Lecture 35.

- Milky Way Galaxy
- Oscillations around disk
- Rotation curves and Dark matter requirement
Milky Way Galaxy (Our Home)

Note: all stars we can see with naked eye (or even telescopes) our stars from our galaxy. Other galaxies appear as "nebular" i.e. stars are not resolved.

Milky Way Side View

- Halo (does not rotate)
- Dark matter is even larger
- Bulge $M \approx 10^{10} \, M_\odot$

Zoom in on Sun

- $V = 7.2 \, \text{km/s}$
- $\sim 80 \, \text{pc}$ in disk plane

Young stars

Thin disk $\Rightarrow M \approx 6 \times 10^{10} \, M_\odot$

$\approx 80 \, \text{pc}$ in $z$

Thick disk $M \approx (0.2-0.6) \times 10^{10} \, M_\odot$

$Z = 1 \, \text{kpc}$, older stars

Thick disk mass $\approx 8 \times 10^{10} \, M_\odot$ luminous mass (disks, bulge, halo)

Total mass $\approx 190 \times 10^{10} \, M_\odot$ — huge error bars, i.e. most of the mass is dark matter

Some could be white/red dwarf which are not very luminous

WIMP — Weakly Interacting Massive Particles

MACHO — Massive Compact Halo Object

$\Rightarrow$ observed transient due to G. lensing
Sun oscillations around disk
and within disk
for any other star

g \approx 50 \, \text{pc}
while disk \( h \approx 50 \, \text{pc} \)

\( h \approx 10^{-3} \)

Gauss law for charges

\[ \oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_i}{\varepsilon_0} \]

Enclosed charge

Similarly for mass (since gravity and Coulomb force \( \sim \frac{1}{r^2} \))

\[ \oint \mathbf{G} \cdot d\mathbf{A} = \frac{GM_\text{inside}}{\varepsilon_0} \]

no flux through sides due to symmetry

\[ -2\pi h \, A_{\text{up}} = -4\pi G M_\odot \]

\[ h = 1 \, 10^{-3} \]

\[ g = 4\pi G h \rho \]

\[ h = \frac{\rho}{\mathbf{G} \rho} \]

\[ \omega = \sqrt{4\pi G \rho} \]

\[ f = 0.15 M_\odot \, \text{pc}^3 \]

\[ P_{\text{osc}} = 9.1 \times 10^4 \, \text{years} \]
Rotation curve

Top view in the book

Recall that circular orbital speed
\[ \frac{m v^2}{\ell} = G \frac{M m}{r^2} \]

Outside central region
\[ \frac{dM_r}{dr} = \frac{U^2}{G} \]

\[ S = \frac{1}{4\pi} \frac{U^2}{G} \sim \frac{1}{r^2} \]

but luminous mass have density which scales as \( \sim \frac{1}{r^{3.5}} \) \( \Rightarrow \) Dark Matter is needed to compensate

Rigid body like
\[ U_z = \omega z \]
\[ M_r = \frac{\omega^2 z^2}{G} \]

\[ \frac{dM_r}{dr} = 4\pi \rho(r) r^2 = \frac{3\omega^2 z^2}{G} \]

\[ \Rightarrow \rho(r) = \frac{3}{4\pi} \frac{\omega^2}{G} \text{ const} \] at center region