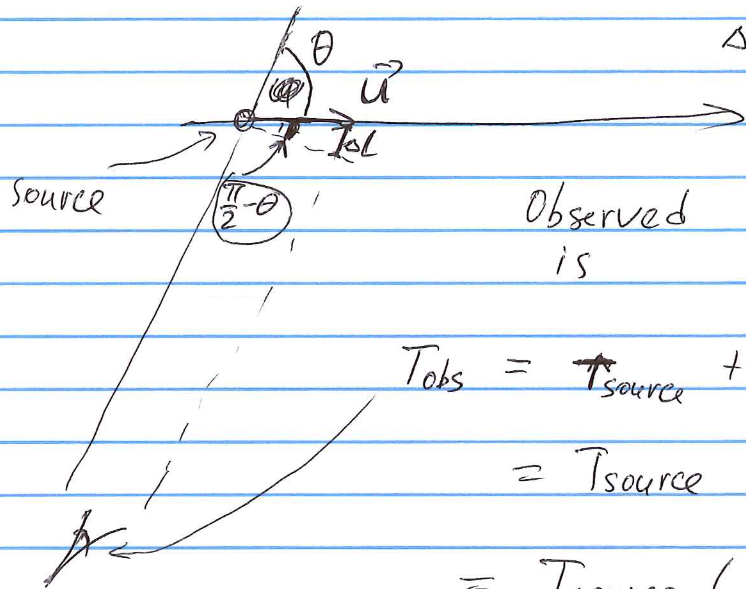
<u>lecture 38</u>

* large scale structures and distance measurements
* <u>Universe expansion</u>
* Hubble law

Doppler shift $\Longrightarrow$ velocity of the object



$$\Delta L = u \cdot \sin\left(\tfrac{\pi}{2}-\theta\right) = u\cos\theta$$

Observed period of an oscillation is

$$T_{obs} = T_{source} + \frac{\Delta L}{c} =$$

$$= T_{source} + \frac{u \cdot \cos\theta}{c} \cdot T_{source} =$$

$$= T_{source}\left(1 + \frac{u\cos\theta}{c}\right)$$

$$= \left/ \; T_{source} = \frac{T_{rest}}{\sqrt{1-\frac{u^2}{c^2}}} \; \right/$$

$$T_{obs} = T_{rest} \cdot \frac{1+\frac{u\cos\theta}{c}}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$\frac{\nu_{obs}}{\nu_{Rest}} = \left(\frac{1}{T_{obs}}\right) / \left(1/T_{rest}\right) = \frac{\sqrt{1-\frac{u^2}{c^2}}}{1+\frac{u\cos\theta}{c}}$$

$$\begin{cases} \text{if } \cos\theta = \pm 1 \quad (\text{away or to observer}) \\ u = v_R \leftarrow \text{radial velocity} \end{cases}$$

$$\frac{\nu_{obs}}{\nu_{Rest}} = \frac{\sqrt{1-\frac{v_R^2}{c^2}}}{1 \pm \frac{v_R}{c}}$$

$+$ to observer

$-$ away from observer

Defining wavelength shift z

$\boxed{\lambda = c/\nu}$

emitted

$$\boxed{z = \frac{\lambda_{obs} - \lambda_{rest}}{\lambda_{rest}}} = \frac{\lambda_{obs}}{\lambda_{rest}} - 1 = \frac{\nu_{rest}}{\nu_{obs}} - 1$$

emitted

$$= \frac{1 \pm v_R/c}{\sqrt{1-\left(\frac{v_R}{c}\right)^2}} - 1 = \frac{1 \pm \frac{v_R}{c}}{\sqrt{\left(1-\frac{v_R}{c}\right)\left(1+\frac{v_R}{c}\right)}} =$$

$$= \boxed{\frac{\sqrt{1 \mp v_R/c}}{\sqrt{1 \pm v_R/c}} - 1 = z_{\substack{\text{to observer} \\ \text{away}}}}$$

For motion away

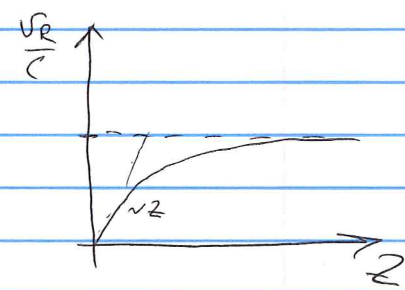
$$z_{away} = \sqrt{\frac{1 + \frac{v_R}{c}}{1 - v_R/c}} - 1$$

Red shift
$\lambda_{obs} > \lambda_{rest}$

$$(z+1)^2 = \frac{1+\frac{v_R}{c}}{1-\frac{v_R}{c}}$$

$$(z+1)^2 \cdot 1 - (z+1)^2 \frac{v_R}{c} = 1 + \frac{v_R}{c}$$

$$\frac{v_R}{c} = \frac{(z+1)^2 - 1}{(z+1)^2 + 1}$$



$$\boxed{H_0 = \frac{100\,h\,\frac{km}{s}}{1\,Mpc}}$$

$h \approx 0.71$
↑
correction

$$z \ll 1 \Rightarrow \boxed{\frac{v_R}{c} \approx z}$$

for small z (i.e. distances d)

Hubble $\Rightarrow \boxed{v_R = H_0 d} \Rightarrow \boxed{d = \frac{c}{H_0}\,\frac{(z+1)^2-1}{(z+1)^2-1}} \approx \boxed{\substack{z \ll 1 \\ \frac{cz}{H_0}}}$

Hubble size of universe

$$v = H_0 \cdot d = c$$

$$d_H = \frac{c}{H_0} = \frac{c \cdot 1 Mpc}{0.7 \cdot 100 \cdot 10^3 \frac{m}{s}} = \frac{3 \cdot 10^8}{7 \cdot 10^4} \simeq$$

$$\approx 4 \cdot 10^3 \, Mpc$$

Recall that $1 pc \simeq 3.2 \, Ly$

$$\frac{d_H}{c} = \frac{4 \cdot 10^3 \, Mpc \cdot 3.2 \frac{Ly}{pc}}{1 Ly / year} \approx \underbrace{13 \cdot 10^9} \, years$$

This is simple estimate for the age of "visible" universe