Lecture 32

* Dipole radiation and rotation slow down of neutron stars

* Gravitational ~~pull~~, and typical energy scales

* Escape velocity $\Rightarrow$ Black holes (classical approach)

* Black hole candidate in the center of our Galaxy with $M = \underline{3.6 \cdot 10^6} \, M_\odot$

Rotation slow down due to dipole radiation

$$\frac{dE}{dt} = -C\omega^4$$

assuming we are taking kinetic energy of rotation

$$\frac{d}{dt}E = \frac{d}{dt}\left(\frac{I\omega^2}{2}\right) = -C\omega^4$$

$$I\omega\frac{d\omega}{dt} = -C\omega^4$$

$$\int_{\omega_i}^{\omega_f}\frac{d\omega}{\omega^3} = -\int_0^t \frac{C}{I}\,dt$$

$$= -\frac{2^{-1}}{\omega^2}\Bigg|_{\omega_i}^{\omega_f} = \frac{2^{-1}}{\omega_i^2} - \frac{2^{-1}}{\omega_f^2} = \frac{C}{I}t$$

$$\omega_f = \omega(t) = \sqrt{\frac{2^{-1}}{2^{-1}/\omega_i^2 + \frac{C}{I}t}} = \sqrt{\frac{1}{1/\omega_i^2 + \frac{2C}{I}t}}$$

Note we know from measurements

$$\frac{dP}{dt} = \frac{d\frac{2\pi}{\omega}}{dt} = 2\pi\left(-\frac{d\omega/dt}{\omega^2}\right)$$

$$\frac{d\omega}{dt} = \frac{-1/2\cdot\frac{2C}{I}}{\left(1/\omega_i^2 + \frac{2C}{I}t\right)^{3/2}}\Bigg|_{t=0} = \frac{2C}{I}\omega_i^3$$

measured

$$\frac{d\omega}{dt}\Bigg|_{t=0} = \frac{C}{I}\omega_i^3 = -\frac{dP}{dt}\cdot\omega_i^2$$

so we will know C/I

Neutron star gravitational pool

$$g = \frac{GM}{R^2}$$

$$g_{\odot \text{,,surface''}} = \frac{G M_\odot}{R_\odot^2} = \frac{6.67 \cdot 10^{-11} \cdot 2 \cdot 10^{30}}{(7 \cdot 10^8)^2} =$$

$$\approx \frac{2}{7} \cdot \frac{10^{19}}{10^{16}} \approx \frac{2 \cdot 10^3}{7} \approx 3 \cdot 10^2 \ \frac{m}{c^2}$$

$$g_{n.s} = \frac{G \cdot 3 M_\odot}{(10^4)^2} = \frac{6.67 \cdot 10^{-11} \cdot 3 \cdot 2 \cdot 10^{30}}{10^8} =$$

$$= 20 \cdot 2 \cdot \frac{10^{+19}}{10^8} = 4 \cdot 10^{12} \ \frac{m}{c^2}$$

So what?

Imagine dropping 1 kg from 1m hight on the surface of N.S.

$$\text{Kinetic energy } E_K = \frac{mv^2}{2} = m(g \cdot h)$$

$$= \underset{1kg}{m} \cdot 4 \cdot 10^{12} \cdot 1m \approx 4 \cdot 10^{12} \text{ J}$$

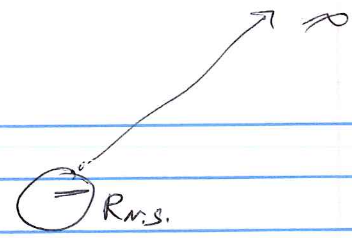
Compare to nuclear fusion, if the mass was all Hydrogen which fused

$$\text{fusion} = E_{release} = 0.007 \cdot mc^2 = 0.007 \cdot 1 \cdot (3 \cdot 10^8)^2$$

$$= 7 \cdot 3^2 \cdot 10^{16} / 10^3 \approx$$

$$\approx 7 \cdot 10^{14} \text{ J}$$

~~nuclea~~ atomic Bomb has yeld
$25 \cdot 10^{12}$ J/kg



Escape velocity

$$\Delta U + \Delta K = 0 \Rightarrow \Delta K = -\Delta U$$

$$0 - \frac{mv^2}{2} = -\frac{GMm}{R} - \Big(-\underbrace{\cancel{\frac{GM}{R_\infty}}}_{0}\Big)$$

$$v = \sqrt{\frac{2GM}{R}}$$

escape velocity

$$v_{sun} = \sqrt{2 \cdot \frac{G \cdot M_\odot}{R_\odot}} = \sqrt{2 \cdot \frac{6.67 \cdot 10^{-11} \cdot 2 \cdot 10^{30}}{7 \cdot 10^{8}}}$$

$$\approx 2 \sqrt{\frac{10^{19}}{10^{8}}} = 2 \cdot \sqrt{10^{11}} = 2 \cdot 10^{5} \cdot \sqrt{10}$$

$$\simeq 6 \cdot 10^{5} \ m/s$$

$$v_{N.S \text{ escape}} = \sqrt{\frac{2 \cdot G \cdot 3M_\odot}{\underbrace{R_{N.S}}_{\simeq 10^4 m}}} = \sqrt{\frac{2 \cdot 6.67 \cdot 10^{-11} \cdot 3 \cdot 2 \cdot 10^{30}}{10^{4}}} =$$

$$= \sqrt{4 \cdot 20 \cdot 10^{15}} = 2 \cdot \sqrt{2} \cdot 10^{8} \ m/s$$

$$= 2 \cdot 1.4 \cdot 10^{8} \simeq \boxed{2.8 \cdot 10^{8} \ m/s \simeq 0.93 \ c}$$

If we need escape velocity > c we are in trouble ⇒ Black hole
the light cannot escape

$$c = \sqrt{\frac{2GM}{R}}$$

$$R_{BH} \leq \frac{2GM}{c^2}$$

$$M = 3M_\odot \Rightarrow \boxed{R_{BH} \simeq \left(\frac{2\cdot G\cdot 3\cdot 2\cdot 10^{30}}{(3\cdot 10^8)^2}\right) = \frac{2\cdot 6.67\cdot 3\cdot 2\cdot 10^{19}}{3^2\cdot 10^{16}} \simeq 9\cdot 10^3\ \text{m}}$$

Mo... ck Hole

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$$... ^2$$

$$R ... \left(... eV \cdot 1.6\cdot 10^{-19}\ J/eV \right) / c^2$$

$$= \frac{2\cdot 6.67\cdot 10^{-11}\cdot 3.2\cdot 10^{12}\cdot 10^{-19}}{(3\cdot 10^8)^4} =$$

$$\approx \frac{4\cdot}{3^2}\ \frac{10^{-18}}{10^{32}} \approx 0.4\cdot 10^{-60}\ \text{m}$$

to small to worry