Homework 09

Problem 1 (5 points):

The model for a more realistic pendulum. Solve numerically (using the built-in `ode45` solver) the following physical problem of a pendulum's motions

$$\theta''(t) = -\frac{g}{L}\sin(\theta)$$

where g is acceleration due to gravity (g=9.8 m/s^2), $L = 1$ is the length of the pendulum, and θ is the angular deviation of the pendulum from the vertical.

Assuming that the initial angular velocity (β) is zero, i.e., $\beta(0) = \theta'(0) = 0$, solve this problem (i.e., plot $\theta(t)$ and $\beta(t)$) for two values of the initial deflection $\theta(0) = \pi/10$ and $\theta(0) = \pi/3$. Both solutions must be presented on the same plot. Make final time large enough to include at least 10 periods. Show that the period of the pendulum depends on the initial deflection. Does it takes longer to make one swing with a larger or smaller initial deflection?

Problem 2 (5 points):

Implement the forth-order Runge-Kutta method (RK4) according to the recipe outlined in the slides for the lecture 20. It should be input compatible to the home-made Euler's implementation `odeeuler.m`. Compare the solution of the above problem done with your own RK4 implementation to the built-in `ode45` solver.

Problem 3 (5 points):

Have a look at the predator and prey model (the `ode_predator_prey_model.m` file provided with the lecture 20 notes).

Find the non trivial solution (i.e. $x_0 \neq 0$ and $y_0 \neq 0$) for which population of wolves and rabbits is independent of time (i.e. $dx/dt = dy/dt = 0$). You should get a system of two linear equations which is super simple. However, I ask you to solve it using MATLAB's numerical solver which we discussed during the lecture 21, i.e., form the matrix A and the column vector B, and find x which satisfies $A \times x = B$. Note: use constants a, b, c, and d provided in the `ode_predator_prey_model.m` file.

So, we see that it possible to have stable populations (or economy without ups and downs) but you need to be smart about initial conditions.

What is the expected shape of the plot of the wolves population vs rabbit's one with calculated above initial conditions? Plot it.

Problem 4 (5 points):

It is possible to draw a parabola through any 3 points in a plane. Using MATLAB' linear equations solver find coefficients a, b and c for parabola described as $y = ax^2 + bx + c$. This parabola passes through the points $p_1 = (-10, 10), p_2 = (-2, 12)$, and $p_3 = (12, 10)$.

Problem 5 (5 points):

Using MATLAB's `interp1` with `spline` method, find where the interpolation line crosses $y = 0$ level. The interpolation is done over the following points [(x,y) notation]: (2,10), (3,8), (4,4), (5,1), (6,-2).

Would it be wise to search crossing with $x = 0$ line using the above data? Why is it so?