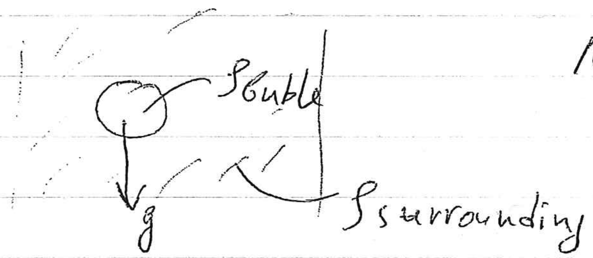
lecture 24 ~~stel~~ gravitational mechanism of stellar pulsation

g-mode — buble slaushing



Note that $P^{(b)} = P^{(s)}$ otherwise buble would compess or expand untill this condition is met

Buoyant force

$$f_{net} = (-\rho^{(b)} g + \rho^{(s)} g) V = (\rho^{(s)} - \rho^{(b)}) g V$$

gravity pull down

once buble displaced dr
we write Taylor expansion

$$\rho^{(s)} = \rho^{(s)}_{initial} + \frac{d\rho^{(s)}}{dr} \delta r$$

$$\rho^{(b)} = \rho^{(b)}_{initial} + \frac{d\rho^{(b)}}{dr} \delta r$$

assuming that $\rho^{(s)}_{initial} = \rho^{(b)}_{initial}$ i.e. they have the same starting point from which buble emerge

$$f_{net} = g\left(\frac{d\rho^{(s)}}{dr} \delta r - \frac{d\rho^{(b)}}{dr} \delta r \right) V =$$

$$= \left(\frac{d\rho^{(s)}}{dr} \delta r - \frac{d\rho^{(b)}}{dP^{(b)}} \cdot \frac{dP^{(b)}}{dr} \delta r \right) g V$$

we require <u>adiabatic</u> process for buble

$$PV^{\gamma} = const$$

since m of the buble is a const,
i.e. we have mass conservation

$$P\left(\frac{V}{m}\right)^{\gamma}_{=V_\rho} = const_2$$

$$d\left(P\rho^{-\gamma}\right) = dP\,\rho^{-\gamma} + (-\gamma)P\rho^{-\gamma-1}d\rho = 0$$

$$\frac{d\rho}{dP} = \frac{\rho^{-\gamma}}{+\gamma\rho^{-\gamma-1}P} = \frac{\rho_i^{(b)}}{\gamma P_i^{(b)}}$$ recall that this is about buble

start cond.

now we can plug it to the fnet eq.

$$f_{net} = Vg\left(\frac{d\rho^{(s)}}{dr} - \frac{\rho_i^{(b)}}{\gamma P_i^{(b)}}\frac{dP^{(b)}}{dr}\right)\delta r$$

since $P^{(b)} = P^{(s)}$ to maintain the buble
and $\rho_i^{(b)} = \rho_i^{(s)}$ we can drop
'b' and 's' subscript, keeping in mind
that it is actually 's' everywhere now

$$f_{net} = \underbrace{Vg\left(\frac{\rho}{\rho}\frac{d\rho}{dr} - \frac{\rho}{\gamma P}\frac{dP}{dr}\right)}_{\text{some coef.}}\delta r = V\underbrace{\left(\frac{1}{\rho}\frac{d\rho}{dr} - \frac{1}{\gamma P}\frac{dP}{dr}\right)}_{A}\rho g\,\delta r$$

$$\boxed{f_{net} = Vg A\rho\,\delta r}$$

$$\boxed{A = \frac{1}{\rho}\frac{d\rho}{dr} - \frac{1}{\gamma P}\frac{dP}{dr}}$$

if $A < 0$ we have a restoring force

think about  So our buble will go ~~here~~ back to equilibrium and thus oscillate around this point.

Since $\rho V = m$ of the buble

$$f_{net} = ma = mgA\delta r$$

$$a = \ddot{(\delta r)}$$

get again equation of harmonic oscillator

$$\boxed{\omega_{g\,mode} = \sqrt{-gA} = \sqrt{\left(\frac{1}{\gamma P}\frac{dP}{dr} - \frac{1}{\rho}\frac{d\rho}{dr}\right)g}}$$

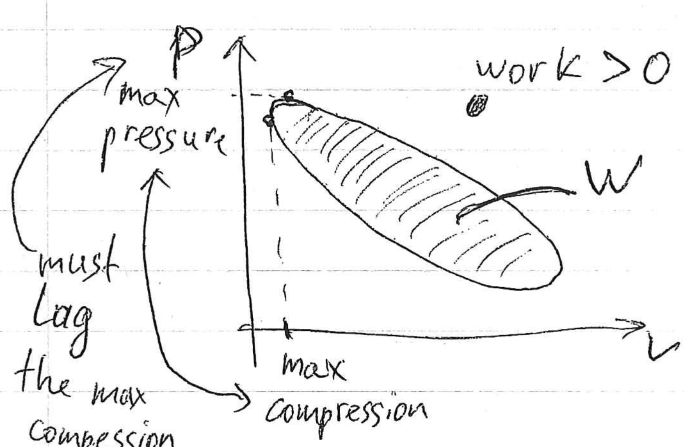
$$T = \frac{2\pi}{\omega}$$

Note that if <u>$A > 0$</u> then there is no restoring force. Once the buble start to move up it will keep going to do it $\Rightarrow$ ~~convection~~ <u>convection condition</u>

Oscillation Driving mechanism

It is not enough to have conditions for oscillations, some mechanism must drive this oscillation since otherwise oscillation energy will dissipate and oscillation will seize.

On top of it whatever the source of energy (fusion) it must be applied right to do a positive work on a slice of a star.



work > 0 if we go and negative if we do direction

$$W = \oint P\,dV$$

Opacity effects, κ and γ - Mechanism

κ ← absorption coef.

Generally $\kappa \sim \frac{\rho}{T^{3.5}}$

We need κ - to be increased with T
can be achieved in partially ionization zone if ~~increase of T~~ injection of heat goes not to kinetic energy of atoms (and thus increase of T) but to increased ionization and thus ↗ κ

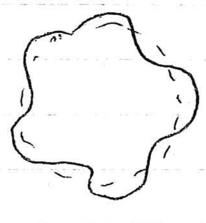
This will introduce delay:
Max pressure will happen after maximum compression

Additionally if some region is heated less during compression (i.e. T is smaller than surrounding)
then heat additionally will go to such region ; γ - mechanism

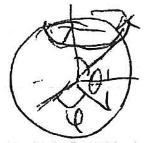
Partial ionization zones thus acts like a pistons. and their location in star defines what mode will oscillate if any

This in turn depends on star temperatures
see fig. 14.14

p-modes. Going back to pressure governed mode. We discussed motion of the sperical layer as a whole, but we can also observe 'riples' on the surface or in dipper part of the star



this described by spherical harmonic function $Y_l^m(\theta, \varphi)$



$l = 0, 1, 2, 3 \ldots$

$m = -l, \ldots 0, \ldots, l$ overall $2l+1$ values

$Y_0^0(\theta,\varphi) = K_0^0$ (const)

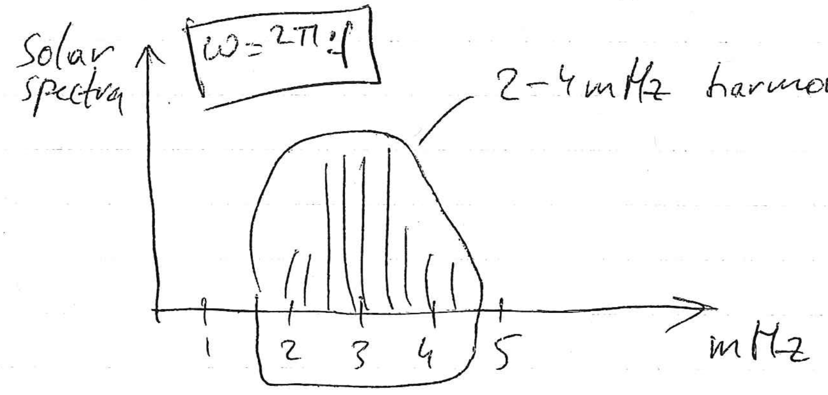
$Y_1^0(\theta,\varphi) = K_1^0 \cos\theta$

$Y_1^{\pm 1}(\theta,\varphi) = K_1^{\pm 1} \sin\theta e^{i\varphi}$

wavelenglt of oscillation $\lambda = \dfrac{2\pi R}{\sqrt{l(l+1)}}$

$$f_l = \frac{v_{sound}}{\lambda} = \sqrt{\frac{\gamma P}{\rho}} \frac{\sqrt{l(l+1)}}{2\pi R}$$

distance from center not the star radius !



3 mHz $\approx$ 300 sec = 5 min oscillation