

## Lecture 39

\* Cosmology

\* Cosmic microwave background

$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{emit}}}{\lambda_{\text{emit}}} = \frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}} - 1$$

$$r = R \cdot \omega$$

$\swarrow$  scale, expansion factor       $\nwarrow$  varpi (distance)

$$\Rightarrow z = R_0 \frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}} - 1$$

$$\frac{R(\text{obs})}{R(\text{emit})} - 1 = \frac{R(z) \omega_{\lambda}}{\omega_{\lambda}} - 1 \Rightarrow \frac{R(\text{obs})}{R(\text{emit})} = z + 1$$

$$R(z) = \frac{1}{z + 1}$$

$$R_{\text{obs}} = R_{\text{now}} = 1$$

We observe objects for  $z \leq 4$

$\Rightarrow$  Quasar (Quasi Star)

## Universe expansion - thermodynamics consideration

$$dU + dW = dQ = 0 \leftarrow \text{no heat to universe}$$

$$\frac{dU}{dt} = - \frac{dW}{dt} = - p \frac{dV}{dt}$$

$$U = u \cdot V$$

↑ energy density

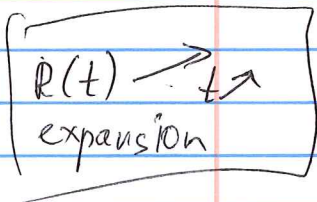
$$\frac{d(u \cdot V)}{dt} = - p \frac{dV}{dt}$$

$$/ V = \frac{4\pi}{3} r^3 /$$

$$\frac{d(ur^3)}{dt} = - p \frac{d(r^3)}{dt}$$

$$/ u = \rho \cdot c^2, \text{ think about } u = \frac{E}{V} = \frac{mc^2}{V} /$$

$$\frac{d(\rho r^3)}{dt} = - \frac{p}{c^2} \frac{d(r^3)}{dt}$$



$$/ r = R(t) \omega \leftarrow \text{varpi} /$$

↑ expansion coef (unitless)      ↑ scale factor [m], constant

$$\frac{d(\rho R^3)}{dt} = - \frac{p}{c^2} \frac{d(R^3)}{dt}$$

in general form

$$P = \omega u = \omega \rho c^2$$

not the same  
as  $\rho c^2$

Recall

$$P = \frac{2}{3} \frac{NkT}{V} = \frac{2}{3} u \text{ for particles}$$

$$P = \frac{1}{3} u \text{ for photons}$$

$$\omega = \frac{2}{3} \text{ or } \frac{1}{3}$$

↑ particles      ↑ photons

pressureless gas  $\omega = 0$

$$\frac{d(\rho R^3)}{dt} = -\omega \rho \frac{d(R^3)}{dt}$$

$$\frac{1}{R^3} \times \left| R^3 \frac{d\rho}{dt} + \rho 3R^2 \frac{dR}{dt} = -\omega \rho 3R^2 \frac{dR}{dt} \right.$$

$$\frac{1}{\rho} \frac{d\rho}{dt} + \frac{3}{R} \frac{dR}{dt} = -\frac{3\omega}{R} \frac{dR}{dt}$$

$$\frac{d(\ln \rho)}{dt} = -3(\omega + 1) \frac{d(\ln R)}{dt}$$

$$\frac{d}{dt} (\ln \rho + 3(\omega + 1) \ln R) = 0$$

$$\Rightarrow \rho R^{3(\omega+1)} = \text{const} = \rho_0 R_0^{3(\omega+1)} = \rho_0$$

Currently we are in pressureless stage, matter is too sparse to interact  $\Rightarrow$  no pressure  
 $\Rightarrow \omega = 0$

$$\rho R^{3(\omega+1)} = \rho R^3 = \rho_0$$

Currently  $\rho_0 = 4.17 \cdot 10^{-28} \frac{\text{kg}}{\text{m}^3}$   
↑  
about 6 H atom per  $1 \text{ m}^3$

To form He:  $\rho \approx 10^{-2} \frac{\text{kg}}{\text{m}^3}$ ,  $T = 10^9 \text{ K}$

$$\Rightarrow R = \left( \frac{\rho_0}{\rho} \right)^{-1/3} = 3.47 \cdot 10^{-9}$$

$\Rightarrow$  At the time formation  
the universe was  $3.47 \cdot 10^{-9}$  times  
smaller!

If we count only radiation

~~At high temperature it is dominated by radiation energy ( $\omega = 1/3$ )~~

$$U_{\text{rad}} = aT^4 = \rho c^2$$

$$\text{Thus } c^2 (\rho R^{3(\omega+1)}) = c^2 \rho_0$$

$$U_{\text{rad}} \propto R^{3(\omega+1)} = U_{\text{rad}_0}$$

$$aT^4 R^{3(1/3+1)} = aT_0^4$$

$$\Rightarrow \boxed{TR = T_0}$$

Expected ( $T_0$ ) temperature now

$$\rightarrow T_0 = \frac{T}{R} = 3.47 \text{ K}$$

$\swarrow$  Me formation  $10^9$

Cosmic background temperature (CB)

Expected  $\lambda_{\text{max}} = \frac{0.00290 \text{ m}\cdot\text{K}}{T_0}$  Wein's law for Black body

$$= 8.36 \cdot 10^{-4} \text{ m}$$

Observed CMB temperature  $\leftarrow$  microwave

is  $\boxed{2.725 \text{ K}}$   $\leftrightarrow$   $\boxed{f_{\text{max}} = 160 \text{ GHz}}$

