Lecture 34

* Gravitational lensing
* travel time to/out of gravitational potential
* luminocity change.

Schwarzschild metric

$$(ds)^2 = \left(c\,dt\sqrt{1-\frac{2GM}{rc^2}}\right)^2 - \left(\frac{dr}{\sqrt{1-\frac{2GM}{rc^2}}}\right)^2 - (r\,d\theta)^2 - (r\sin\theta\,d\varphi)^2$$

$\frac{2GM}{c^2} = R_s$ Schwarzschild radius, recall expession for Black hole radius

where dr, dt are measured by remote observer

Another view on gravitational lensing

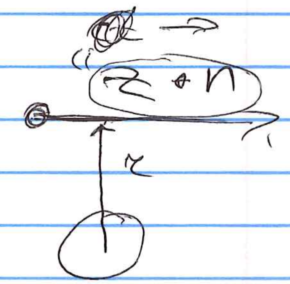
$\theta, \varphi = \text{const}$

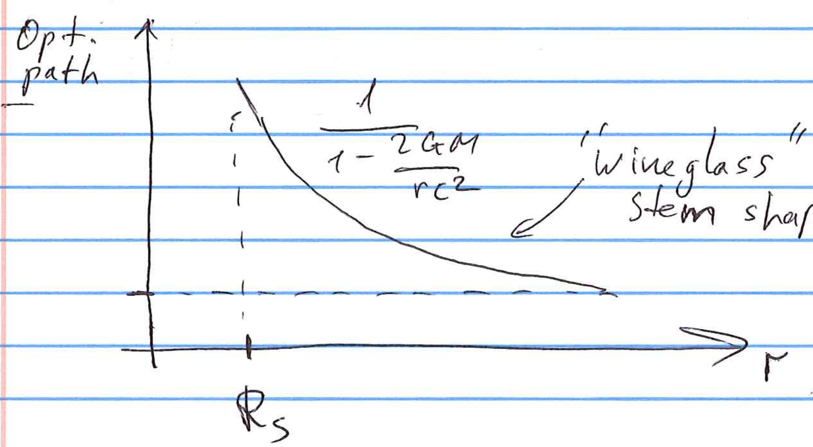
$(ds)=0$ for light

$$\frac{dr}{dt} = c\cdot\left(1-\frac{2GM}{rc^2}\right) = c/n \quad \text{"index of refraction"}$$

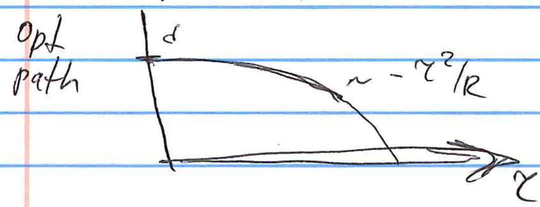
$$n = \left(1-\frac{2GM}{rc^2}\right)^{-1}$$

$$\text{optical path} = n\cdot \mathcal{Z}$$





Compare for a lense



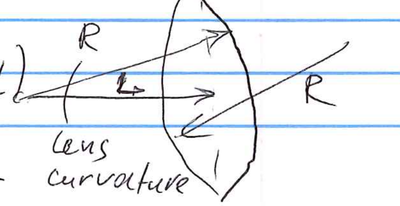
optical path

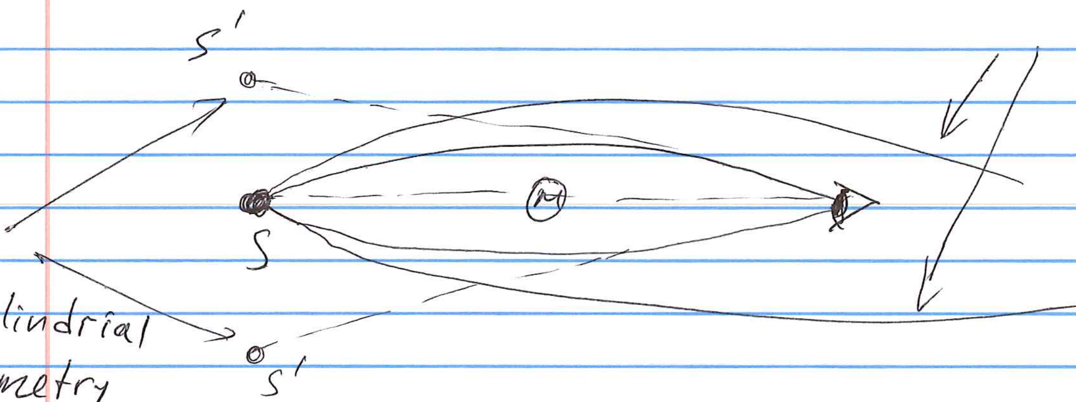
$$2\left(\sqrt{R^2-r^2}-L\right) =$$

$$= |r \ll r| =$$

$$= 2\left(R - \frac{1}{2}\frac{r^2}{R} - L\right)$$

$$\simeq d - r^2/R$$

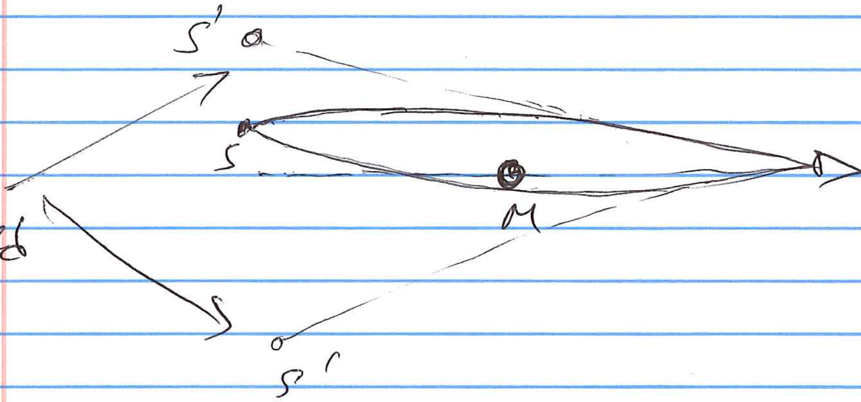




missing observer

due to cylindrial symmetry the image form a ring

If object not on axis of eye-mass line



Skewed images

But this also breaks cyllindrial symmetry so image does not look like a ring anymore light To B. h. or back

$$(ds)^2 = (c\,d\tau)^2 - (d\tilde{r})^2 =$$

$$= \left(c\,dt\sqrt{1-\frac{r_s}{r_E}}\right)^2 - \left(\frac{dr}{\sqrt{1-r_s/r_E}}\right)^2$$

for light $ds = 0$

$$\cancel{\frac{dr}{dt}} = \underbrace{\frac{d\tilde{r}}{c\,d\tau}}_{1 = \text{for falling photon}} = \frac{dr}{c\,dt\,(1 - r_s/r_E)}$$

$\downarrow\uparrow$

depends on direction

$$\int dt = \cancel{\int_{r_1}^{r_2} \frac{dr}{c(1-r/r_s)}} = \int_{r_1}^{r_2} \frac{\pm dr}{c(1-r_s/r)} =$$

$$= \int \frac{r\,dr}{c(r-r_s)} = \int \frac{1}{c}\,dr + \int \frac{r_s\,dr}{c(r-r_s)} =$$

$(+r_s - r_s)$

$$\boxed{\Delta t = \frac{r_1 - r_2}{c} + \frac{r_s}{c}\ln\left(\frac{r_2 - r_s}{r_1 - r_s}\right)}$$

if r_1 is $r_s = \frac{2GM}{c^2}$ $\quad \Delta t = \infty$

reverse is always true
light never reaches event horizon $r = r_s$

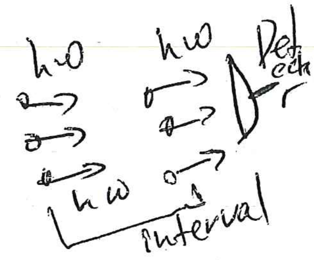
Gravity corrections

~~This~~ also results in luminocity change

$$L_\infty = L \cdot \left(1 - \frac{2GM}{Rc^2}\right)$$

near
Star ref frame

L!



two effects : time runs slower near mass meaning that photons are emited less often from our point of view

$$L_\infty = \cancel{\frac{\Delta E_R}{\Delta t_R}} = \cancel{\frac{\Delta E_R}{\Delta t_\infty}} \quad \frac{\Delta E_\infty}{\Delta t_\infty} =$$

$$= \frac{\Delta E_\infty}{\Delta t_R / \sqrt{1 - \frac{2GM}{Rc^2}}} =$$

second effect each photon has less energy

$$\frac{\nu_\infty}{\nu_R} = \sqrt{1 - \frac{2GM}{Rc^2}} = \frac{\Delta E_\infty}{\Delta E_R}$$

$$\boxed{L_\infty = L_R \cdot \left(1 - \frac{2GM}{Rc^2}\right)}$$

it also changes position λ_{max} thus T_{eff}
for most energetic λ