Lecture 26

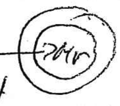
Collapse – Hydrodynamic eq.

~~Hydrostatic~~ time dependense.

$$\rho \frac{d^2 r}{dt^2} = - \frac{G M_r \rho}{r^2} - \cancel{\frac{dP}{dr}}$$

assuming it small
in other words free collapse

M_r = M of inner part = const



$$\Rightarrow \frac{d^2 r}{dt^2} = - \frac{G M_r}{r^2} = - \frac{G \left(\frac{4\pi}{3} r_0^3 \rho_0\right)}{r^2}$$

$$\frac{dr}{dt} = v$$

$$v \times \left| \frac{dv}{dt} = - G \frac{M_r}{r^2} \right| \times v$$

$$\frac{v\,dv}{dt} = - G \frac{M_r}{r^2} v = - G \frac{M_r}{r^2} \frac{dr}{dt}$$

$$\int dt \times \left(\frac{1}{2} \frac{d(v^2)}{dt} = + G M_r \frac{d\left(\frac{1}{r}\right)}{dt} \right)$$

$$\frac{1}{2} v^2 = + G M_r \frac{1}{r} + C$$

$v = 0$ when $r = r_0$

$$C = - G \frac{M_r}{r_0}$$

$$\frac{1}{2} v^2 = + G M_r \left(\frac{1}{r} - \frac{1}{r_0} \right)$$

$$v = + \ominus \sqrt{2 \frac{G M_r}{r_0} \left(\frac{r_0}{r} - 1 \right)}$$

$\ominus$ ← fall down case

$$v = \frac{dr}{dt} = -\sqrt{\frac{2GM_r}{r_0}\left(\frac{r_0}{r}-1\right)}$$

$$r_0\,\frac{d(r/r_0)}{dt} = -\sqrt{\frac{2GM_r}{r_0}\left(\frac{r_0}{r}-1\right)}$$

$$r/r_0 = \theta \qquad r\in(0,r_0) \rightarrow \theta = (0,1)$$

$$-\frac{d\theta}{\underbrace{\sqrt{2G\frac{M_r}{r_0^3}}}_{\parallel}\sqrt{\left(\frac{1}{\theta}-1\right)}} = dt$$

$$\sqrt{\frac{8\pi}{3}G\rho_0} = \chi$$

$$-\frac{1}{\chi}\frac{d\theta}{\sqrt{\frac{1}{\theta}-1}} = -\frac{1}{\chi}\frac{\sqrt{\theta}\,d\theta}{\sqrt{1-\theta}} = dt$$

$$\theta = \cos^2\zeta \Rightarrow +\frac{1}{\chi}\frac{\cos\zeta\;2\cos\zeta\,\cancel{\sin\zeta}\,d\zeta}{\cancel{\sin\zeta}} = dt$$

$$\cos^2\zeta\,d\zeta = +\frac{\chi}{2}\,dt$$

$$\frac{1+\cos 2\zeta}{2}\,d\zeta = \frac{\chi}{2}\,dt \Rightarrow \boxed{\zeta+\frac{1}{2}\sin 2\zeta\,\Big|_0^{\zeta_{final}} = \frac{\chi}{\cancel{2}}t}$$

$$\zeta+\frac{1}{2}\sin 2\zeta\,\Big|_{r=r_0 \Leftrightarrow \zeta=0}^{r=0 \Rightarrow \zeta=\pi/2} = \chi\cdot t + C_2 \qquad C_2 = 0$$

since $\zeta = 0$ since at $t=0$ $r=r_0$

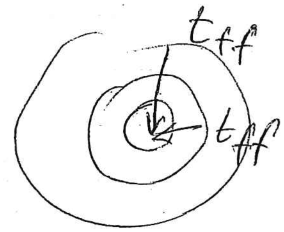
$$\Rightarrow \quad \frac{\pi}{2} = \chi t$$

$$t_{\substack{\text{free}\\\text{fall}}} = \frac{\pi}{2\chi} = \frac{\pi}{2\sqrt{\frac{8\pi}{3} G\rho_0}}$$

$$\boxed{t_{ff} = \sqrt{\frac{3\pi}{32\, G\rho_0}}}$$

Homologous collapse time

Note does not depend on initial size so all layers (with the same ρ) will arrive to center at the same time



This will not be true if there we original density distribution. For example if center is more dense it will collapse first.

Overall it cannot be the description of the whole process since we compress to '0' radius which require infinite densities let's see how long does it take to collapse inter stellar medium (ISM) Cloud.

We will discuss dense clouds which are prone to collapse $\Updownarrow$ globules $\Leftrightarrow$ $M = 1 \div 10^3 M_\odot$

They consist mostly of Molecular hydrogen H_2 with densities $n_{H_2} = 10^{10} m^{-3}$

$$\rho_0 = 2 m_H \cdot n_{H_2} = 2 \cdot 1.67 \cdot 10^{-27} \cdot 10^{10} \simeq$$

$$\simeq 3.3 \cdot 10^{-17} \frac{kg}{m^3}$$

Their overall mass $\approx 10 M_\odot$

$$t_{ff} = \sqrt{\frac{3\pi}{32 \cdot G \cdot \rho_0}} = \sqrt{\frac{3\pi}{32 \cdot 6.67 \cdot 10^{-11} \cdot 3.3 \cdot 10^{-17}}}$$

$$= 1.15 \cdot 10^{13} s \approx 3.7 \cdot 10^5 \text{ years}$$

Much smaller than Kelvin-Helmholtz time scale

if we want to see how r/r_0 depends on time

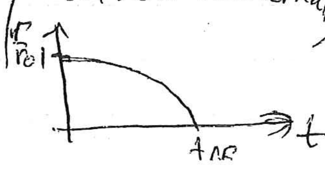
$$\zeta + \frac{1}{2}\sin 2\zeta \Big|_0^{\zeta(r)} = \xi \chi t$$

$$\zeta(r) \Rightarrow \cos^2\zeta = r/r_0 \Rightarrow \zeta(r)$$

$$\boxed{\zeta(r) + \frac{1}{2}\sin 2\zeta(r) = \chi t(r)}$$

can be solved numerically

$$r(t)$$



Star luminocity

Recall Kelvin-Helmholtz energy time frame

$$E = -\frac{3GM^2}{10R}$$

ΔE for $R_J \to R_\odot$, for the t_{ff} would imply that they outshine sun by orders of magnitude

MW: Calc. how much

See bunch of typical Nebulas / clouds on web site.

Globules where star form,
Dark means $\Rightarrow$ dense
Emission - due to highlight of inner stars
Reflection - due to light sourse at the side