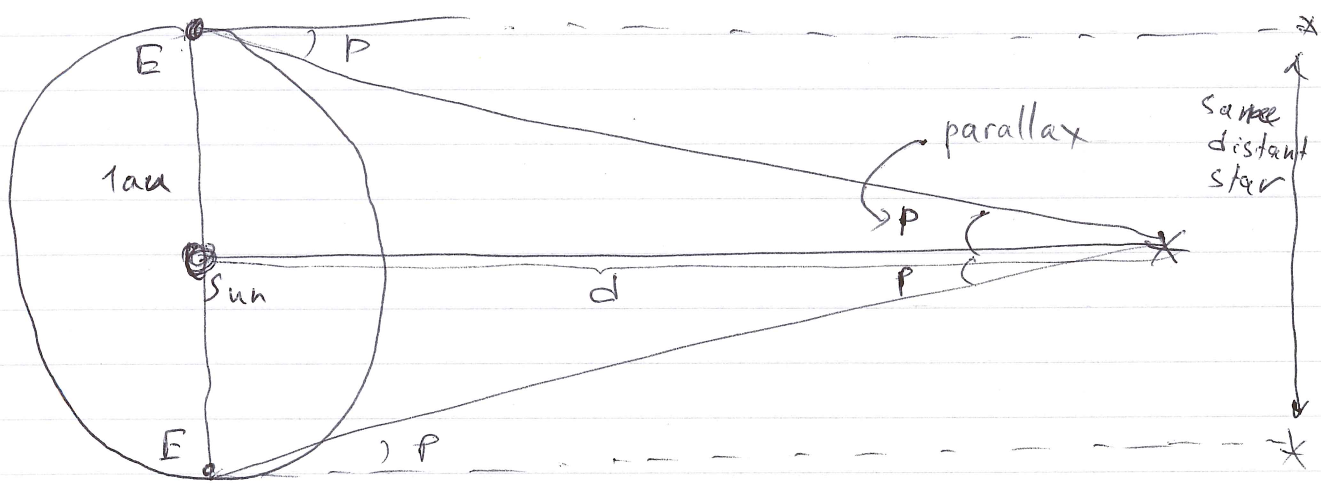
(p0)

lecture 3

Definition of par sec
parallax second = pc

the largest base available to Earth based observer is the Diameter of Earth orbit.



~~tan~~ $d = \frac{1au}{\tan p}$ = / p is very small /

Side note:
Closest star: Proxima Centauri has distance to us = 1,3pc

$\Rightarrow d = \frac{1au}{p}$, since p is small it is measured in arc second

$\Rightarrow d = \frac{1\,au}{p[''] \cdot \frac{1^\circ}{3600''} \cdot \frac{\pi}{180}} = \frac{1au}{1/(2.06 \cdot 10^5)\, p['']}$

$d(p = 1'') = \boxed{\text{parsec} = pc} = \frac{1.5 \cdot 10^{11}}{1} \cdot 2.06 \cdot 10^5$

$\simeq 3.08 \cdot 10^{16}$ m

$\simeq 3.26$ Lightyears $= 3.26$ Ly

Generally distance

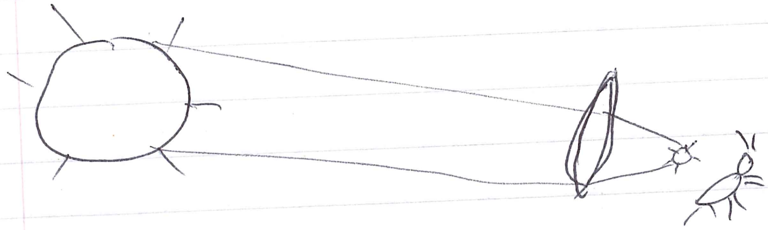
$\boxed{d = \frac{1pc}{p}}$ ← parallax

Beyond naked eye observation.

Telescopes:

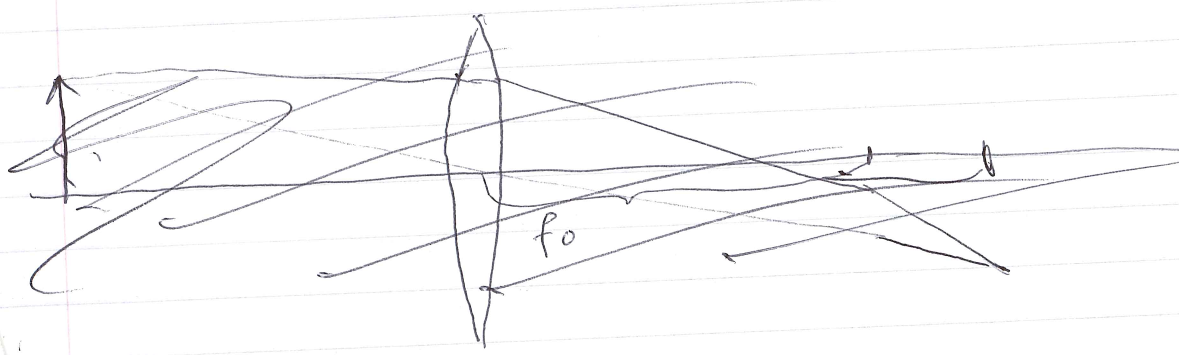
One lens is not enough though it often refers as a magnifying glass.

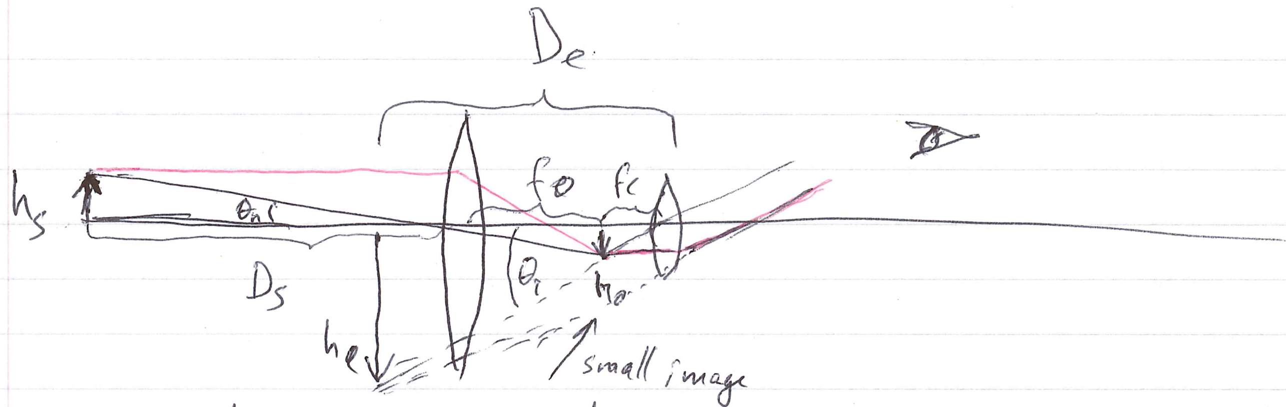
Why? it makes very small image of a distant objects. Recall burning ants adventures



Side not.
Negative lenses form "imaginary" image so we need 2nd element

So we need two element system, one form image, 2nd magnifies





$$\frac{h_o}{h_s} = \frac{f_o}{D_s} \ll 1 \ !$$

$$\frac{h_e}{h_o} = \frac{D_e}{\cancel{D_o}\, f_e} \overset{?}{\gg} 1$$

but by itself it is not very useful

Note though that

$\theta_{\text{naked eye}} = \dfrac{h_s}{D_s}$ - was old (unaided) angular size of the star

$\theta_{\text{telescope}} = \dfrac{h_e}{D_e}$ - is new angular size

So angular magnification

$$M = \frac{\theta_T}{\theta_n} = \frac{h_s}{D_s} \cdot \frac{D_e}{h_e} = \frac{\cancel{h_o}}{f_o} \cdot \frac{f_e}{\cancel{h_o}} = \frac{f_e}{f_o}$$

$$\boxed{M = \frac{f_e}{f_o}}$$

So we can boost angular resolution with a telescope

A few practical limitation

M - can be large but we need to hold lens assembly together

$\updownarrow$

telescope

so $f_e + f_o \approx 10\,m$

$f_e \gtrsim \lambda \approx 1\,\mu m$ ← but this is hard to operate

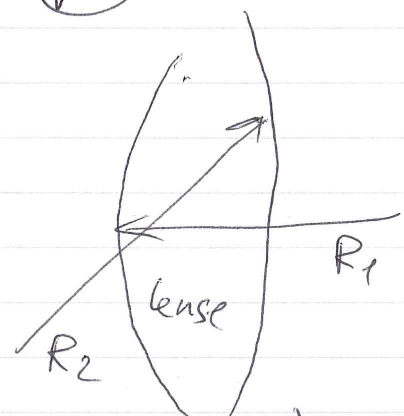
so max $M \approx \cancel{\frac{10m}{1\mu m}}\ \frac{10m}{1\mu m} = 10^7$.

So theoretically we can do the smalles angular resolution of

$\theta_{detect} / M = 0.5' / 10^7$

naked eye

sounds quete good but is it all to consider?



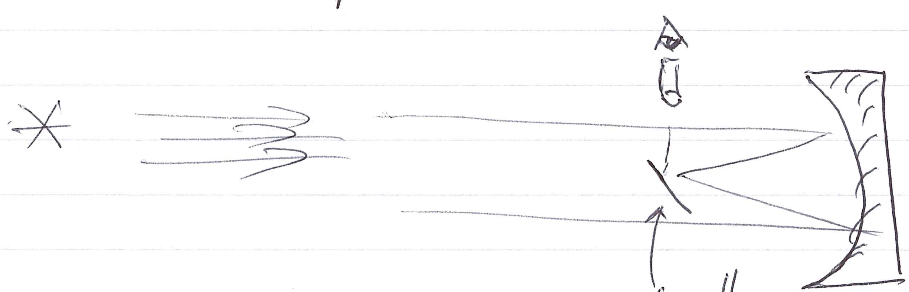
Chromatic aberations

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Recall that n is a function of wavelength $n = n(\lambda)$
~~otherwise~~ we will see set of green, red, and blue images on top of each other.

Solutions: * color filter ⇒ less light to detect

* Reflector telescope suggested by Newton i.e., replase lenses with mirror.



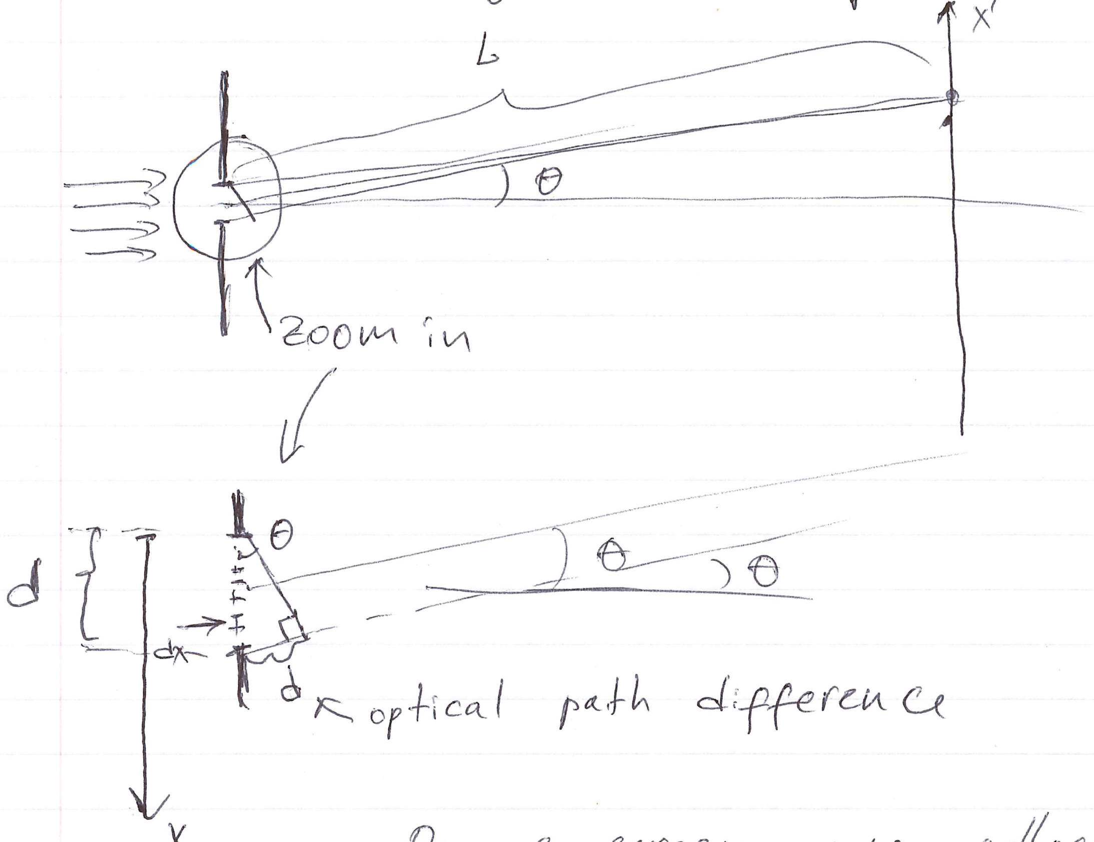
This techique was mastered by <u>Herschel</u> who build very large mirrors by literally polishing horse manure.

→ 1781 – find new planet <u>Uranus</u>

1803-1804 – binary star system (20 years of observations)
Same guy "map" of our galaxy as brightness over distance estimate.

A larger issue is difraction!

Recall single slit experiment



On a screen we collect all light from the slit i.e. from every element dx

Electromag field (light) will sum up on the screen

$$E_{screen} = \frac{E}{d}\cdot\int_0^d e^{i\frac{2\pi(L+x\cdot\sin\theta)}{\lambda}}dx = \underbrace{\overbrace{\frac{E}{d}e^{i\frac{2\pi L}{\lambda}}}^{E'}}_{\text{boring common factor}}\int_0^d e^{\frac{2\pi}{\lambda}x\sin\theta}dx$$

proportionality coef field per "pixel"

$$= \Big/ \theta \ll 1 \text{ but we will keep it} \Big/ = \frac{E'}{d}\int_0^d e^{i\frac{2\pi}{\lambda}x\sin\theta}dx$$

$$E_s = \frac{E'}{d}\int_0^d e^{i\frac{2\pi}{\lambda}\sin\theta\, x}\, dx =$$

$$= \frac{E'}{d}\,\frac{1}{i\frac{2\pi}{\lambda}\sin\theta}\, e^{i\frac{2\pi}{\lambda}\sin\theta\, x}\Big|_0^d =$$

$$= E'\,\frac{1}{i\frac{2\pi}{\lambda}\sin\theta d}\cdot\left(e^{i\frac{2\pi}{\lambda}\sin\theta d} - 1\right)\cdot\frac{e^{-\frac{1}{2}i\frac{2\pi}{\lambda}\sin\theta d}}{e^{-\frac{1}{2}i\frac{2\pi}{\lambda}\sin\theta d}}$$

$$= \frac{\overbrace{\underbrace{E'\, e^{-\frac{1}{2}i\frac{2\pi}{\lambda}\sin\theta\cdot d}}_{\text{another boring factor}}}^{E''}}{i\frac{2\pi}{\lambda}\sin\theta d}\;\underbrace{\boxed{\frac{e^{i\frac{\pi}{\lambda}\sin\theta d} - e^{i\frac{\pi}{\lambda}\sin\theta d}}{2i}}}_{=\sin\left(\frac{\pi}{\lambda}\sin\theta d\right)}\cdot 2i$$

$$= \frac{2\cancel{i}E''}{2\cancel{i}}\cdot\frac{\sin\left(\frac{\pi}{\lambda}(\sin\theta)\cdot d\right)}{\frac{\pi}{\lambda}(\sin\theta)\cdot d}$$

const phase φ

$$\boxed{E_s = \frac{E\cdot e^{i\varphi}}{\cancel{2i}}\;\frac{\sin\left(\frac{\pi}{\lambda}d\cdot(\sin\theta)\right)}{\frac{\pi}{\lambda}d\,(\sin\theta)}}$$

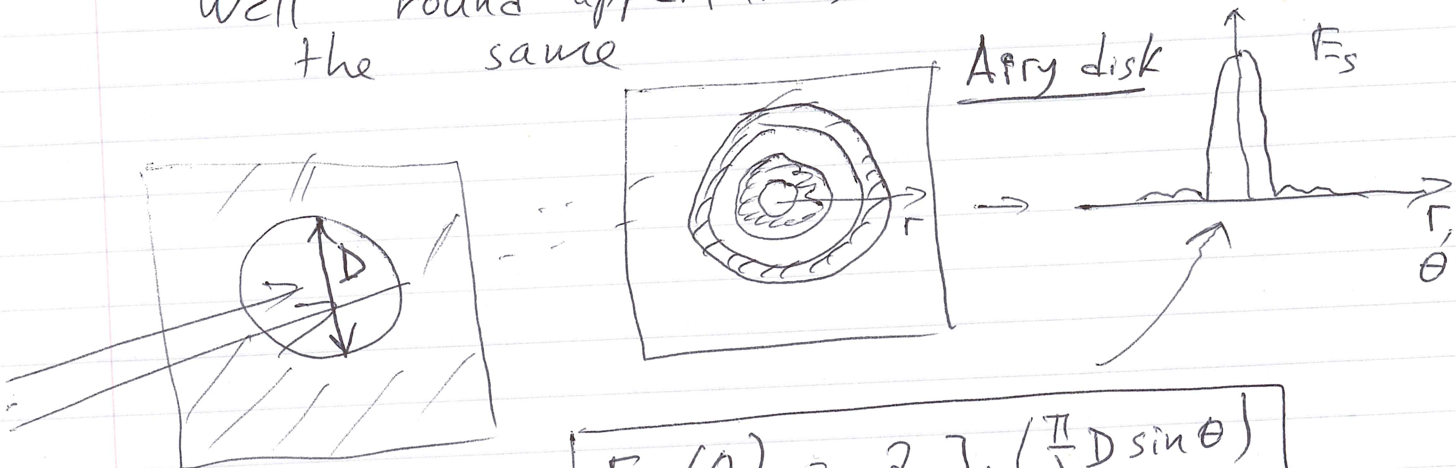
Minima condition $E_s = 0$

$$\frac{\pi}{\lambda}d\cdot\sin\theta = m\pi$$

$$\boxed{\sin\theta = m\frac{\lambda}{d}}$$

main maxima angle $\theta_m = \frac{\lambda}{d}$

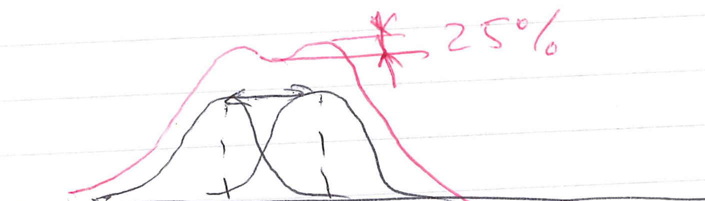
Well round appertures are almost the same



$$\boxed{E_s(\theta) = 2\frac{J_1\left(\frac{\pi}{\lambda} D \sin\theta\right)}{\frac{\pi}{\lambda} D \sin\theta}}$$

Bessel function

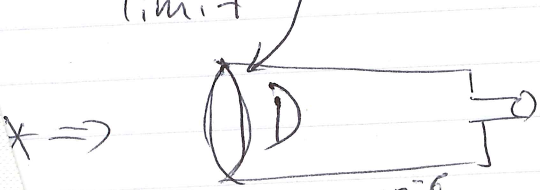
Now if we have two objects close to each other, they form two overlaping Airy disks. Which in cross section looks like.



$$\boxed{\theta_{min} = 1.22 \frac{\lambda}{D}}$$

Rayleigh criterion

So size of the objective is the actual limit



$D \sim 10\,m$

$\lambda \sim 1\,\mu m$

~~so it is nicely matches physical building construction limit~~

$\theta_m = 1.2 \frac{\lambda}{D} = 1.2 \cdot \frac{10^{-6}\,m}{10\,m} \simeq 10^{-7}\,rad$

Recall that aided eye limited by ~~to~~ maximum M - achivable by construction

$$\theta_m = \theta_{eye}/M = \frac{2\mu m}{2cm} \Big/ \frac{10m}{1\mu} =$$

$$= 10^{-4}/10^{7} = 10^{-11}\,\text{rad}$$ which

is not achivable due to diffraction

with $\theta_m \simeq \frac{\lambda}{D} = \frac{10^{-6}m}{10m} \simeq 10^{-7}\,\text{rad}$

$$\boxed{1\,\text{rad} = 2 \cdot 10^{5}\ \text{arc sec}}$$