Homework 01

Prerequisites: Read chapter 8.

Problem 1 (4 points):

Prove that the reduced mass

$$\mu = \frac{m_1 \times m_2}{m_1 + m_2} \tag{1}$$

is indeed smaller than either m_1 or m_2.

Problem 2 (4 points):

Show that in the center of mass (CM) reference frame of a two-body system the total momentum is zero. Actually, this is true for multi-body system as well.

Problem 3 (4 points):

We have two bodies with known mass-ratio m_2/m_1.

The velocity of the first body has the following $\{x, y, z\}$ components in the CM reference frame $\{v_{1_x}, v_{1_y}, v_{1_z}\}$. Find the $\{x, y, z\}$ components of the second body velocity.

Problem 4 (8 points):

The expression for the potential energy of a two-body system is

$$U(r) = U_0 \times \left(\frac{1}{\sin(r)} + r^2 \right) \tag{2}$$

Assume that masses m_1 and m_2 are known.

What is the magnitude of the force acting on the first body? At some point of time, the first body starts moving from the rest along the positive y-axis direction (in the CM reference frame). What is the direction of the force acting on the second body?

Can you make a statement that the second body was at rest prior the discussed point of time? Why is it so?