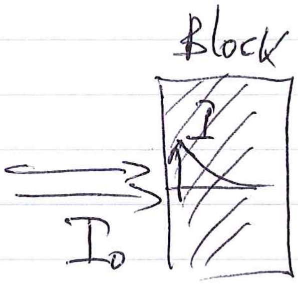
lecture 16

light source function

Imagine



Naively we expect nothing here

$I_\lambda \sim I_0 e^{-\kappa_\lambda \rho s} \to 0$

From other hand if energy delivered than it should go somewhere including forward.

Same with the stars they absorbe but they are shiny.

So we introduce a correction a "source"

$$dI_\lambda = \underbrace{-\kappa_\lambda \rho I_\lambda ds}_{\text{absorption}} + j_\lambda \rho ds$$

emission coef.

$$\underbrace{\frac{1}{\kappa_\lambda \rho} \frac{dI_\lambda}{ds}}_{d\tau_\lambda} = -I_\lambda + \frac{j_\lambda}{\kappa_\lambda \rho}$$

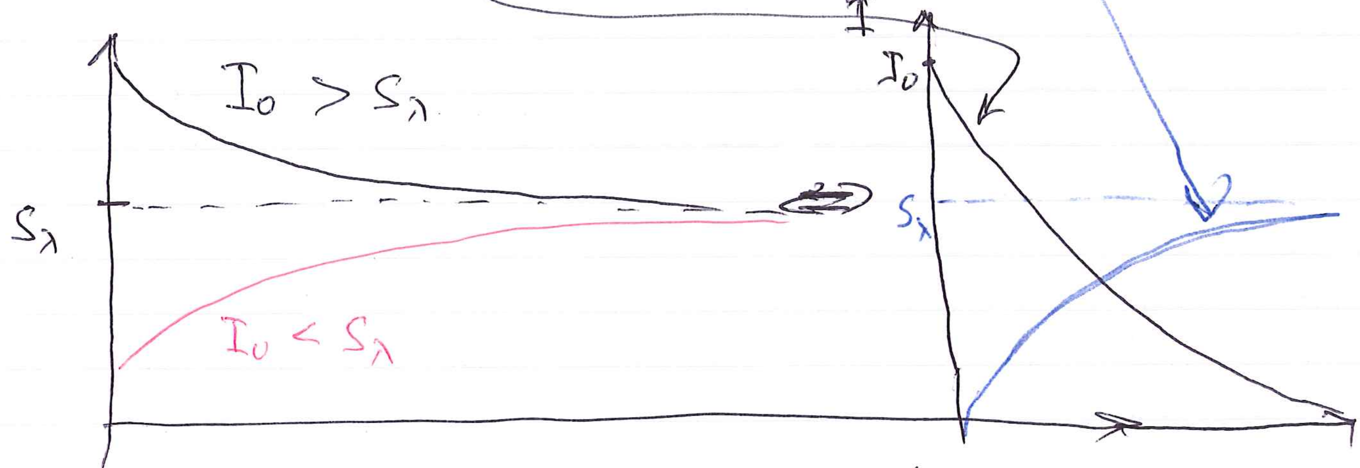
S_λ source

$$\frac{dI_\lambda}{d\tau_\lambda} = -I_\lambda + S_\lambda$$

Assuming K_λ = const and S_λ = const

$$+\frac{1}{k_\lambda \rho}\frac{dI_\lambda}{ds} = -I_\lambda + S_\lambda$$

$$I = I_0 e^{-k_\lambda \rho s} + S_\lambda\left(1 - e^{-k_\lambda \rho s}\right)$$



looks like feedback stabilization at S_λ level.

In equilibrium $\frac{dI_\lambda}{ds} = 0 \Rightarrow \boxed{I_\lambda = S_\lambda}$

So what is S_λ ?

We know that in Black body

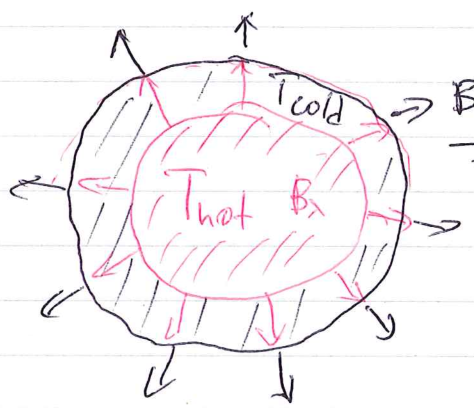
$I_\lambda = B_\lambda$ = and thus $\boxed{S_\lambda = B_\lambda}$

note this is "crazy" =
= specific intensity $\left[\frac{W}{m^2}\frac{1}{sr}\right]$

So we see if light propagates long enough ~~it~~ it will "forget" its initial intensity distribution and will be replaced with "source" function of the underlying medium!

$$I_\lambda = I_0 e^{-\kappa_\lambda \rho s} + S_\lambda (1 - e^{-\kappa_\lambda \rho s})$$

This is due to the fact that $\underline{\underline{\kappa_\lambda > 0}}$



$\rightarrow$ $\underline{\underline{B_\lambda(T_{cold})}}$!

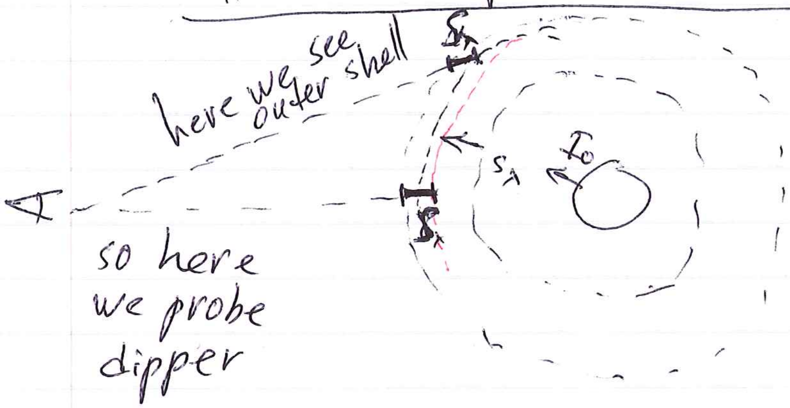
So when we look at the star we see properties of outer layer only, recall that for sun $\frac{1}{\kappa_\lambda \rho} \simeq 160\ km = 1.6 \cdot 10^5$

when $R_\odot = 7 \cdot 10^8\ m$

In reality ~~life~~ analysis is much harder

Since S_λ depends on position i.e $\neq$ const and also the $\kappa_\lambda \neq$ const

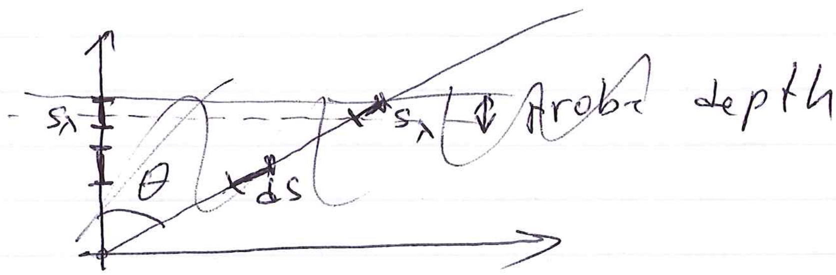
Still simple const model

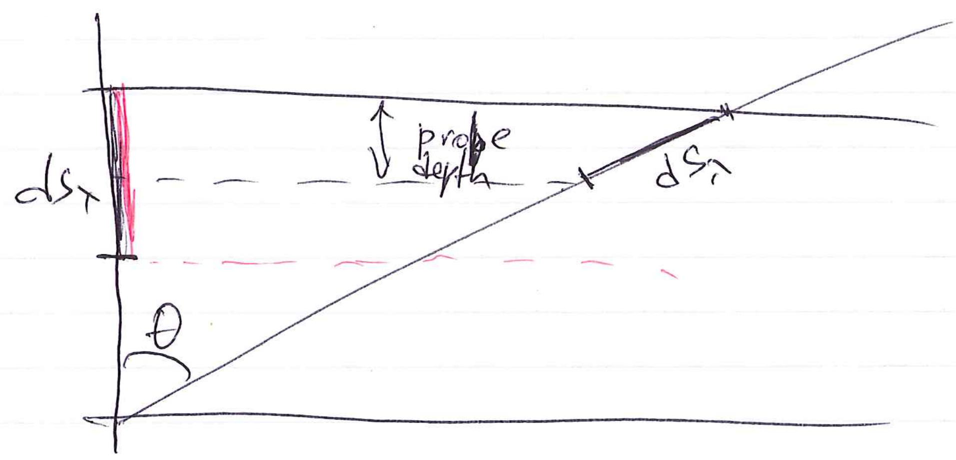


$$\boxed{S_\lambda = \frac{1}{\kappa_\lambda \rho}}$$

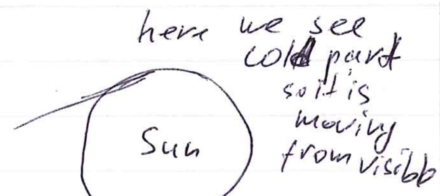
probing depth

$\Rightarrow e^{-\kappa_\lambda \rho S_\lambda} \simeq e^{-1}$



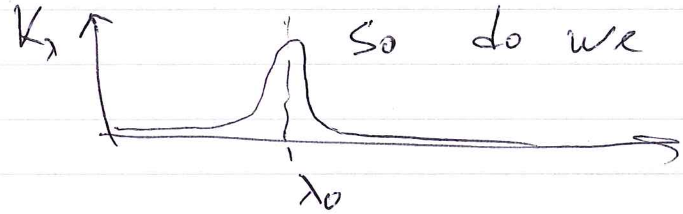


So when we look at the sun we have quite sharp edges though it is a gas object with no well defined edges



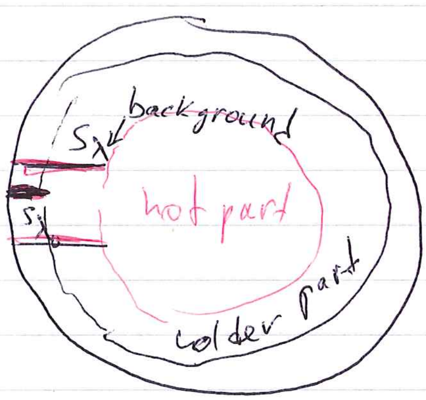
Another amusing fact

K_λ is a function of λ



So do we see deeper in the sun at λ_0 or not?

$s_\lambda \simeq \frac{1}{K_\lambda \rho}$ so if $K_\lambda \nearrow$ then $s_\lambda \searrow$
i.e. we can probe less deep

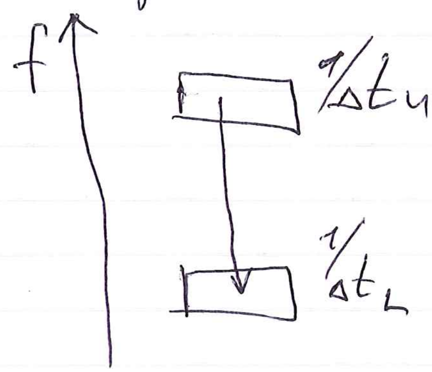


What constitute K_λ

It is $\sim$ to probability to catch / emit photon

$$\Delta f = \frac{1}{\pi}\left(\frac{1}{\Delta t_u} + \frac{1}{\Delta t_L}\right)$$

$$= \frac{1}{\pi}\,\frac{1}{\Delta t}$$



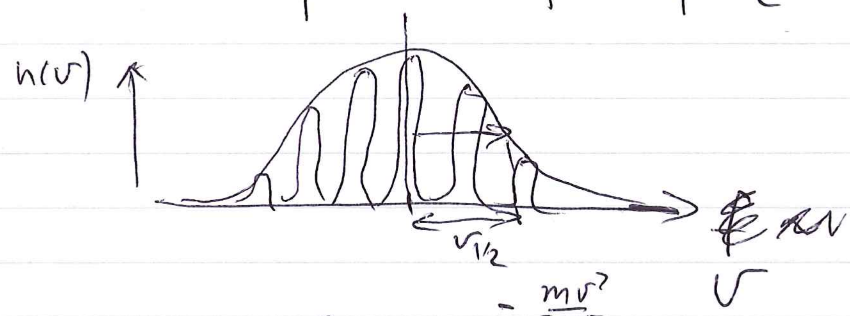
H_α line $\quad \lambda = 656.3$ nm

$\Delta t = 10^{-8}$ s

P1

Doppler Broadening

D. shift $\quad \Delta f = f_0 \frac{v}{c}$



$$n(v) \sim e^{-\frac{mv^2}{2kT}}$$

$$v_{1/2} \Leftrightarrow e^{-\frac{m v_{1/2}^2}{2kT}} = 1/2$$

$$\frac{m v_{1/2}^2}{2kT} = \ln 2$$

$$v_{1/2} = \sqrt{\frac{2kT}{m}\ln 2}$$

typical doppler shift, i.e. broadening $\quad \Delta f_{1/2} = \underset{1/\lambda}{\frac{f_0}{c}} \sqrt{\frac{2kT}{m}\ln 2} \approx \underbrace{12\ \mathrm{GHz}}$

compare to natural Linewidth

$$\mathrm{FWHM} = 2 \cdot \Delta f_{1/2}$$

$$= 2\sqrt{2}\,\frac{f_0}{c}\sqrt{\frac{kT}{m}\ln 2}$$

$m_H = 1.67 \cdot 10^{-27}$ kg

$k = 1.38 \cdot 10^{-23}$ J/K

$T = 5800$ K

For heavier elements D. broadening is not that bad