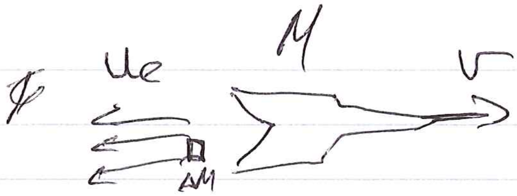
lecture 37

How to get to the sky.

* Propulsion motion
* Solar sail



$$(M + \Delta m)\,v = (M)(v + \Delta v) + \Delta m (v - u_e)$$

$$\cancel{Mv} + \cancel{\Delta m v} = \cancel{Mv} + M\Delta v + \cancel{\Delta m v} - \Delta m u_e$$

$$M\Delta v = u_e \Delta m$$

$$F_{rocket} = \frac{M\Delta v}{\Delta t} = u_e \frac{\Delta m}{\Delta t} + F_{ext}$$

$$\Delta m = -dM$$

$$F_{ext} = 0$$

$$dv = -u_e \frac{dM}{M} + \frac{F_{ext}}{M} dt$$

$$v_f = u_e \ln \frac{M_0}{M_0 + M_{fuel}} + \frac{F_{ext}}{M} t$$

$F_{ext} = 0$

$$M_{fuel} = M_0 \left(e^{v_f / u_e} - 1 \right)$$

$F_{ext} \neq 0$

$$M_{fuel} = M_0 \left(e^{\frac{v_f - \frac{F_{ext}}{M} t}{u_e}} - 1 \right)$$

<u>liquid fuel rocket</u> $u_e \approx 4000\ m/s$

To leave Earth we need $\simeq 12\ km/s$

Let's say we want to send

$M_0 = 1kg$ $M_{fuel} = 1 \cdot \left(e^{\frac{12}{4}} - 1\right) =$

$\simeq 19\ kg$

What if we need to go against gravity $\Rightarrow -\frac{F_{ext}}{M} = g$

If we send Humans we cannot handle $a > 10g$ and more or less comfortably $3g$

$$t = \frac{v_f - v_i}{a} = \frac{v_f}{6g} = \frac{12 \cdot 10^3}{60} \simeq 200\ sec$$

$$\Rightarrow \frac{F_{ext}}{M} \cdot t = g \cdot 200 \simeq 2000\ m$$

Possible hack: send robots $a \gg g$

$$\Rightarrow M_{fuel} = 1kg\left(e^{\frac{12+2}{4}} - 1\right) = 31\ kg$$

Note we are skiping air drag.

Compare to Saturn V rocket $M_0 \simeq 120T$ ← to low orbit $M_{rock} = 2800T$

$M_{moon} \simeq 45T$ ratio $\frac{M_{rock}}{M_0} \simeq 23 - 62$

In order to move comfortably ~~along~~ in Solar system

we need to reach 2nd space velocity

$\approx \sqrt{2}\, v_1 \simeq \sqrt{2} \cdot 30 \frac{km}{s} \approx 43$ km/s

$$M_{fuel} = M_0 \left(e^{\frac{v_f - v_{Earth\ escape}}{u_e}} - 1 \right)$$

M_0 ↑ to reach 12 km/s

$$= 31\,kg \cdot \left(e^{\frac{43-12}{4}} - 1 \right) =$$

$= 310\,1807$ kg $\approx$ **56 · 10³ kg for each 1kg of load** !!!

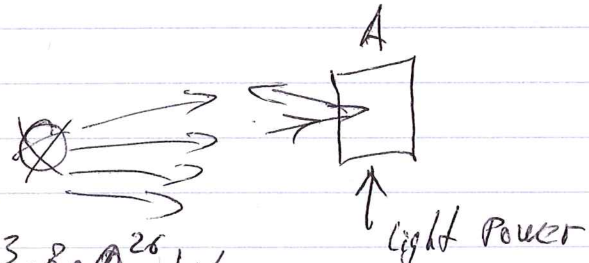
Taking price of gasoline 4$ per galon $\approx$ $1/kg

we are in big trouble here.

~~this does not counts~~

Possible solution – Solar sail

we still need to get to the Earth orbit somehow but then



$L_\odot$ $3.8\cdot10^{26}$ W

$$I = \frac{L_\odot}{4\pi R_E^2} = \frac{3.8\cdot10^{26}}{4\pi\cdot1.5\cdot10^{11}} = 1344\ \frac{W}{m^2}$$

recall light momentum $= \hbar k = \hbar\frac{\omega}{c} \approx \frac{E}{c}$

so ~~mo~~ carried ~~k~~ momentum per area per second → Pressure

$$\frac{\Delta K}{\Delta t\cdot A} \approx \frac{I}{c} = 4.5\cdot10^{-6}\ \frac{m\cdot kg}{s^2 m^2} = \frac{N}{m^2} = P_s$$

Depending if we reflecting or absorbing this light we can get ~~some~~ transfer coef $\alpha \simeq 1 \div 2$

So ~~max~~ acceleration which we can get

$$a = \frac{P_s\cdot A}{m} = \cancel{\frac{P_s\cdot A}{\rho\cdot h\cdot A} = \frac{P_s}{\rho h}}$$

Let's say we want $a = g$ then for each kg we need ↗ $A = \frac{g\cdot 1kg}{P_s} = \frac{10}{4.5e^6} \approx 2\cdot10^6\ m^2$

let's estimate the weight of the solar sail

Assuming that $\rho = \frac{10^3 kg}{m^3}$ and thickness

$h = 1\mu m = 10^{-6} m$ $\simeq 1.4 g/cm^2 = \frac{1.4 \cdot 10^{-3} kg}{10^{-4} m^2} \simeq$

Mylar $= 14 \frac{kg}{m^3}$

pretty much the same stuff as in plastic botles

mass of sail =

$= \rho \cdot A \cdot h \simeq 2 \cdot 10^6 \cdot 14 \cdot 10^{-6} =$

$= 28\ kg$ ← Ups sail is heavier then expected pay load.

OK. how fast sail alone can accelerate

$$a = \frac{P_s \cdot A}{m_{sail}} = \frac{P_s \cdot A}{\rho \cdot h \cdot A} = \frac{P_s}{\rho h} =$$

$$= \frac{4.5 \cdot 10^{-6}\ Pa}{14 \cdot 10^{-6}} = \frac{4.5}{14} \frac{m}{s^2} = 0.3 \frac{m}{s^2}$$

actually not bad given that we can use it for very long time (solar fuel is free)

Note
$g_{sun} = G \frac{M_\odot}{(1AU)^2} =$
$= \frac{6.6 \cdot 10^{-11} \cdot 2 \cdot 10^{30}}{(1.5 \cdot 10^{11})^2} =$
$= 6 \cdot 10^{-3} \frac{m}{s^2}$

How to solve low u_e

↙ ion trusters $u_e \simeq 10^5$ m/s
but the mass Δm is tiny so
it will take ~~force~~ long time to
accelerate (not good in presence
of large F_{ext})

Radical solution nuclear blast rocket