Lecture 3~~9~~

Neutron stars

$$p^+ + e^- \longrightarrow n + \nu_e$$

n is heavier tha (p + e)
so we need extra energy

Such energy is in Kinetic energy of e^-

$$K = m_e c^2 \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 \right)$$ special theory of relativity

$$\cancel{m_e c^2} + m_e c^2 \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \cancel{-1} \right) + m_p c^2 = m_n c^2$$

$$\frac{m_e}{m_n - m_p} = \frac{9.1 \cdot 10^{-31}}{1.6749 \cdot 10^{-27} - 1.6726 \cdot 10^{-27}} \simeq 0.4 =$$

$$= \sqrt{1 - \frac{v^2}{c^2}}$$

$$\Rightarrow \frac{v^2}{c^2} = 1 - 0.4^2 = 0.84$$

$$\frac{v}{c} = 0.916.$$

$$PV = NkT = \frac{2}{3} \underbrace{\left(N \tfrac{3}{2} kT \right)}_{\text{averaged per partic}} = E$$

~~Recall~~

Recall $\Rightarrow P = \frac{2}{3} \frac{E}{V} = \frac{1}{3} n_e p v =$

$$= \frac{1}{3} n_e m_e v^2$$

note non-relativistic

$$\boxed{P = \frac{(3\pi^2)^{2/3}}{5} \frac{\hbar^2}{m_e} n_e^{5/3} = \frac{1}{3} n_e m_e v^2}$$

$$\frac{3(3\pi^2)^{2/3}}{5} \frac{\hbar^2}{m_e^2} \frac{n_e^{2/3}}{c^2} = \frac{v^2}{c^2} = 1 - \left(\frac{m_e}{m_n - m_p} \right)^2$$

5.74

$$n_e = \left[\frac{m_e^2 c^2}{\hbar^2} \cdot \frac{5}{3(3\pi^2)^{2/3}} \left(1 - \left(\frac{m_e}{m_n - m_p}\right)^2\right)\right]^{3/2}$$

$$= \left(\frac{m_e c}{\hbar}\right)^3 \left(\frac{5}{3}\right)^{3/2} \frac{1}{(3\pi^2)} \left(1 - \left(\frac{m_e}{m_n - m_p}\right)^2\right)^{3/2}$$

$$= \frac{A}{Z} \frac{Z}{A} \frac{\rho}{m_H}$$

$m_H = 1.67353 \cdot 10^{-27}$ kg
$m_n = 1.67492 \cdot 10^{-27}$ kg
$m_p = 1.67262 \cdot 10^{-27}$ kg
$m_e = 9.1094 \cdot 10^{-31}$ kg

$$\rho = \frac{A}{Z} m_H \left(\frac{m_e c}{\hbar}\right)^3 \underbrace{\left(\frac{5}{3}\right)^{3/2} \frac{1}{(3\pi^2)}}_{\approx 0.072} \left(1 - \left(\frac{m_e}{m_n - m_p}\right)^2\right)^{3/2}$$

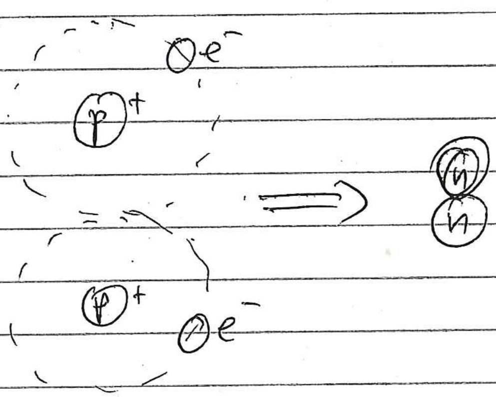
$\frac{A}{Z} \approx 2$ For $^{56}_{26}Fe$, since $\frac{A}{Z} = \frac{56}{26} \approx 2$

$$\boxed{\rho_{critical} = 3.28 \cdot 10^9 \ \frac{kg}{m^3}}$$

note that the book claims that the "actual" value is $1.2 \cdot 10^{10} \ \frac{kg}{m^3}$

at which electrons are fast enough to provide enough energy for binding

When $\rho > \rho_{critical}$ neutrons start to form from p and e^-



density is much higher at the final stage

however there are still some p and e^- due to β-decay $n \to p^+ + e^- + \bar{\nu}_e$

$n:p:e = 8:1:1$

Using chandrasekhar limit, followed from hydrostatic equilibrium

$$\frac{2}{3}\pi G \rho^2 R^2 = P \quad \text{non relativistic}$$

$$= \frac{(3\pi^2)^{2/3}}{5} \frac{\hbar^2}{m_f} [n_f]^{5/3}$$

$$M = \frac{4\pi}{3}\rho R^3$$

for neutrons $n_f = \frac{\rho}{m_H}$

$$R = \frac{(18\pi)^{2/3}}{10} \frac{\hbar^2}{G \underset{\underset{m_n \approx m_H}{\parallel}}{m_f} M^{1/3}} \left[\frac{1}{m_H}\right]^{5/3}$$

$$= \frac{(18\pi)^{2/3}}{10} \frac{\hbar^2}{G M^{1/3}} \left[\frac{1}{m_H}\right]^{8/3}$$

For $M \approx 1.4 M_\odot$

$R \approx 4400$ m accepted value ≈ 10 km

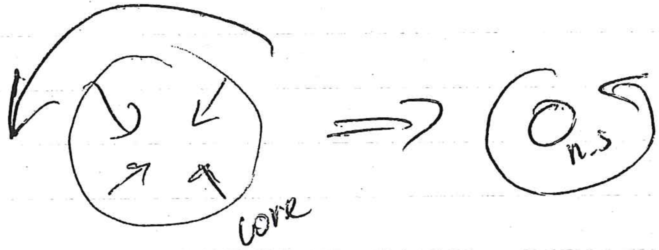
↑ neutron star

$$\rho = \frac{1.4 \cdot M_\odot}{\frac{4\pi}{3}(10^4)^3} = \frac{1.4 \cdot 2 \cdot 10^{30}}{\frac{4\pi}{3} \cdot 10^{12}} \approx$$

$$\approx 6.6 \cdot 10^{17} \frac{kg}{m^3}$$

$\boxed{\text{again } \rho_{nuclear} \approx 10^{17}\ kg/m^3}$

Rotation Neutron star



$I\omega = const$ angular mom

$I \sim M_0 R^2$

$\cancel{M_0} R_f^2 \cdot \omega_f = \cancel{M_0} R_i^2 \cdot \omega_i$

$\omega_f = \omega_i \left(\frac{R_i}{R_f}\right)^2$ R_i: $3 \times 10^6\,m$ for W.D R_f: $10\,km$

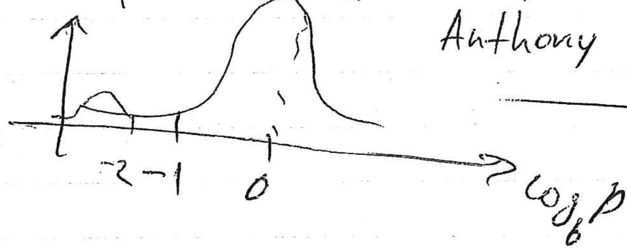
$P_{core} = 1350\,s$ for observed W.D.
$\Rightarrow \frac{1}{\omega_i}$

$R_f \approx 10\,km$

n.s. ~~[illegible]~~

$\Rightarrow$ Periods of Pulsars Jocelyn Bell Anthony Hewish

$\Rightarrow$



$\omega = \frac{2\pi}{P}$

$\hookrightarrow P_f = P_i \left(\frac{R_f}{R_i}\right)^2 \approx P_i \cdot 10^{-5}$

$\approx 1350\,s \cdot 10^{-5} \approx 13\,ms$

Upper limit on Rotation

$$\frac{mv^2}{r} \leq \frac{GMm}{r^2}$$

$$m(\omega r)^2 \leq \frac{GMm}{r}$$

$$\omega \leq \sqrt{\frac{GM}{r^3}} \Rightarrow P \geq \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R^3}{GM}}$$

$$= 2\pi\sqrt{\frac{1}{4\pi/3\, G\rho}}$$

↑ maximum rotating speed allowed before breakdown

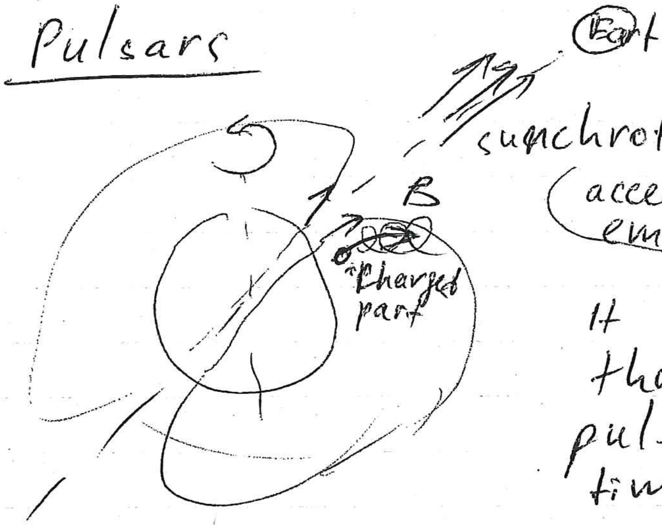
$$\rho_{W.D} \approx 10^9\ kg/m^3 \Rightarrow P \geq 12\ sec$$

$$\rho_{n.s} = 6 \cdot 10^{17}\ kg/m^3 \Rightarrow P \geq 2\pi\sqrt{\frac{1}{\frac{4\pi}{3} \cdot 6.6 \cdot 10^{-11} \cdot 6 \cdot 10^{17}}}$$

$$\approx 4.8 \cdot 10^{-4}\ s$$

Fastest known Pulsar $P = 0.00139$

Pulsars



synchrotron radiation

acceleration particle emit radiation!

It was noticed that Periods of pulsar grows with time

$$\frac{dP}{dt} \approx 10^{-15} \text{ typically}$$

but one known to be $\approx 10^{-19}$

If period is growing that means that rotation slows down.

some speeding up but what is the mehanism?

moment of inertia

$$\frac{dE}{dt} = \frac{d\,\frac{1}{2}I\omega^2}{dt} = \frac{1}{2}I\frac{d\,(2\pi/P)^2}{dt} =$$

$$= 2\pi^2 I\left(-\frac{dP}{dt}\right)\cdot\frac{2}{P^3} =$$

$$= \frac{1}{2}I\omega^2\left(-\frac{2\,dP}{P\,dt}\right)$$

$$I = \frac{2}{5}MR^2$$

$M = 1.4\,M_\odot$, $P = 0.03$ — Crab nebula

$R = 10\,km$, $\dot{P} = 4.2\cdot 10^{-13}$

$$\frac{dE}{dt} = \frac{1}{5}M\cdot R^2\left(\frac{2\pi}{P}\right)^2\cdot\left(-\frac{2}{P}\frac{dP}{dt}\right) = -6.87\cdot 10^{+31}\ \frac{J}{s}$$

outshine sun $\approx 10^5 L_\odot$

Energy carried away by dipole radiation $\sim \omega^4$

We stopped here

$$\frac{dE}{dt} = -C\omega^4$$

$$\frac{d\frac{1}{2}I\omega^2}{dt} = -c\omega^4$$

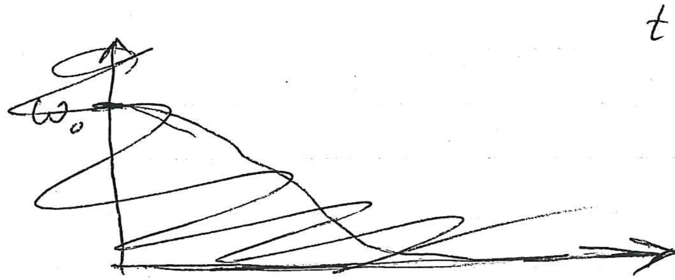
$$\frac{2}{2}I\,\omega\frac{d\omega}{dt} = -c\,\omega^4$$

$$\frac{d\omega}{dt} = -\frac{C}{I}\omega^3 \Rightarrow \frac{1}{3}\frac{1}{\omega^2} = \frac{c}{I}t + C_0$$

$$\cancel{\omega(t) = -\frac{c}{4I}\omega^4 +} \omega_0$$

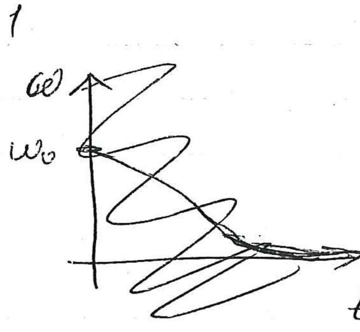
$t=0 \Rightarrow \omega = \omega_0$

$C_0 = \frac{1}{3\omega_0^2}$



$$\frac{1}{3}\left(\frac{1}{\omega^2} - \frac{1}{\omega_0^2}\right) = \frac{c}{I}t$$

$$\omega^2 = \left(3\frac{c}{I}t + \frac{1}{\omega_0^2}\right)^{-1}$$



$$\cancel{\omega = 0 \Rightarrow t =}$$