# lecture 20

(p1)

Last lecture summary.

To overcome coulomb barrier thermal protons (H-atoms) would need

$$\boxed{T \approx 10^{10} K}$$

We want to relate it to pressure in the center of a star which we find to be

$$\boxed{P_c = \frac{3}{8\pi} G \frac{M_\odot^2}{R_\odot^4}}$$

under constant $\rho$ assumption $= 1300 \frac{kg}{m^3}$

remember also that $P(r) \sim -\rho^2 \int_{R_\odot}^{r} r' dr'$

$\Rightarrow 1.3 \cdot 10^{14}$ Pa

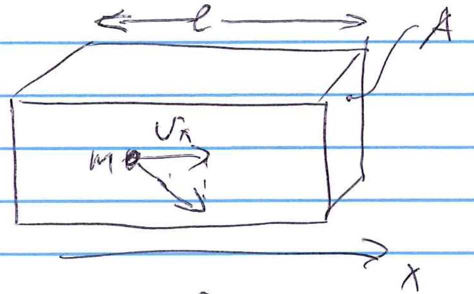
Since we know pressure, we should be able to find temperature of thermal ensemble creating this pressure.

Let's think what kind of particles are there

matter
i.e. atoms, nuclei, electrons, molecullas

photons
(light pressure)

Pressure in the box




$$F_x = \frac{\Delta P_x}{\Delta t} = \frac{m v_x - m(-v_x)}{\Delta t} =$$

$$= \frac{2 m v_x}{(2l/v_x)} = \frac{m v_x^2}{l}$$

round trip time

$$v_x^2 + v_y^2 + v_z^2 = v^2$$

average $v_x^2 = v_y^2 = v_z^2$

$$\Rightarrow v_x^2 = \frac{v^2}{3}$$

$$F_x = \frac{1}{3}\frac{m v^2}{l} \Rightarrow P = P_x = \frac{F_x}{A} = \frac{1}{3}\frac{m v^2}{l \cdot A} \Rightarrow$$

momentum

$$P = \frac{1}{3}\frac{m v^2}{V} = \frac{1}{3}\frac{1}{V} \cdot p v$$

pressure per particle

concentration

average velocity

$$N = n \cdot V$$

number of particles

$$\Rightarrow P_T = \frac{1}{3} n \, m\overline{v}^2 \cdot \alpha$$

friendly coef :)

momentum

$$P_{gas} = \int_{\substack{\text{all particles}\\ \text{momentum}}} \frac{1}{3}\, p \cdot v \; n_p \, dp$$

density $\frac{N_p}{V}$

Note that $\int_{all} n_p \, dp = \frac{N}{V}$

$$\int n_p \, dp = \int n_v \, dv = \Big/ \; v = \frac{p}{m} \Big/ =$$

$$= \int n_v \cdot \frac{dp}{m} = \int \frac{n_v}{m} \, dp$$

$$\Rightarrow \boxed{n_p = \frac{n_v}{m}}$$

$n_v$ is governed by Maxwell - Boltzmann distribution

$$n_v dv = n \left( \frac{m}{2\pi k T} \right)^{3/2} e^{-\frac{m v^2}{2kT}} \; 4\pi v^2 \, dv$$

$$HW \div \int_{\substack{\text{allpart}\\ \text{momentum}}} \frac{1}{3} \, p \cdot v \cdot n_p \cdot dp = \boxed{P_{gas} = n k T} \; \boxed{= \frac{\rho}{\mu m_H} k T}$$

$$N = \frac{N_{elect} + N_H + N_{He} + N_{other\ atoms}}{V}$$

$$\rho = \frac{m_e N_{elect} + m_H \cdot N_H + m_{He} \cdot N_{He} + \ldots}{V} = \frac{M}{V}$$

$$= \frac{N \cdot m_H \cdot \mu}{V_n} \Rightarrow n = \frac{\rho}{\mu m_H}$$

$$\boxed{\mu = \frac{\bar{m}}{m_H}}$$

$\boxed{\text{for Sun} \quad \mu \approx 0.62}$

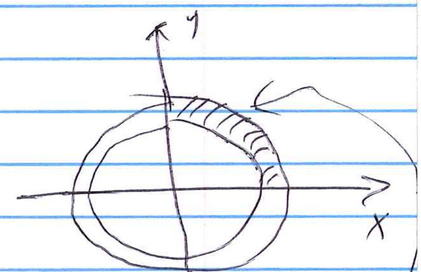
Some tricks to do M-B integral

$$\underbrace{\int_0^\infty d(e^{-x^2}\cdot x)}_{0} = \int_0^\infty -e^{-x^2}\, 2x^2\,dx + \int_0^\infty e^{-x^2}\,dX$$

$$\boxed{\int e^{-x^2} x^2\,dX = \frac{1}{2}\int_0^\infty e^{-x^2}\,dx}$$

$$\int_0^\infty e^{-x^2}dx \cdot \int_0^\infty e^{-y^2}dy = / x^2+y^2 = r^2 /$$

$$= \iint_0^\infty e^{-r^2}\,dx\,dy =$$




$$= \int_0^\infty e^{-r^2}\,\frac{2\pi r\,dr}{4}$$

we integrate over only one quadrant

$$= \int_0^\infty e^{-r^2}\,dr^2 \cdot \frac{\pi}{4} = \frac{\pi}{4}$$

$$= \left(\int e^{-x^2}dx\right)^2 = \frac{\pi}{4}$$

$\cancel{\sqrt{\pi}/2}$

$$\boxed{\int e^{-x^2} x^2\,dx = \frac{1}{2}\sqrt{\frac{\pi}{4}} = \frac{\sqrt{\pi}}{4}}$$

What about light/photon pressure?

Well, Pressure equation is the same

$$P_{ph} = \frac{1}{3}\int_0^\infty \underbrace{p}_{\frac{h\nu}{c}} \cdot \overset{c}{v}\, n_p\, dp =$$

energy per photon

$$= \frac{1}{3}\int_0^\infty h\nu\, n_p\, dp = \frac{1}{3}\int_0^\infty \underbrace{\overset{E_\nu}{h\nu} \cdot n_\nu\, d\nu}_{U = \int u_\nu d\nu = \int u_\lambda d\lambda}$$

energy of photon gas by definition

$$\boxed{P_{rad} = \frac{1}{3} U}$$

HW: $\boxed{P_{rad} = \frac{1}{3} a T^4}$, where $a = \frac{4\sigma}{c} = 7.56\cdot 10^{-16} \frac{J}{m^3 K^4}$

Stefan-Boltzmann constant

note that $\boxed{u_\lambda d\lambda = \frac{4\pi}{c} B_\lambda\, d\lambda}$

Black body radiation spectrum

P6

$$P = P_{gas} + P_{radiation} =$$

$$= \frac{\rho k T}{\mu m_H} + \frac{1}{3} a T^4$$

$\mu_{sun} = 0.62$, note $\mu = \frac{\bar{m}}{m_H}$

if ~~there~~ the hydrogen fully ionized

$$\cancel{\mu = \frac{1 \cdot m_H + 1 \cdot m_e}{2} \frac{1}{m_H} = \frac{1}{2}}$$

$$\mu = \underbrace{\frac{1 \cdot m_H + 1 \cdot m_e}{2}}_{\bar{m}} \frac{1}{m_H} = \frac{1}{2}$$

so with $\mu_{sun} \simeq 0.62$ it looks that hydrogen quite ionized

Ommiting $P_{radiation}$

$$T_c = \frac{P_c \cdot \mu m_H}{\rho \cdot k} = \frac{\overbrace{1.3 \cdot 10^{14}}^{P_c} \, 0.62 \cdot 1.67 \cdot 10^{-27}}{1.3 \cdot 10^{3} \; 1.38 \cdot 10^{-23}}$$

$\approx \underline{0.75 \cdot 10^7 K}$, which is way smaller than required $T_{fusion} = 10^{10} K$

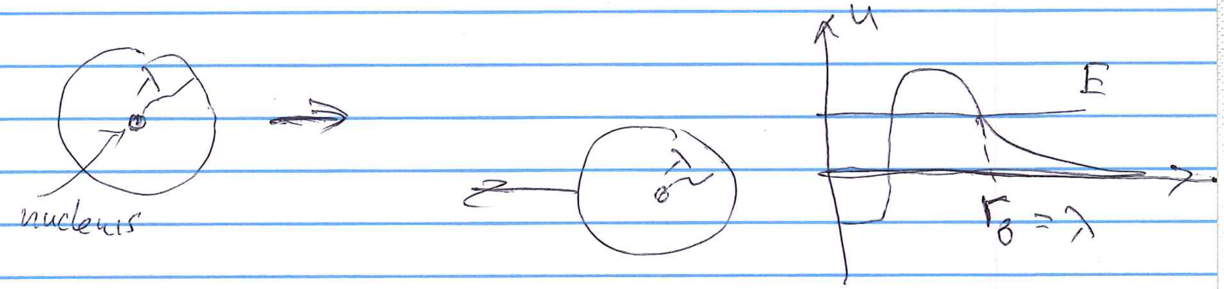
The improved pressure models give $P_c = 2.34 \cdot 10^{16} Pa$ but even then $T_c$ is not hot enough!

Since classical ~~mota~~ mechanics leaves us in suspence.

Let's have a look at Q.M.

Wave length of a particle

$$\lambda = \frac{h}{p}$$ (De Broglie wavelength)




So U repulsion at $r = \lambda$, should be equal to particle energy

$$\frac{K_e z_1 z_2 e^2}{r} = \frac{K_e z_1 z_2 e^2}{\lambda} = E = \frac{p^2}{2\mu} = \frac{(h/\lambda)^2}{2\mu}$$

$\mu$ reduced mass $= \frac{m_1 m_2}{m_1 + m_2}$

$$\lambda = \frac{h^2}{2\mu} \frac{1}{K_e z_1 z_2 e^2} =$$

$$= \frac{(6.6 \cdot 10^{-34})^2}{2 \cdot \frac{1.6 \cdot 10^{-27}}{2} \cdot 9 \cdot 10^9 \cdot 1 \cdot 1 \cdot (1.6 \cdot 10^{-19})^2}$$

$= 1.2 \cdot e^{-12}$ m ← much weaker condition than for nuclei strike $r_n \approx 10^{-15}$ m

$$K_c \cdot \frac{z_1 \cdot z_2 e^2}{\lambda} = \frac{3}{2} K_B T$$

$$T = \frac{K_c}{K_B} \cdot \frac{2}{3} \frac{z_1 z_2 e^2}{\lambda} = \frac{9\cdot 10^{9}}{1.38\cdot 10^{-23}} \cdot \frac{2}{3} \cdot \frac{(1.6\cdot 10^{-19})^2}{1.2\cdot 10^{-12}}$$

$$= 9.2\cdot 10^{6}\ K$$

So looks like $T_c$ in the star is enough for fusion