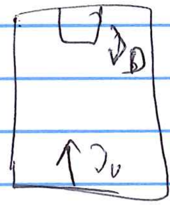
lecture 33

General relativity

In all floating (free fall) ~~refe~~ and non rotating reference frames laws of physics are the same.

$\boxed{\text{Equivalence principle}}$ ~~principle~~

* Time dilation
* Light bending



falling frame

$\nu_D = \nu_0$

lab frame

$\downarrow g$ $\quad \nu_D = \nu_0 \left(1 + \frac{v}{c}\right) =$

$$= \nu_0 \left(1 + \frac{gt}{c}\right)$$

$$\Delta \nu = \nu_D - \nu_0 = \nu_0 \frac{gt}{c}$$

$$\frac{\Delta \nu}{\nu_0} = \frac{gt}{c} = \frac{g \cdot (h/c)}{c} = \frac{gh}{c^2}$$

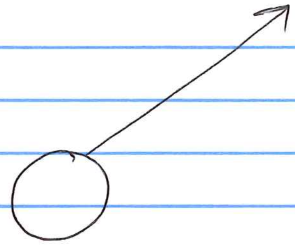
increase of ν_0 i.e. Blue shift

In floating frame: But $\nu_D = \nu_0 \Rightarrow \Delta \nu = 0$

So we need to introduce Red shift to compensate.

$$\frac{\Delta \nu}{\nu_0} = \ominus \frac{gh}{c^2}$$

i.e. as photon moves up its frequency decreases ~~since gravity takes away energy~~



dr ↑ ↓ g

$$\int_{\nu_0}^{\nu_\infty} \frac{d\nu}{\nu} = -\frac{g\,dr}{c^2} = \int_{R_0}^{\infty} -\frac{GM}{r^2c^2}\,dr$$

$$\ln \nu \Big|_{\nu_0}^{\nu_\infty} = +\frac{GM}{r c^2}\Big|_{R_0}^{\infty}$$

$$\ln\left(\frac{\nu_\infty}{\nu_0}\right) = -\frac{GM}{c^2 R_0}$$

$$\nu_\infty = \nu_0\, e^{-\frac{GM}{c^2R_0}}$$

if $\frac{GM}{c^2R_0} \ll 1$

$$\nu_\infty \approx \nu_0\left(1 - \frac{GM}{c^2R_0}\right)$$

2 correction to account space change

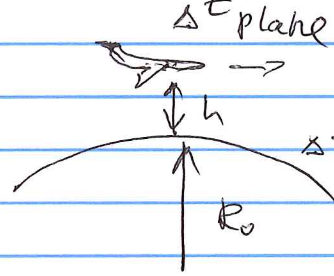
$$\boxed{\nu_\infty = \nu_0\left(1 - 2\frac{GM}{c^2R_0}\right)^{1/2}}$$

$$\frac{\nu_\infty}{\nu_0} = \frac{\Delta t_0}{\Delta t_\infty} = \left(1 - \frac{2GM}{c^2R_0}\right)^{1/2}$$

$\Delta t_0 < \Delta t_\infty$

i.e. "less time passes" for a given time Δt_∞

time dilation!



$$\Delta t_0 = \Delta t_\infty \left(1 - 2\frac{GM}{R_0 c^2}\right)^{1/2}$$

$$\Delta t_{plane} = \Delta t_\infty \left(1 - \frac{2GM}{\underbrace{R_p}_{(R_0+h)} c^2}\right)^{1/2}$$

ground ref

$$\Delta t_{plane} = \Delta t_0 \frac{\left(1 - \frac{2GM}{(R_0+h)c^2}\right)^{1/2}}{\left(1 - \frac{2GM}{R_0 c^2}\right)^{1/2}} =$$

$$\simeq \Delta t_0 \frac{\left(1 - \frac{GM}{(R_0+h)c^2}\right)}{\left(1 - \frac{GM}{R_0 c^2}\right)} \simeq$$

$$\simeq \Delta t_0 \left(1 - \frac{GM}{(R_0+h)c^2}\right)\left(1 + \frac{GM}{R_0 c^2}\right)$$

$$= \Delta t_0 \left(1 - \frac{GM}{\underbrace{(R_0+h)}_{R_0\left(1+\frac{h}{R_0}\right)} c^2} + \frac{GM}{R_0 c^2} + \cancel{\left(\frac{GM}{R_0 c^2}\right)^2}\right)$$

Good atomic clock are stable to the level $10^{-15} \div 10^{-17}$

measurable

$$= \Delta t_0 \left(1 - \frac{GM}{R_0 c^2}\left(1 - \frac{h}{R_0}\right) + \frac{GM}{R_0 c^2}\right) \simeq$$

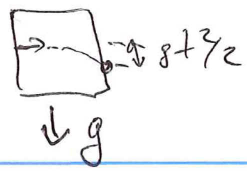
$$\simeq \Delta t_0 \left(1 + \overbrace{\frac{GM}{R_0 c^2}\frac{h}{R_0}}^{=g}\right)$$

$$\simeq \Delta t_0 \left(1 + \frac{gh}{c^2}\right) \Rightarrow$$ while on ground Δt_0 passes in the plane a longer interval

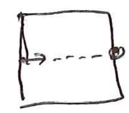
$$\frac{\Delta t_p - \Delta t_0}{\Delta t_0} \simeq \frac{g \overbrace{h}^{10\,km}}{c^2} \simeq \frac{10 \cdot 10^4}{(3\cdot 10^8)^2} \simeq 10^{-12}$$

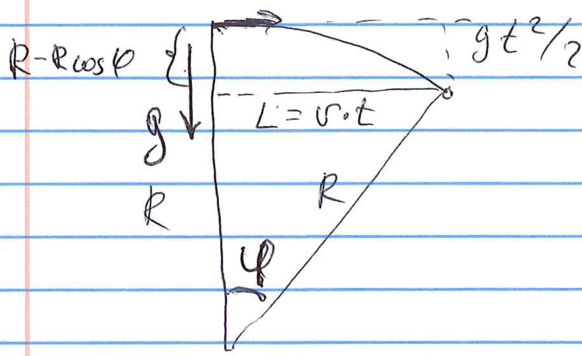
extra 5 hour flight = 18000

$\Rightarrow$ you shave from you life $10^{-12} \cdot 18 \cdot 10^3 \simeq 2 \cdot 10^{-8}$ s



Floating Frame





$$\begin{cases} R(1-\cos\varphi) = gt^2/2 \\ R \cdot \sin\varphi = v \cdot t \end{cases}$$

$$\varphi \ll 1$$

$$R \approx \frac{v \cdot t}{R}$$

$$R(1 - \cos\varphi) = gt^2/2$$

$$R\left(1 - \sqrt{(1-\varphi^2)}\right) = gt^2/2$$

$$R\left(1 - 1 + \tfrac{1}{2}\varphi^2\right) = gt^2/2$$

$$R \cdot \frac{1}{\cancel{2}}\left(\frac{v \cdot \cancel{t}}{R}\right)^2 = g\cancel{t^2}/\cancel{2}$$

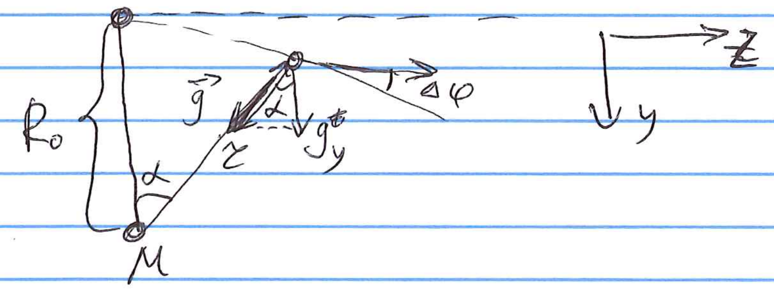
$$R \cdot \frac{v^2}{R^2} = g \Rightarrow \boxed{R_{curv} = \frac{v^2}{g}} \quad \text{eq1}$$

for small φ $\boxed{\varphi \approx \frac{v \cdot t}{R_{curv}}}$ eq2

Now replace $\boxed{v \Rightarrow c}$

curvature of light

Light Bending near mass



from eq. 2 $\qquad$ from eq. 1

$$\Delta\varphi = \frac{v_0 \Delta t}{R_{curv}}, \qquad R_{curv} = \frac{v^2}{g_y}$$

$$\Delta\varphi = \frac{v\Delta t}{v^2} \cdot g_y \overset{v \Rightarrow c}{=} \frac{\overbrace{c\Delta t}^{=\Delta z}}{c^2} \frac{GM}{r^2} \cdot \cos\alpha$$

$$= \frac{\Delta z}{c^2} \frac{GM}{r^2} \cos\alpha \Rightarrow$$

$\Rightarrow$

$$d\varphi = \frac{dz}{c^2} \frac{GM}{r^2} \cos\alpha =$$

$r = \frac{R_0}{\cos\alpha}$

asuming small "sag"

$z = R_0 \cdot \tan\alpha$

$dz = \frac{R_0}{\cos^2\alpha} d\alpha$

$$= \frac{R_0\, d\alpha}{c^2 \cos^2\alpha} \cdot \frac{GM}{R_0^2/\cos^2\alpha} \cdot \cos\alpha =$$

$$\int_0^{\varphi_B} d\varphi = \frac{GM}{c^2 R_0} \int_{-\pi/2}^{\pi/2} \cos\alpha \, d\alpha = \boxed{2 \frac{GM}{R_0 \cdot c^2} = \varphi_{Bending}}$$

we need extra (2) to account time dillation, photon spend more time when it is near the star

$$\boxed{\varphi_B = 4 \frac{GM}{R_0 c^2}}$$