can be calculated by knowing the energy of the electrons when they leave the "electron gun":

$$\frac{1}{2}mv^2 = \frac{p^2}{2m} = eV_a, \tag{1.1}$$

where V_a is the accelerating potential. The condition for constructive interference is that the path length difference for the two waves shown in Fig. 1.1 be a multiple of a wavelength. This leads to Bragg's Law:

$$n\lambda = 2d\sin\theta \tag{1.2}$$

where $n = 1, 2, \ldots$ is integer. In this experiment, only the first order diffraction $n = 1$ is observed. Therefore, the intra-atomic distance in a crystal can be calculated by measuring the angle of electron diffraction and their wavelength (*i.e.* their momentum):

$$d = \frac{\lambda}{2\sin\theta} = \frac{1}{2\sin\theta}\frac{h}{\sqrt{2em_eV_a}} \tag{1.3}$$

where h is Planck's constant, e is the electronic charge, m_e is the electron's mass, and V_a is the accelerating voltage.

Experimental Procedure

Equipment needed: Electron diffraction apparatus and power supply, ruler and thin receipt paper or masking tape.

Safety

You will be working with high voltage. The power supply will be connected for you, but inspect the apparatus when you arrive **before** turning the power on. If any wires are unplugged, ask an instructor to reconnect them. Also, **before** turning the power on, identify the high voltage contacts on the electron diffraction tube, make sure these connections are well-protected and cannot be touched by accident while taking measurements.
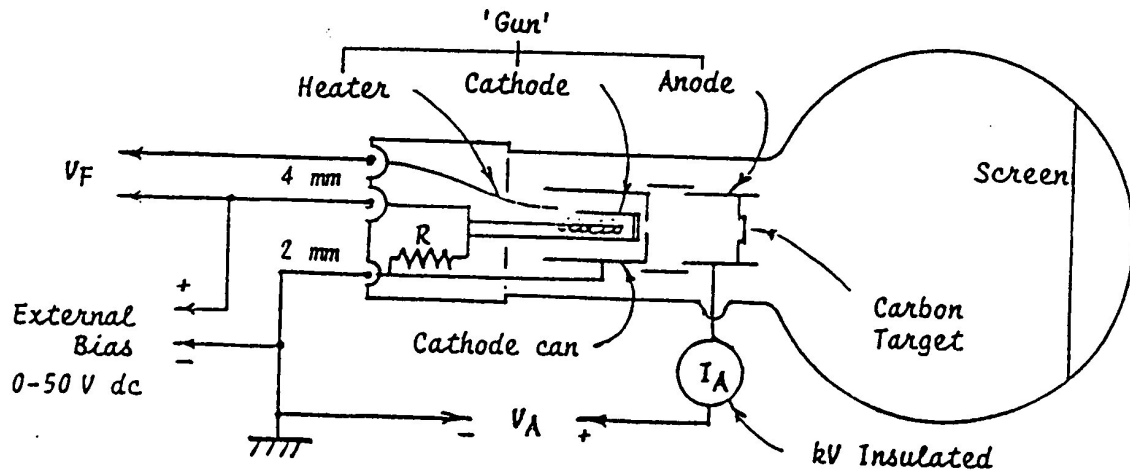

Figure 1.2: Electron Diffraction Apparatus.

Setup

The diagram of the apparatus is given in Fig.1.2. An electron gun (consisting of a heated filament to boil electrons off a cathode and an anode to accelerate them, similar to the e/m experiment) "shoots" electrons at a carbon (graphite) target.

The electrons diffract from the carbon target and the resulting interference pattern is viewed on a phosphorus screen.

The graphitized carbon is not completely crystalline but consists of crystal sheets in random orientations. Therefore, the constructive interference patterns will be seen as bright circular rings. For the carbon target, two rings (an outer and inner, corresponding to different crystal planes) will be seen, corresponding to two spacings between atoms in the graphite arrangement (see Fig. 1.3).
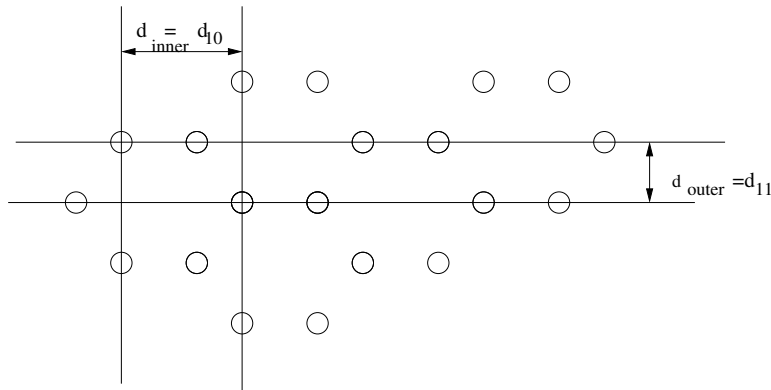
Figure 1.3: Spacing of carbon atoms. Here subscripts *10* and *11* correspond to the crystallographic directions in the graphite crystal.

Data acquisition

Acceptable power supply settings:

Filament Voltage	V_F	6.3 V ac/dc (8.0 V max.)
Anode Voltage	V_A	1500 - 5000 V dc
Anode Current	I_A	0.15 mA at 4000 V (0.20 mA max.)

1. Switch on the heater and wait one minute for the oxide cathode to achieve thermal stability.

2. Slowly increase V_a until you observe two rings appear around the direct beam. Slowly change the voltage and determine the highest achievable accelerating voltage, and the lowest voltage when the rings are visible.

3. Measure the diffraction angle θ for both inner and outer rings for 5-10 voltages from that range, using the same thin receipt paper (see procedure below). Each lab partner should repeat these measurements (using an individual length of the thin paper).

4. Calculate the average value of θ from the individual measurements for each voltage V_a. Calculate the uncertainties for each θ.

Measurement procedure for the diffraction angle θ

To determine the crystalline structure of the target, one needs to carefully measure the diffraction angle θ. It is easy to see (for example, from Fig. 1.1) that the diffraction angle θ is 1/2 of the angle between the beam incident on the target and the diffracted beam to a ring, hence the 2θ appearing in Fig. 1.4. You are going to determine the diffraction angle θ for a given accelerated voltage from the approximate geometrical ratio

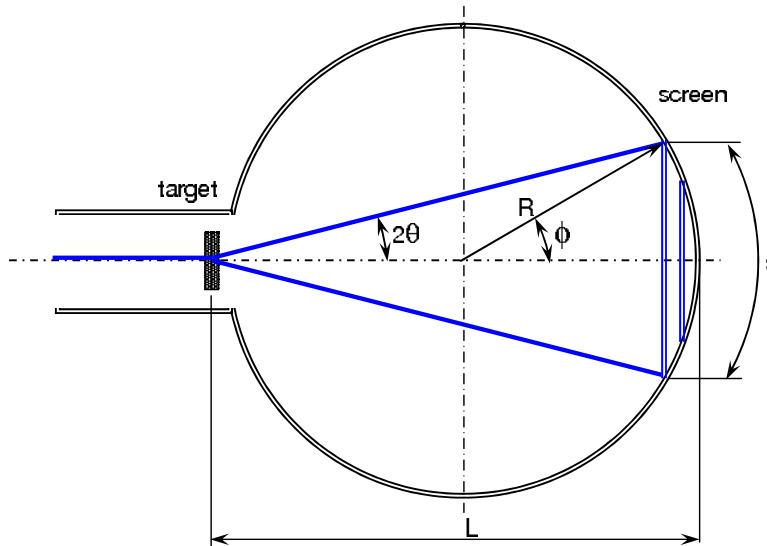$$L\sin 2\theta = R\sin \phi, \tag{1.4}$$

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Figure 1.4: Geometry of the experiment.

where the distance between the target and the screen $L = 0.130$ m is controlled during the production process to have an accuracy better than 2%. *Note, this means that the electron tubes are not quite spherical.*

The ratio between the arc length s and the radius of the curvature for the screen $R = 0.066$ m gives the angle ϕ in radians: $\phi = s/2R$. To measure ϕ carefully place a piece of thin receipt paper on the tube so that it crosses the ring along the diameter. Mark the position of the ring for each accelerating voltage, and then remove the paper and measure the arc length s corresponding to each ring. You can also make these markings on masking tape placed gently on the tube.

Data analysis

Use the graphical method to find the average values for the distances between the atomic planes in the graphite crystal d_{11} (outer ring) and d_{10} (inner ring). To determine the combination of the experimental parameters that is proportional to d, one needs to substitute the expression for the electron's velocity Eq.(1.1) into the diffraction condition given by Eq.(1.2):

$$\sin\theta = \frac{h}{2\sqrt{2m_e e}}\frac{1}{d}\frac{1}{\sqrt{V_a}} \tag{1.5}$$

Make a plot of $\sin\theta$ (y-axis, with uncertainty) vs $1/\sqrt{V_a}$ (x-axis) for the inner and outer rings (both curves can be on the same graph). Fit the linear dependence and measure the slope for both lines. From the values of the slopes find the distances between atomic layers d_{inner} and d_{outer}.

Compare your measurements to the accepted values: $d_{inner} = d_{10} = .213$ nm and $d_{outer} = d_{11} = 0.123$ nm.

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Seeing with Electrons

The resolution of ordinary optical microscopes is limited (the diffraction limit) by the wavelength of light ($\approx$ 400 nm). This means that we cannot resolve anything smaller than this by looking at it with light (even if we had no limitation on our optical instruments). Since the electron wavelength is only a couple of angstroms (10^{-10} m), with electrons as your "light source" you can resolve features to the angstrom scale. This is why "scanning electron microscopes" (SEMs) are used to look at very small features. The SEM is very similar to an optical microscope, except that "light" in SEMs is electrons and the lenses used are made of magnetic fields, not glass.