Lecture 17

(P1)

Voigt profile.

$$K_f = \int K_{natural} \cdot K_v(v)\, dv$$

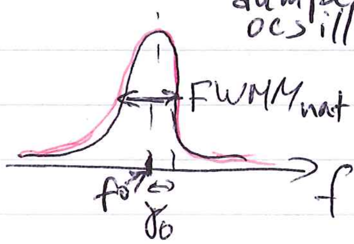
Convolution of Doppler and natural lineshapes

$K_{natural}$: $K_{nat}(f - f\frac{v}{c})$

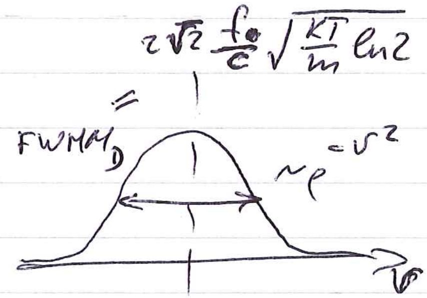
K_v: speed

$$K_{natural}(f) = K_0 \frac{1}{\pi} \frac{\gamma_0^2}{\gamma_0^2 + (f - f_0)^2} \Longleftrightarrow \text{lorentzian shape}$$

dumped oscillator



$$2\gamma_0 = FWMM \Longleftrightarrow K_0(\gamma_0 = f) = \frac{K_0}{2}$$



$K_v \Rightarrow$ Doppler profiler

$$K_v = K_D \cdot e^{-\frac{mv^2}{2KT}}$$

K_D: normalization

$$K_D = \left(\frac{m}{2\pi KT}\right)^{1/2} \Longleftrightarrow \boxed{\int K_v\, dv = 1} \quad \text{velocity distribution}$$

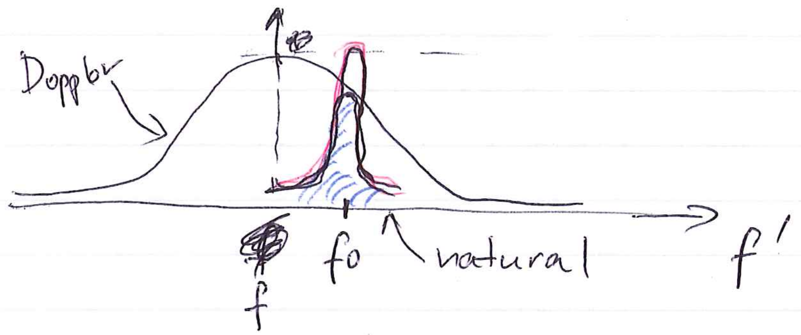
$$K_f = \int K_{nat}\left(f - f\frac{v}{c}\right) \cdot e^{-\frac{mv^2}{2KT}}\, dv$$

$$= \int K_{nat}(f - \Delta f)\, e^{-(\Delta f / g(T))^2}\, d(\Delta f)$$

$$\Delta f = f\frac{v}{c} \Longrightarrow v = \frac{\Delta f}{f} \cdot c$$

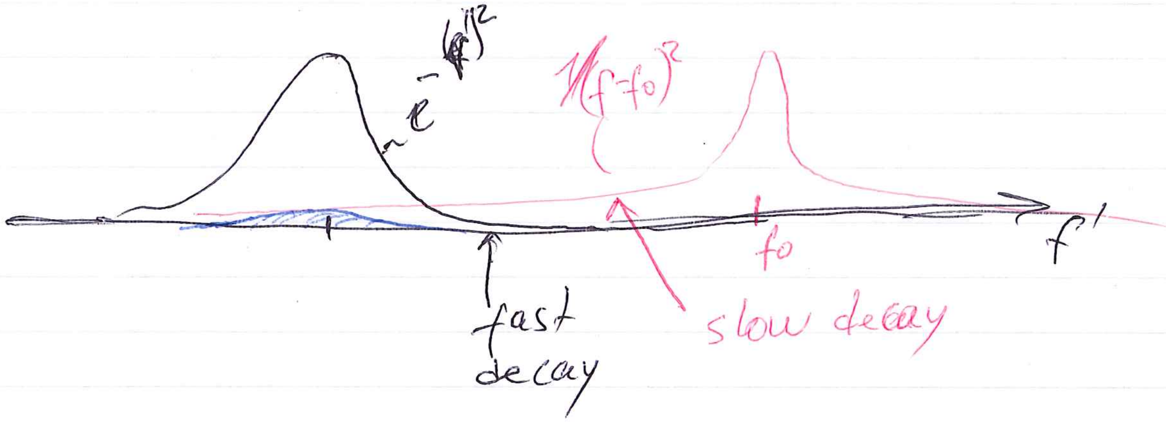
$$\Delta f \to f'$$

$$= \int K_{nat}(f - f')\, e^{-(f'/g(T))^2}\, df'$$



$$\begin{cases} (f - f_0) < FWMM_D, & \text{we treat natural line as delta function } \delta(f - f_0) \\ K_f \approx K_{nat}\, e^{-(f')^2/g(T)} & \end{cases}$$

$(f - f_0) > FWMM_D$

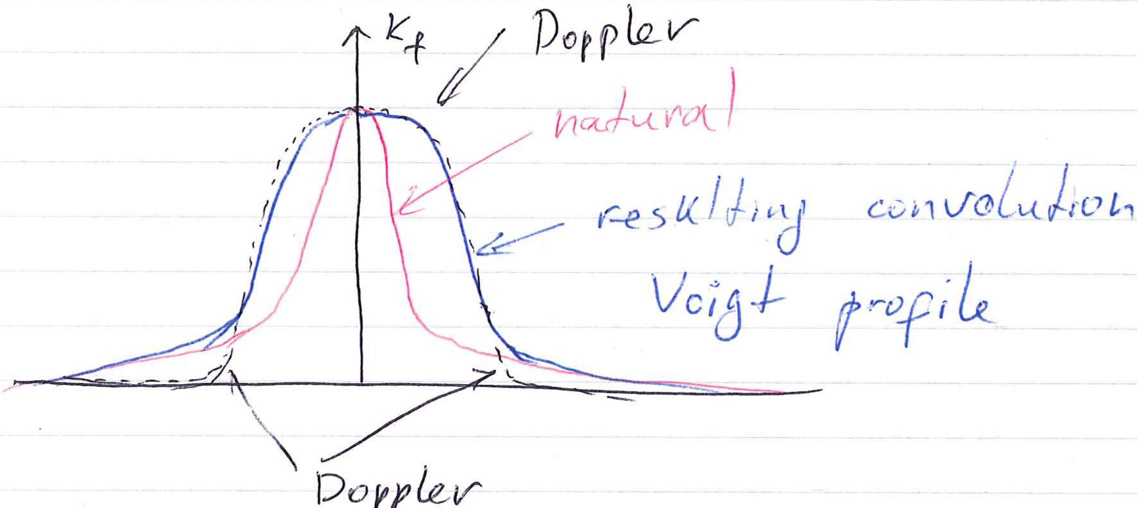


Now we treat Doppler profile as delta functio at $f' = 0$

$$\int K_{nat}(f - f')\, \delta(f')\, df' =$$

$$= K_{nat} \frac{\gamma_0^2}{\gamma_0^2 + (f - f_0)^2}$$

So κ_f ~~too~~ follows Doppler (Gaussian) shape at $(f - f_0) \ll FWHM_{Doppler}$ and Lorentzian shape at $(f - f_0) \gg FWHM_{Doppler}$



Now recall that

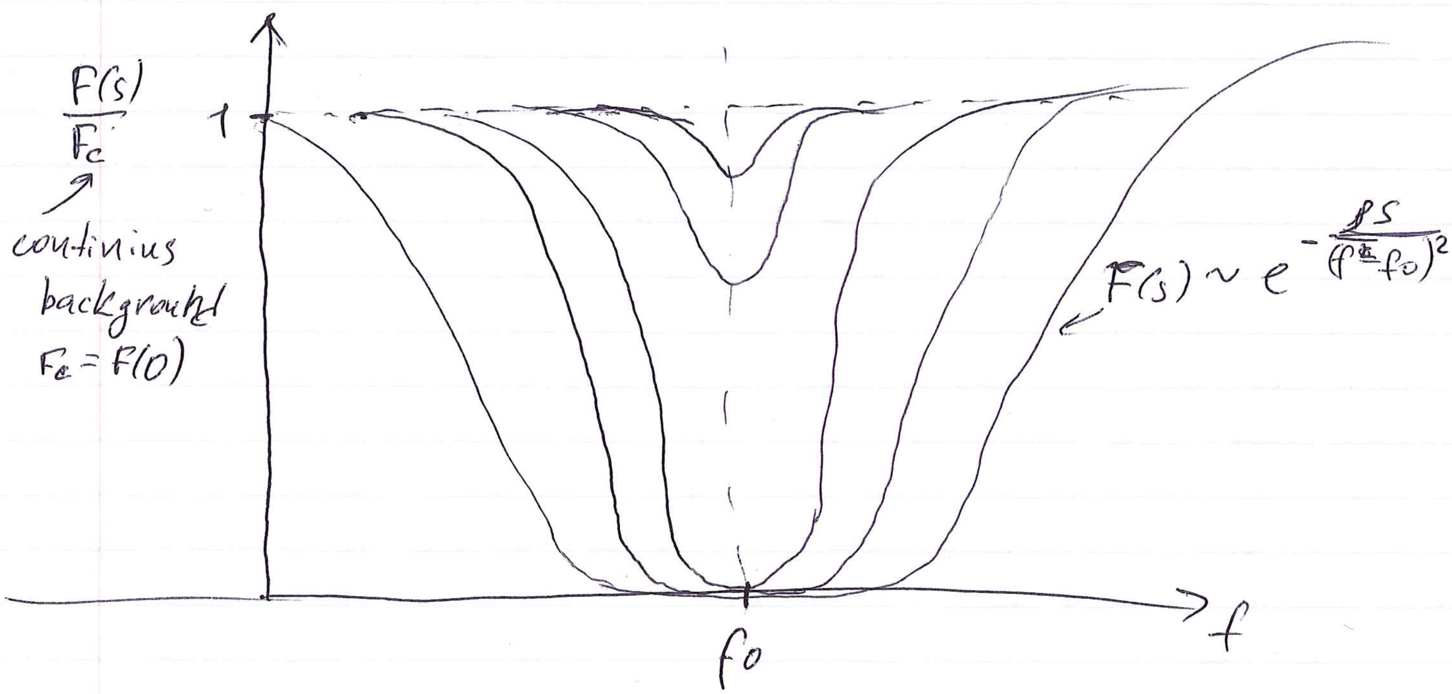
intensity $I(s) = I_0 \, e^{-\kappa_\lambda \rho \, ds} = I_0 \, e^{-\kappa_f \rho \, s}$

flux $\dfrac{F(s)}{F(0)} = \dfrac{I(s)}{I(0)} = e^{-\kappa_\lambda \rho \, s}$

When $\kappa_f \rho s \ll 1$ small optical depth τ

$\tau \ll 1$ at small ρ

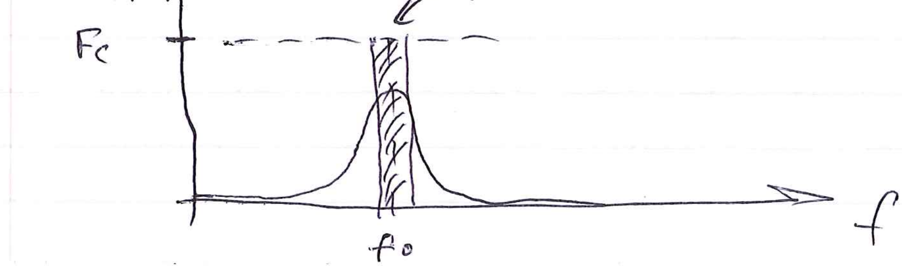
$$\frac{F(s)}{F(0)} = e^{-\kappa_f \rho s} \approx 1 - \kappa_f \rho s$$



Overall absorption =

$$\cancel{\int F(s,f)\,df} \quad \int \Big[\underbrace{F_c}_{=\,const} - F(s,f)\Big]\,df = F_c \cdot W$$

we can equal it to some haracteristic linewidth W times F_c



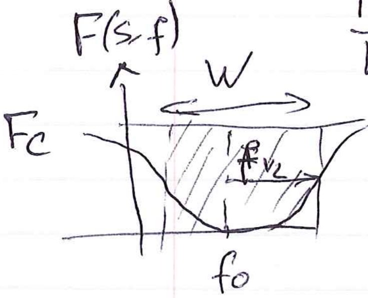
large density

$$K_x \sim \frac{\gamma_0^2}{\gamma_0^2+(f-f_0)^2}$$

const

$$|\gamma_0| < |(f-f_0)|$$

$$\frac{F(s)}{F(0)} = e^{-C_0 \frac{\gamma_0^2}{\gamma_0^2+(f-f_0)^2}\cdot \rho s} \approx e^{-\frac{c'\rho}{(f-f_0)^2}}$$



$$\cancel{\frac{F(s, f=f_{1/2})}{F(0)}}$$

$$\frac{F(s, \overset{(f-f_0)}{f = f_{1/2}})}{F(0)} = \frac{1}{2} = e^{-\frac{c'\rho}{(f-f_0)^2}}, \quad (f-f_0) = f_{1/2}$$

$$\frac{c\rho}{(f-f_0)^2} = \ln 2, \quad (f-f_0) = f_{1/2}$$

$$\cancel{f-f_0}$$

$$f_{1/2}^2 = \left(\frac{W}{2}\right)^2 = \frac{c}{\ln 2}\cdot\rho$$

$$\Rightarrow W \sim \sqrt{\rho} \sim \sqrt{N_{atoms}}$$

when $\boxed{K_f \rho \cdot s \gg 1}$

Recall that small ρ

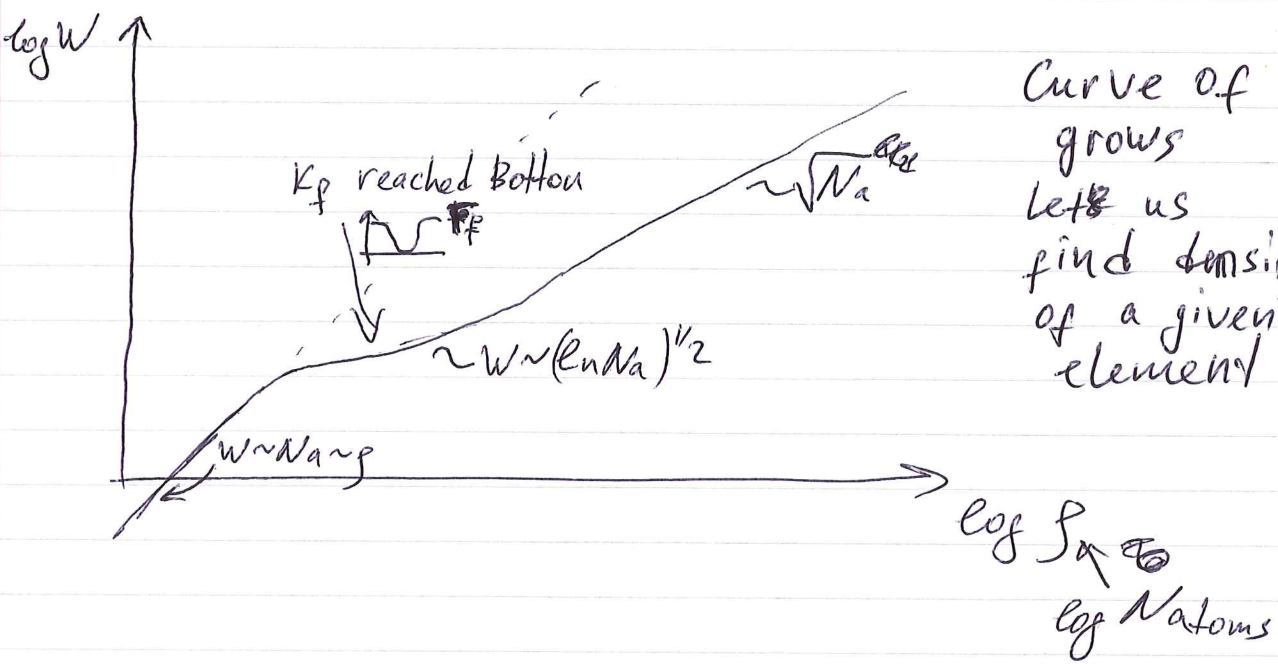
$$\frac{F_s}{F(0)} = 1 - K_\rho \rho S \qquad F(0) = F_c$$

$$F_c - F_s \approx F_c - F_c(1 - K_\rho \rho S) =$$

$$= F_c \cdot K_\rho \cdot \rho S$$ in vicinity of ρ_0

$$W = \frac{\int [F_c - F_s] \, d\rho}{F_c} \approx \left[\int K_\rho \, d\rho\right] \rho \cdot S$$

so width grows linearly with density at small ρ



Curve of W grows
Let us find density of a given element

At very large densities.

we will be affected by pressure broadening (PB) of the absorption line profile

$$\Delta f_{P.B} = \frac{1}{\pi} \frac{1}{\Delta t}$$ time between collision of atoms

$$\Delta t = \frac{\ell_{m.f.p}}{v_{mean}}$$

let's consider hydrogen

$$\ell_{mfp} = \frac{1}{n\sigma} = \frac{m_H}{\rho \cdot \sigma}$$ cross section $\pi (2a)^2$ radius of atom

$$v_{mean} = \sqrt{\frac{8KT}{\pi m_H}}$$ 3D corrections

$$\Delta f_{P.B} = \frac{1}{\pi} \frac{\sqrt{\frac{8KT}{\pi m_H}}}{m_H/\rho\sigma} = \frac{\rho\sigma}{\pi} \sqrt{\frac{8KT}{\pi m_H^3}}$$

$\sigma_{Hydrogen} = 3.52 \cdot 10^{-20}\ m^2$

at $T = 5800K$

$m_H = 1.6 \cdot 10^{-27}$

$K = 1.38 \cdot 10^{-23}\ J/K$

$$\Delta f_{p.B.} = \rho \cdot \frac{3.52 \cdot 10^{-20}}{\pi} \sqrt{\frac{8 \cdot 1.38 \cdot 10^{-23} \cdot 5800}{\pi \cdot (1.6 \cdot 10^{-27})^3}}$$

$$= \rho \cdot 7.9 \cdot 10^{10} \left[\frac{Hz}{kg/m^3} \right]$$

$$= \rho \cdot 80 \cdot \frac{GHz}{kg/m^3}$$

Recall that in the Sun photosphere

$\rho \simeq 2.1 \cdot 10^{-4} \frac{kg}{m^3}$

So ~~$\rho \cdot 80$~~ $\Delta f_{p.B} = 2.1 \cdot 10^{-4} \cdot 80$ GHz $=$

$= 16$ MHz

much less than doppler governed linewidth of ~ 12 GHz (see prev. lecture)

But it is comparable with natural linewidth $\Delta t = 10^{-8}$ $\Delta f = \frac{1}{\pi} \frac{1}{10^{-8}} =$

$\simeq 3.2$ MHz

which means that we need to replace γ_0 with pressure broadening one

$$\Delta f = \frac{1}{\pi} \left(\frac{1}{\Delta t_{natural}} + \frac{1}{\Delta t_{p.B}} \right)$$