Homework 03 - it is all about α

Problem 1 (4 points):

Assume that $m_1 \ll m_2$. Show that a change of the reduced mass due to changes in m_1 (as long as the above condition is satisfied) does not change parameters of the orbit, i.e. the semimajor axis size (a) and the eccentricity (e). Think about the case of spacecraft which lost some air, which quickly dispersed.

Problem 2 (5 points):

As far as we know, the gravitational constant is indeed a constant but what if it is abruptly changed: $G_{new} = \alpha G_{old}$. Find new parameters of the orbit a and e. Assume that initially it was a circular orbit and $m_1 \ll m_2$.

Problem 3 (2 points):

Forget fancy trajectory math. What should be the new trajectory if $G_{new} = 0$, i.e. $\alpha = 0$ in the above problem? Why so?

Now prove it, show that the modified trajectory is indeed matches it.

Problem 4 (4 points):

Solve problem 8.29 about a half of the Sun mass disappearance. Assuming that the Earth was moving on a circular orbit with the semimajor axis value a and the eccentricity $e = 0$. What is the new value of the eccentricity? How long is one new year?

Hint: if you did the second problem in a symbolic form, this problem is almost the same.

Problem 5 (5 points):

You are the commander of the spacecraft which need to dock with the international space station moving along the circular orbit with the speed $v_0 = 7.67$ km/s . Due to "unforeseen" weather conditions the launch was delayed and now you are on the same orbit but behind the space station by $\phi_0 = \pi/100$ radians. The mass of the Earth is $M_E = 6 \times 10^{24}$ kg, the gravitational constant is $G = 6.67 \times 10^{-11}$ m^3/kg/s^2.

While you and the rest of your crew doing paper and pencil calculation trying to find the proper change of the angular component of the velocity $v_\phi = \alpha v_0$, some biology major cowboy, whose only job was to feed Drosophila flies, decided that he knows what to do. That person give the spacecraft a boost of v_ϕ so the $\alpha = 1.1$.

Calculate the time after which your spacecraft comes to the same radial separation with the Earth, i.e. back to the international station orbit.

Calculate the new angular separation between you and the space station after one period of your new orbit. Will you ever let this person outside of the cargo area?

Problem 6 Bonus (4 points):

What is the proper value of α (in the above problem) to meet with the space station at the shortest time? Should you speed up or slow down? Why so?