## Homework 12

Prerequisites: Read chapter 10. Skip (if you wish) sections 10.9 Euler angles and 10.10 Motion of Spinning Top.

## Problem 1 (4 points):

Find tensor of inertia (be smart about axes choice) of a solid spheroid with an uniform density and total mass $M$. Spheroid surface is described by the following shape:

$$
\begin{equation*}
\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}+\frac{z^{2}}{c^{2}}=1 \tag{1}
\end{equation*}
$$

## Problem 2 (4 points):

What is the rate of precession of an axially symmetrical top in the Earth gravitational field. The top is a spheroid on an axle passing through its long axis of symmetry. $c>a, b$ and $a=b$. The distance between the support point and the CM of the top is $d>c$. The top is spinning with angular velocity $\omega$ initially directed along the axle.

## Problem 3 (4 points):

We discussed the Dzhanibekov Effect (review section 10.7 for crucial equations). Some people make claims that the Earth flipped in the past due to this effect and this is why we had massive extinction of species. These people also claim that sooner or later it will happen again. One of the posted movies even has a real demo with with a spherically looking object which does the flip in space.
Shall we worry that the Earth flips due to this effect? Why? Assume that the Earth is an isolated spinning body with a spheroid shape. Do some research and find which dimension is shorter? What about other two? Also, assume (incorrectly) that the Earth has uniform density.

## Problem 4 (4 points):

Let's model pliers as two straight rods (with negligible cross section size) each of length $L$ and mass $M / 2$. The pivot point is $L / 4$ distance away from the end of a rod. Assume that pliers lay on a table in 'xy' plane and axis 'z' sticks out of the table and passes through the axle of pliers, the 'y' axis is the symmetry axis and point in direction from handles to jaws.

- Find the center of mass position of pliers with respect to the pivot point as a function of angle between jaws $(\theta)$.
- Where is the origin of principal axes and what is their direction? Use symmetry as much as you can.
- Find the inertia tensor with respect to the primary axes as function of the opening angle $\theta$.
- Review the video with the spaceman playing with pliers in space. What is the range of $\theta$ at which pliers will experience instability for rotation along $x$ axis.


## Problem 5 (4 points):

Someone extended an axle of pliers from the above problem and suspended pliers at this axle. What is the period of their oscillatory motion around this axle in the Earth field of gravity? Note: the period is the function of $\theta$. Assume that pliers can maintain the opening angle.

