Numerical integration

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Lecture 09

Integration problem statement

Suppose we are given function

$$f(x)$$

our goal is to find

$$\int_a^b f(x)dx$$

Not all function can be easily integrated analytically in the elementary enough form.

Example

$$\int_0^y exp(-x^2)dx$$

So we must use numerical methods.

The Rectangle method

Recall the Riemann integral definition our goal is to find

$$\int_a^b f(x)dx = \lim_{N\to\infty} \sum_{i=1}^{N-1} f(x_i)h$$

where N is the number of points, $h = (b-a)/(N-1)$ is the distance between points.

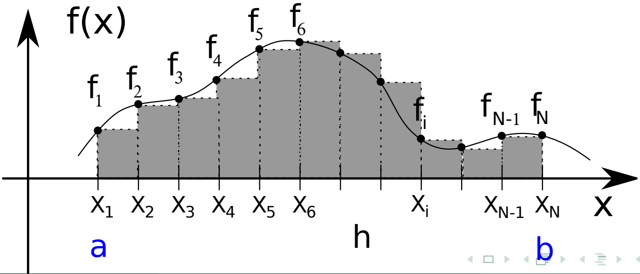
The Rectangle method continued

Riemann rule is almost direct recipe.

Rectangle method

$$\int_a^b f(x)dx \approx \sum_{i=1}^{N-1} f(x_i)h, \quad \text{where} \quad h = \frac{b-a}{N-1} \quad \text{and} \quad x_i = a + (i-1)h$$

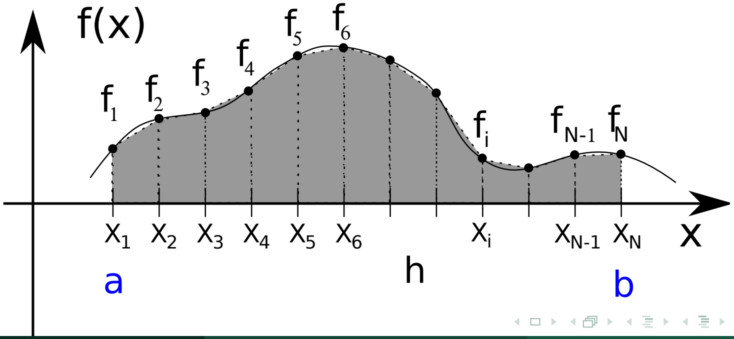
We just need to remember about round off errors so *h* should not be too small or equivalently *N* should not be to big.



Trapezoidal method

$$\int_a^b f(x)dx \approx h \times (\frac{1}{2}f_1 + f_2 + f_3 + \cdots + f_{N-2} + f_{N-1} + \frac{1}{2}f_N) = h\sum_{i=1}^{N} f(x_i)w_i,$$

$$\text{where } h = \frac{b-a}{N-1} \text{ and } x_i = a + (i-1)h$$



Simpson method

Simpson method - approximation by parabolas

$$\int_a^b f(x)dx \approx h\frac{1}{3} \times (f_1 + 4f_2 + 2f_3 + 4f_4 + \cdots + 2f_{N-2} + 4f_{N-1} + f_N)$$

$$= h\sum_{i=1}^{N} f(x_i)w_i, \text{ where } h = \frac{b-a}{N-1} \text{ and } x_i = a + (i-1)h$$

note that N must be in special form $N = 2k + 1$, i.e. odd