

## lecture 36

- \* Galaxies classification  
→ Mubble fork.

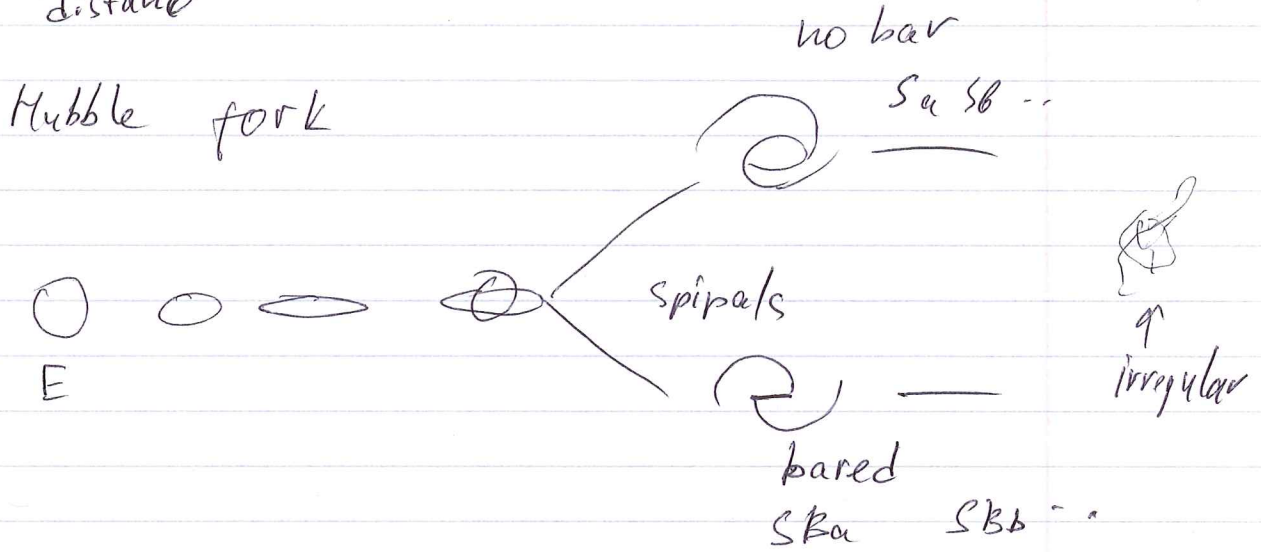
- \* Galaxy mass estimate

- \* Interaction of galaxies

  - \* Rapid passage

  - \* "Friction drag" - dynamic friction

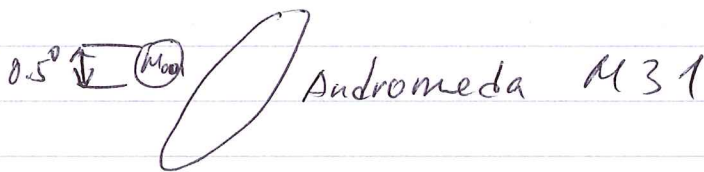
1 Galaxy debate are their part of Milky Way?  
 $\Rightarrow$  sol. Hubble measured variable stars and set distance



Dust disk often visible

2. How big they are

M31 is  $\approx 2.5\text{Mly}$  away  
 $\approx 600\text{kpc}$



lets do crude estimate

MW  $\approx 50\text{kpc}$

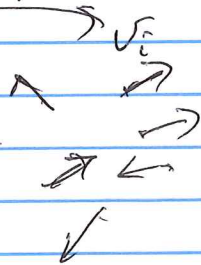
let put similar Galaxy at 10 times  
 this size

$$\theta = \frac{50}{500} = 0.1 \cdot \frac{180}{\pi} \approx 6^\circ \text{ a bit too large probably}$$

But there are neighbors  
 see Magellan clouds LMC and SMC

# A Galaxy mass estimate

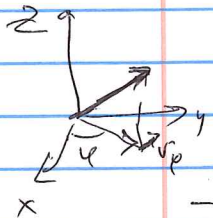
with respect  
to center  
of mass



Virial theorem

$$-2 \langle K \rangle = \langle U \rangle$$

$$-2 \sum_i \frac{m_i v_i^2}{2} = U$$



$$\langle v_i^2 \rangle = \langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle =$$

= / cylindrical coordinates /

$$= \langle v_R^2 \rangle + \langle v_\phi^2 \rangle + \langle v_z^2 \rangle =$$

$$= 3 \langle v_R^2 \rangle$$

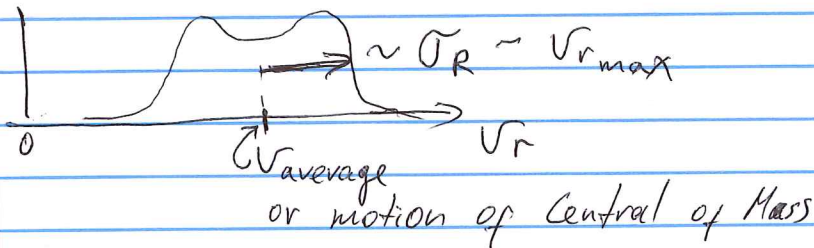
↑ all components are the same

assuming  $m_1 = m_2 = \dots = m_N$

$$\Rightarrow -2 \langle K \rangle = (N \cdot m) \cdot 3 \langle v_R^2 \rangle = U = -\frac{3}{5} \frac{GM^2}{R}$$

(assuming constant  $\rho$ )

Rotation curve



↑ standard variation  
=  $\sigma_R^2$  when taken around C.M.  
↑ standard deviation of radial speed

$$\sigma_R^2 = \frac{1}{5} \frac{GM}{R}$$

$$M = \frac{5 \cdot R}{G} \cdot \sigma_R^2$$

↑  
Mass inside Radius R

$\sigma_R = \text{const}$   
due to flat rotation curve

lets assume constant mass to luminosity ratio  $\frac{M}{L} = \frac{1}{C_{ML}}$

$$L = C_{ML} \cdot M = C_{ML} \cdot 5 \frac{\sigma_R^2 R}{G}$$

Big assumption

$$\frac{L}{R^2} = C_{SB} \leftarrow \text{constant surface brightness}$$

$$L = C_{ML} \cdot 5 \frac{\sigma_R^2 \sqrt{\frac{L}{C_{SB}}}}{G}$$

(Recall Stefan Boltzmann  
 $L = \sigma \cdot A \cdot T^4$   
 $= \frac{L}{A} \sim \sigma T^4$   
in e. galaxy has same T)

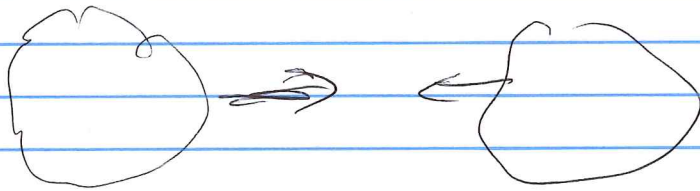
$$L = \left( \frac{C_{ML}^2 \cdot 5^2}{C_{SB} \cdot G} \right) \sigma_R^4$$

const

So we connected luminosity of a galaxy to its rotation speed

Fast passage of the objects  
through each other

~~mean velocity~~  $\rightarrow$  ~~than change in~~  
~~speeds for the interaction~~  
~~time~~  
 Stars did not shift from their initial position around an object C.M.



but velocity did change.

So energy exchange is kinetic energy change

After interaction objects separates so we can treat them as stand alone.

let's say we gain some energy  $\Delta E$  and wait long time to reach equilibrium

$$K_i + \Delta K + U_i + \Delta U = E_i + \Delta E$$

Virial theorem  $-2K = U \Rightarrow -2\Delta K = \Delta U$

$$\Delta K + \Delta U = \Delta E$$

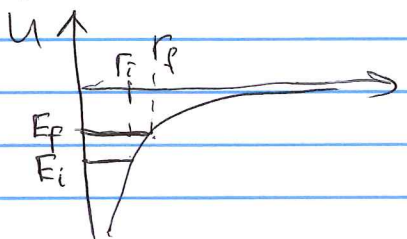
$$\Delta K - 2\Delta K = \Delta E$$

$$K_f - K_i = \boxed{\Delta K = -\Delta E}$$

i.e. after equilibrium reached the kinetic energy decreases

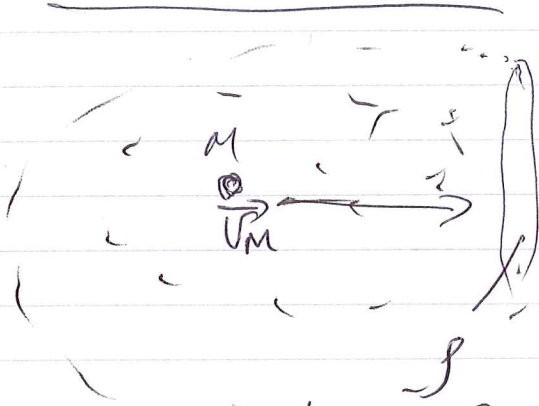
So to drop kinetic energy you need to decrease potential.

$$\boxed{\Delta U = 2\Delta E}$$



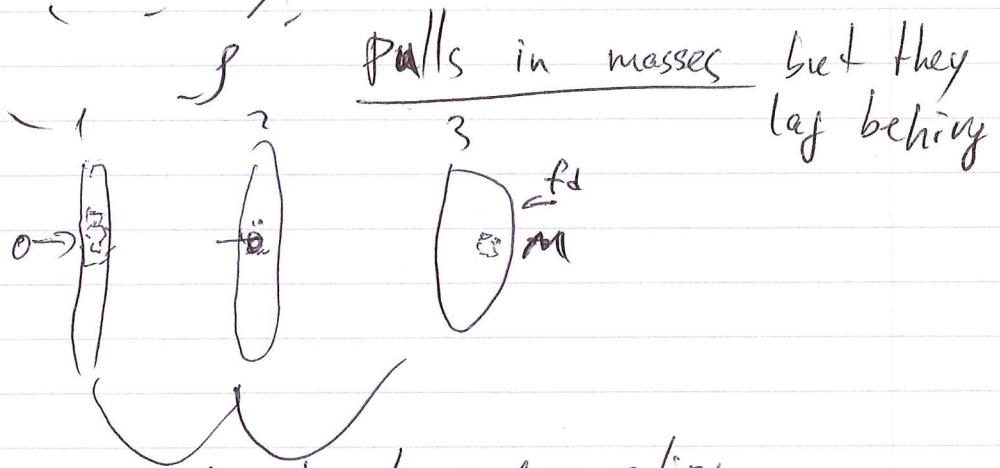
\*) expand  $r_f > r_i \Rightarrow$  rings  
 2) through away parts  $\Rightarrow$  tails

# Dynamic friction



force

$$f_d = c \frac{G^2 M^2 \rho}{v_M^2}$$



pulls in masses but they lag behind

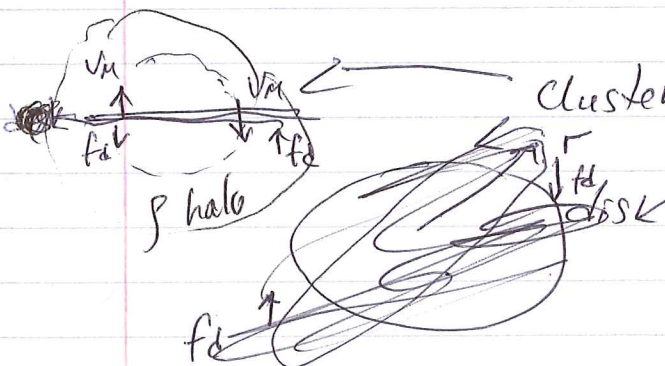
density clump formation due to G pull  $\Rightarrow f_d$

# Dark Matter density

$$\rho(r) \approx \frac{v_{rot}^2}{4\pi G r^2}$$

rotation speed

$$f_d = c \frac{G^2 M^2 \rho}{v_M^2} = c G^2 M^2 \frac{v_M^2}{v_M^2 4\pi G r^2} = c G \frac{M^2}{4\pi r^2}$$



ang moment

$$\frac{dL}{dt} = \frac{d(M v_M r)}{dt} = f_d \cdot r$$

torque

$$M v_M \frac{dr}{dt} = -G C \frac{M^2}{4\pi r}$$

$$\int_{r_c}^0 r dr = \int_0^{t_c} \frac{G C M}{4\pi v_M} dt$$

$$r_{\max} = \sqrt{\frac{t_{\max} C G M}{2\pi v_M}}$$

↑  
capture radius

galaxy age