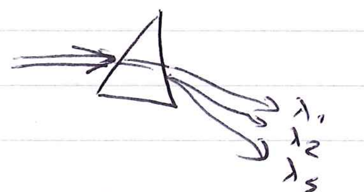
lecture 12

Stars spectroscopy

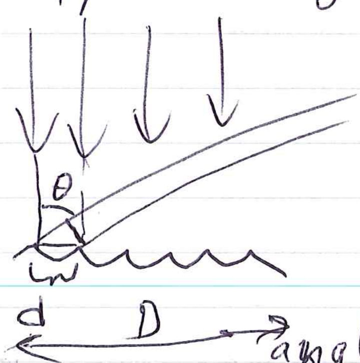
To see a spectrum we need an ellement which separate different wavelength.

Newton $\rightarrow$ prism

but dispersion is limited by nature $\rightarrow$ $n(\lambda)$ ↖ refractive index



Diffraction gratings are better



$\begin{cases} \text{constructive interference} \\ \Delta l = d\sin\theta = m\lambda \end{cases}$ (eq.1.)

Spectral resolution $\Delta\lambda$ is connected to the smallest angle θ difference we can resolve

$$\Delta\theta = \theta_2 - \theta_1 = \arcsin\left(\frac{m\lambda_2}{d}\right) - \arcsin\left(\frac{m\lambda_1}{d}\right)$$

$$\simeq /\ \lambda_2 = \lambda_1 + \Delta\lambda\ / \Rightarrow /\text{Taylor expansion}/$$

$$= \cancel{\arcsin\left(\frac{m\lambda_1}{d}\right)} + \frac{1 \cdot \frac{m}{d}\Delta\lambda}{\sqrt{1-\left(\frac{m\lambda_1}{d}\right)^2}} - \cancel{\arcsin\left(\frac{m\lambda_1}{d}\right)}$$

$$\approx \frac{1}{\sqrt{1-\left(\frac{m\lambda_1}{d}\right)^2}} \cdot \frac{m}{d}\Delta\lambda = \underbrace{\frac{1}{\cos\theta}}_{\approx 1} \frac{m}{d}\Delta\lambda$$

see eq1

From other hand we are diffraction limited and the smallest angle we can see is $\geq \frac{\lambda}{D}$ ← grating length

$$\frac{\lambda}{D} \cancel{\approx} \frac{m}{d} \Delta\lambda \quad \Rightarrow \quad \boxed{\Delta\lambda \geq \frac{\lambda}{m} \underbrace{\left(\frac{d}{D}\right)}_{=N \text{ number of groves}} = \frac{\lambda}{m \cdot N}}$$

typically gratings have $\approx 1000 \frac{\text{groves}}{\text{mm}}$

Gratings about $D = 1m$ are known

$$\Delta\lambda = \frac{500\,nm}{(m=1)} \cdot \frac{10^{-3}\,mm}{1\,m} =$$

$$= 500\,nm \cdot 10^{-6} \approx \underbrace{500\,fm}_{\text{sounds quite small}}$$

But recall that we want to see velocities ≈ 60 cm/s which gives Doppler shift $\Delta f \approx f \frac{v}{c}$

$$\Delta\lambda = \frac{c}{f_1} - \frac{c}{f_2} = \frac{c}{f_1} - \frac{c}{f_1 + \Delta f} \approx \underbrace{\left(\frac{c}{f_1}\right)}_{\lambda} \left(\frac{\Delta f}{f_1}\right)$$

$= f_1 \frac{v}{c}$

$$\Rightarrow \frac{\Delta\lambda}{\lambda} = \frac{\Delta f}{f_1} = \boxed{\frac{1}{m} \frac{d}{D} = \frac{v}{c}}$$

$$\cancel{D = \frac{\Delta f}{f_1} \frac{m}{d} = \frac{v}{c} \cdot \frac{m}{d} = \frac{0.6}{3e8} \cdot \frac{1}{10^{-6} m}}$$

$$D = \frac{d}{m} \cdot \frac{c}{v} = \frac{10^{-6}\,m}{1} \cdot \frac{3 \cdot 10^{8}}{0.6} =$$

$\simeq 500\,m$ which sound impractical.

But there are other methods!

<u>See my lab</u> :)

But what wavelength do we want to see?

Recall that H is the most common element, so let's focus on it.

Q.M. $\quad E_n = -13.6\,eV \frac{1}{n^2}$

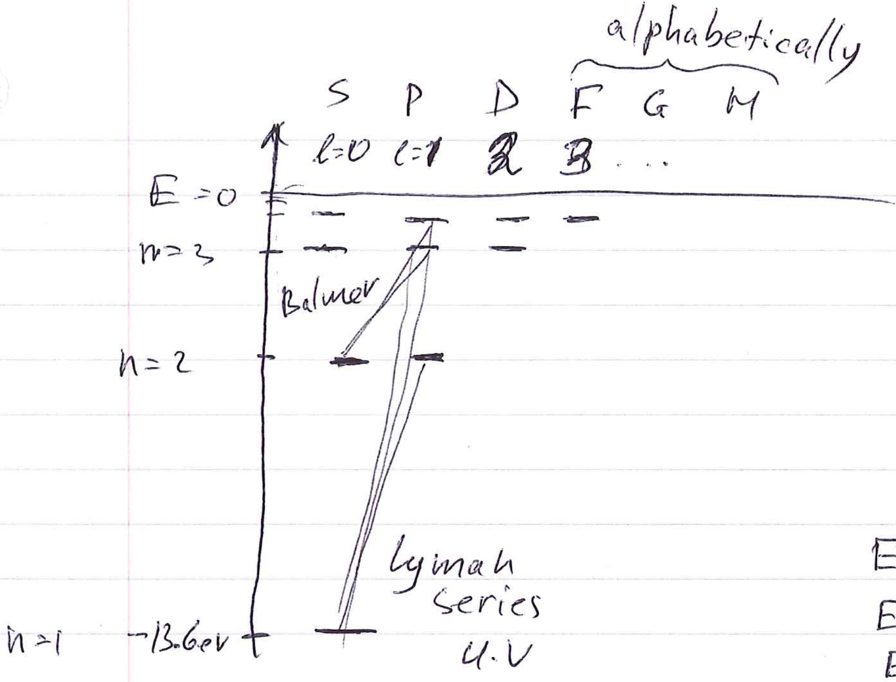
principal quantum number

For other element with core charge Z and <u>only one electron</u>

$$\boxed{E_0 \Rightarrow E_0 \cdot Z^2}$$

So we label energy levels with n and ℓ (angular momentum) m_ℓ (projection of ℓ) and m_s (spin of electron) $m_s = \pm 1/2$

Hydrogen energies



transition rules

Δn = anything

$\Delta l = \pm 1$

$\Delta m = 0, \pm 1$

$E_j \to 1$	Lyman
$E_j \to 2$	Balmer
$E_j \to 3$	Pashen

$$(\lambda_{n \to m})^{-1} = \frac{\Delta E}{h} \frac{1}{c} = R_H \left(\frac{1}{n^2} - \frac{1}{m^2} \right)$$

Rydberg constant for Hydrogen

$$R_H = \frac{13.6\,eV}{hc} = 1.096 \cdot 10^{7}\ m^{-1}$$

Series are numbered as $\alpha, \beta, \gamma \ldots$ starting from the smallest λ

$$\lambda(Ly_\alpha) = \left[1.096 \cdot 10^7 \left(\frac{1}{1^2} - \frac{1}{2^2} \right) \right]^{-1} =$$

$$= \left[1.096 \cdot 10^7 \cdot \frac{3}{4} \right]^{-1} \approx$$

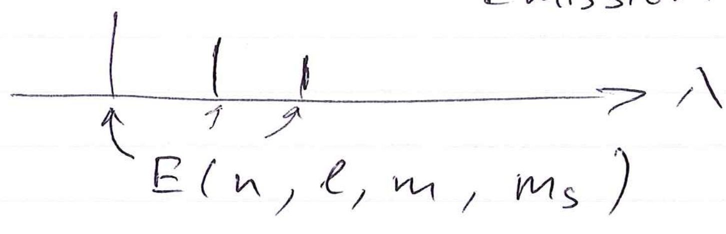
$$\approx 1.2 \cdot 10^{-7}\ m \simeq \underline{120\ nm}$$

$$\lambda(\text{Balmer } \alpha) = \left[1.096 \cdot 10^7 \left(\frac{1}{2^2} - \frac{1}{3^2} \right) \right]^{-1} =$$

$$= 657\ nm$$

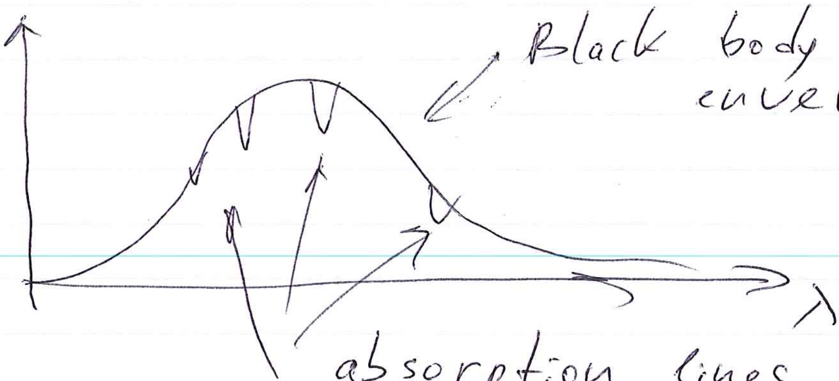
Similar ~~so~~ energy structures
$\Rightarrow$ spectra exist for other atoms and moleculas

Emission in gas disharge

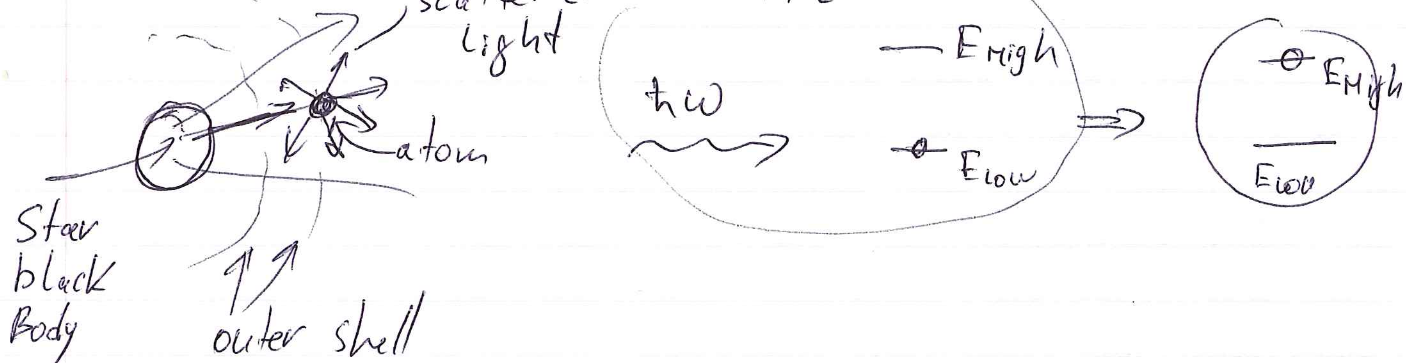


Q: So how the star spectrum ~~to~~ looks like?

Black body radiation envelope



absorption lines in the location of energy transition.



outer shell of gas which absorbs according to location of energy transition