Lecture 39

* Cosmology
* Cosmic microwave background

$$z = \frac{\lambda_{obs} - \lambda_{emit}}{\lambda_{emit}} = \frac{\lambda_{obs}}{\lambda_{emit}} - 1$$

$$r = R\varpi$$

scale, expansion factor

varpi (distance)

$$\Rightarrow \quad z = R_0 \frac{\lambda_{obs}}{\lambda_{emit}} - 1$$

$$\frac{R(obs)}{R(emit)} - 1 = \frac{R(z)\varpi_\lambda}{R(0)\varpi_\lambda} - 1 \qquad \Rightarrow \frac{R(obs)}{R(emit)} = z+1$$

$$R_{obs} = R_{now} = 1$$

$$\boxed{R(z) = \frac{1}{z+1}}$$

We observe objects for $z \leq 4$

$\Rightarrow$ Quasar (Quasi Star)

Universe expansion - thermodynamics consideration

$$dU + dW = dQ = 0 \Leftarrow \text{no heat to universe}$$

$$\frac{dU}{dt} = -\frac{dW}{dt} = -p\frac{dV}{dt}$$

$$U = \underbrace{u}_{\text{energy density}} \cdot V$$

$$\frac{d(u \cdot V)}{dt} = -p\frac{dV}{dt}$$

$$/\ V = \frac{4\pi}{3} r^3\ /$$

$$\frac{d(ur^3)}{dt} = -p\frac{d(r^3)}{dt}$$

$$/\ u = \rho_0 c^2,\ \text{think about } u = \frac{E}{V} = \frac{mc^2}{V}\ /$$

$$\frac{d(\rho r^3)}{dt} = -\frac{p}{c^2}\frac{d(r^3)}{dt}$$

$\boxed{\dot{R}(t) \nearrow\ t \nearrow \text{ expansion}}$

$$/\ r = \underbrace{R(t)}_{\substack{\text{expansion} \\ \text{coef (unitless)}}}\ \underbrace{\varpi}_{\substack{\text{scale factor [m],} \\ \text{constant}}}\ /$$

varpi

$$\frac{d(\rho R^3)}{dt} = -\frac{p}{c^2}\frac{d(R^3)}{dt}$$

in general form

$P = \omega u = \omega \rho c^2$

not the same as ϖ

Recall

$P = \frac{2}{3} \frac{NkT}{V} = \frac{2}{3} u$ for particles

$P = \frac{1}{3} u$ for photons

$\omega = \frac{2}{3}$ or $\frac{1}{3}$

particles / photons

presureless gas $\omega = 0$

$$\frac{d(\rho R^3)}{dt} = -\omega\rho \frac{d(R^3)}{dt}$$

$$\frac{1}{\rho R^3} \times \Big| \quad R^3 \frac{d\rho}{dt} + \rho 3R^2 \frac{dR}{dt} = -\omega\rho\, 3R^2 \frac{dR}{dt}$$

$$\frac{1}{\rho}\frac{d\rho}{dt} + \frac{3}{R}\frac{dR}{dt} = -\frac{3\omega}{R}\frac{dR}{dt}$$

$$\frac{d(\ln\rho)}{dt} = -3(\omega+1)\frac{d(\ln R)}{dt}$$

$$\frac{d}{dt}\left(\ln\rho + 3(\omega+1)\ln R\right) = 0$$

$$\Rightarrow \boxed{\rho R^{3(\omega+1)} = \text{const} = \rho_0 \underset{1}{R_0^{3(\omega+1)}} = \rho_0}$$

Currently we are in pressure less stage, matter is to sparce to interact $\Rightarrow$ no pressure $\Rightarrow \omega = 0$

$$\boxed{\rho R^{3(0+1)} = \rho R^3 = \rho_0}$$

Currently $\rho_0 = 4.17 \cdot 10^{-28} \frac{kg}{m^3}$

about 6 M atom per $1 m^3$

To form He : $\rho \simeq 10^{-2} \frac{kg}{m^3}$, $T = 10^9 K$

$$\Rightarrow R = \left(\frac{\rho_0}{\rho}\right)^{-1/3} = 3.47 \cdot 10^{-9}$$

$\Rightarrow$ At the time formation the universe was $3.47 \cdot 10^{-9}$ times smaller!

If we count only Radiation

~~At high temperature u is dominated by radiation energy ($\omega = 1/3$)~~

$$u_{rad} = aT^4 = \rho c^2$$

Thus $$c^2\left(\rho R^{3(\omega+1)}\right) = c^2 \rho_0$$

$$u_{rad} \cdot R^{3(\omega+1)} = u_{rad\,0}$$

$$aT^4 R^{3(1/3+1)} = aT_0^4$$

$$\Rightarrow \boxed{TR = T_0}$$

Expected (T_0) temperature now

$$T_0 = T R = 3.47\ K$$

R: $3.47 \cdot 10^{-9}$

T: He formation 10^9

Cosmic background (CB) temperature

Expected $$\boxed{\lambda_{max} = \frac{0.00290\ m\cdot K}{T_0} = 8.36 \cdot 10^{-4}\ m}$$

Wein's law for Black body

Observed CMB (microwave) temperature

is $\boxed{2.725\ K} \longleftrightarrow \boxed{f_{max} = 160\ GHz}$