Lecture 28

Collapse - Hydrodynamic eq.

Hydrodynamics

time dependence.

\[ \int \frac{dv}{r^2} = - \frac{GM_\odot M}{r^2} - \frac{dP}{dt} \]

assuming it in other words free collapse

\[ = \frac{d^2r}{dt^2} = - \frac{GM_\odot M}{r^2} = - \frac{G}{r^2} \]

\[ \frac{dr}{dt} = v \]

\[ \sqrt{\frac{1}{2}} \frac{d(v^2)}{dt} = - \frac{GM_\odot M}{r^2} \]

\[ = - \frac{GM_\odot M}{r^2} - \frac{d\rho}{dt} \]

\[ \int dt \left( \frac{1}{2} \frac{d(v^2)}{dt} + \frac{G M_\odot M}{r} \right) \]

\[ \frac{1}{2} v^2 = - \frac{GM_\odot M}{r} + C \]

\[ v = 0 \text{ when } r = r_0 \]

\[ C = \frac{GM_\odot M}{r_0} \]

\[ \frac{1}{2} v^2 = - \frac{GM_\odot M}{r} + \frac{1}{2} \frac{GM_\odot M}{r_0} \]

\[ v = \sqrt{2 \frac{GM_\odot M}{r_0} \left( \frac{r_0}{r} - 1 \right)} \]
\[
U = \frac{dr}{dt} = -\sqrt{\frac{2GMr}{r_0}} \left( \frac{r_0}{r} - 1 \right)
\]
\[
\frac{r_0}{d(r/r_0)} = -\sqrt{\frac{2GMr}{r_0}} \left( \frac{r_0}{r} - 1 \right)
\]
\[
r/r_0 = \theta \quad r \in (0, r_0) \rightarrow \theta = (0, 1)
\]
\[
-\frac{d\theta}{\sqrt{\frac{2GMr}{r_0} \left( \frac{r_0}{r} - 1 \right)}} = dt
\]
\[
\sqrt{\frac{8\pi}{3} G \rho_0} \approx \gamma
\]
\[
-\frac{1}{\gamma} \frac{d\theta}{\sqrt{\theta - 1}} = -\frac{1}{\gamma} \frac{\sqrt{\theta} \, d\theta}{\sqrt{\theta - 1}} = dt
\]
\[
\theta = \cos^2 \gamma \rightarrow \frac{1}{\gamma} \frac{\cos^2 \gamma \sin \gamma \, d\gamma}{\sin \gamma} = dt
\]
\[
\cos^2 \gamma \, d\gamma = -\frac{\gamma}{2} \, dt
\]
\[
\frac{1 + \cos 2\gamma}{2} \, d\gamma = \frac{\gamma}{2} \, dt \rightarrow \int_{\gamma = \frac{\pi}{2}}^{\gamma + \frac{1}{2} \sin 2\gamma} = \gamma_0 + C
\]
\[
\gamma + \frac{1}{2} \sin 2\gamma = \gamma_0 + C\quad r_0 = r_0, \quad r = r_0
\]
\[
\Rightarrow \quad \frac{77}{2} = \kappa \cdot t
\]

\[
t_{\text{free fall}} = \frac{77}{2\kappa} = \frac{77}{2 \sqrt{\frac{877}{3} G \rho_0}}
\]

\[
t_{\text{eff}} = \sqrt{\frac{377}{32 G \rho_0}} \quad \text{Homologous collapse time}
\]

Note: does not depend on initial size, so all layers (with the same \( \rho \)) will arrive to center at the same time.

This will not be true if there were original density distribution, for example if center is more dense it will collapse first.

Overall, it cannot be the description of the whole process since we compress to 0 radius which require infinite densities.
Let's see how long does it take to collapse interstellar medium (ISM) cloud.

We will discuss dense clouds which are prone to collapse \( \psi \) globules \( M = 1.6 \times 10^3 M_\odot \).

They consist mostly of molecular hydrogen \( H_2 \) with densities \( N_{H_2} = 10^{10} \text{m}^{-3} \).

\[
\psi_0 = 2M_\odot N_{H_2} = 2 \times 1.6 \times 10^{-24} \times 10^{10} \approx
\]
\[
\approx 3.2 \times 10^{-14} \frac{\text{Kg}}{\text{m}^3}
\]
Their overall mass \( \approx 10 M_\odot \).

\[
\ell_{ff} = \sqrt{\frac{3\pi}{32 \cdot 6 \cdot \psi_0}} = \sqrt{\frac{3\pi}{32 \cdot 6 \cdot 1.6 \times 10^{-11} \cdot 3.2 \times 10^{-14}}
\]
\[
= 1.15 \times 10^{13} \approx 3.7 \times 10^5 \text{ years}
\]

Much smaller than Kelvin-Helmholtz time scale.

If we want to see how \( \ell_{ff} \) depends on time,

\[
\frac{\partial}{\partial t} \left( \frac{1}{2} \sin 2\varphi \right) = \varphi \approx t
\]

\[
\varphi(r) \Rightarrow \cos^2 \varphi = r/r_0 \Rightarrow \varphi(r)
\]

\[
\varphi(r) + \frac{1}{2} \sin 2\varphi(r) = \kappa \ell(t)
\]

(can be solved numerically

\[
\ell(t) \Rightarrow
\]

\[
\ell_{ff}
\]
Star Luminosity

Recall Kelvin-Helmholtz energy time frame

\[ E = \frac{-3 GM^2}{10 R} \]

\( \Delta E \) for \( R_3 \rightarrow R_0 \), for the tff would imply that they outshine Sun by orders of magnitude

HW: calc. how much

\[ \boxed{} \]

See bunch of typical Nebulas/clouds on website.

Globules where star form,
Dark means \( \Rightarrow \) dense
Emission - due to highlight of inner stars
Reflection - due to light source at the side