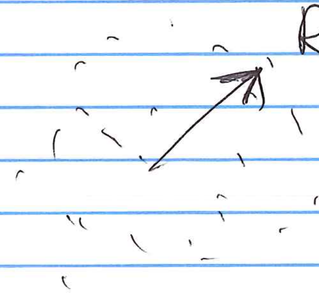
Lecture 25 Proto star formation

1st we discuss clouds in the sky.

Once we convince our self that there is available matter let's see the ~~[illegible]~~ necessary conditions for cloud to collide.



R_c - cloud size.

Particles in the cloud moves with random velocity, so we will use temperature (T) to describe average / typical energy / speed.

Kinetic energy $K = \underbrace{\frac{3}{2} N K_B T}_{\text{ideal monaatomic gas}} = \frac{3}{2} \frac{\overbrace{M_c}^{\text{cloud mass}}}{\underbrace{\mu}_{\text{mean molecular mass}} \underbrace{m_H}_{\text{Hydrogen mass}}} K_B T$

We will make use of <u>the virial theorem</u> (we did it before in lecture devoted to Possible star energy sources)

$$\boxed{2\overline{K} = -\overline{U}_g}$$

in equilibrium averaged values $(\overline{K}, \overline{U_g})$ of energy

$$U_g = -\int_0^{R_c} G \frac{M_r\,dm}{r} =$$

$$= -\int_0^{R_c} \frac{G M_r \, 4\pi \rho r^2\, dr}{r} =$$ — shell mass

$$= \Big/ \begin{matrix}\text{assuming}\\ \rho = \text{const}\end{matrix} \Big/ = -\int_0^{R_c} G \frac{\frac{4\pi}{3}\rho r^3 \cdot 4\pi \rho r^2}{r} dr$$

$$= -G \frac{16\pi^2}{3} \rho^2 \int_0^{R_c} r^4 dr =$$

$$= -G \frac{16\pi^2}{15} \rho^2 R_c^5 =$$

$$= -G \frac{\frac{4\pi}{3}\rho R_c^3 \cdot \frac{4\pi}{3}\rho R_c^3}{R_c} \cdot \frac{3}{5} =$$

$$\boxed{U_g = -G \frac{3}{5} \frac{M_c^2}{R_c}}$$

~~From Vi~~

if there is no equilibrium
the cloud can expand or compress.

<u>Compression / shrinkage condition</u>

$$2K < -U_g$$

$$\cancel{2}\cdot\frac{3}{\cancel{2}}\frac{M_c}{\mu m_H}k_BT < (+)\frac{3}{5}\frac{GM_c^2}{R_c} \qquad \text{(eq.1)}$$

$$M_c > \frac{5k_BT}{G\mu m_H}R_c = \qquad R_c = \left(\frac{M_c}{\rho_0}\frac{3}{4\pi}\right)^{1/3} \quad \text{(eq. 2)}$$

$$= \frac{5k_BT}{G\mu m_H}\cdot\left(\overset{=V}{\frac{M_c}{\rho_0}}\cdot\frac{3}{4\pi}\right)^{1/3} =$$

starting density

$$M_c^{2/3} > \left(\frac{5k_BT}{G\mu m_H}\right)\cdot\left(\frac{3}{4\pi}\frac{1}{\rho_0}\right)^{1/3}$$

Collapse condition

$$\boxed{M_c > \left(\frac{5k_BT}{G\mu m_H}\right)^{3/2}\cdot\left(\frac{3}{4\pi}\frac{1}{\rho_0}\right)^{1/2} = M_J}$$

$\Updownarrow$

The Jeans criterion

the Jeans mass

So This is a bit counterintuitive

$$M_c > M_J \sim 1/\sqrt{\rho}$$

Since the dense the cloud the higher the gravitational atraction, but it also increases the # of particle ($\sim \rho$) and thus kinetic energy (K).

using eq. 1 and 2 we can rewrite

$$\frac{K_B T}{\mu m_H} < \frac{1}{5} G \frac{4\pi}{3} \rho_0 R_c^2$$

Collapse criterion

$$\boxed{R_c > R_J \equiv \left(\frac{15 K_B T}{4\pi G \mu m_H} \frac{1}{\rho_0} \right)^{1/2}}$$

the Jeans radius

Typical number $T \simeq 50\,K$

$n \simeq 5 \cdot 10^8 \frac{1}{m^3}$

Hydrogen cloud

$\rho = \mu m_H \cdot n$

$\rho = / \mu = 1 / = 8.4 \cdot 10^{-19} \frac{kg}{m^3}$

$\Rightarrow M_J \simeq 1500 M_\odot \Rightarrow$ Too much Hydrogen is stable

molecular cloud

$T \simeq 10K$, $n_{H_2} = 10^{10}\ m^{-3} \Rightarrow \rho = 2 \cdot m_H n_{H_2}$

$\simeq 3 \cdot 10^{-17}\ kg/m^3$

$\Rightarrow M_J \simeq 8 M_\odot$

isothermal sound speed ($\gamma = 1$)

$$v_T = \sqrt{k_B T / m_p} = \sqrt{\frac{k_B T}{\mu m_H}}$$

$$P_0 = \frac{N}{V} k_B T = \frac{\rho_0}{\mu m_H} k_B T = \rho_0 v_T^2$$

pressure

$$M_J = \left(\frac{5}{G} \overbrace{\frac{k_B T}{\mu m_H}}^{= v_T^2} \right)^{3/2} \left(\frac{3}{4\pi \rho_0} \right)^{1/2} =$$

$$= \left(\frac{5}{G} v_T^2 \right)^{3/2} \left(\frac{3}{4\pi} \frac{v_T^2}{P_0} \right)^{1/2} =$$

$$= \left(\frac{5}{G} \right)^{3/2} \left(\frac{3}{4\pi} \right)^{1/2} \frac{v_T^4}{\sqrt{P_0}}$$

$$\boxed{M_J = \underbrace{5.46}_{= C_J} \cdot \frac{v_T^4}{G^{3/2} P_0^{1/2}}}$$

under assumption of no external pressure

More alaborative calculation gives

the Bonnor - Ebert mass

$$\boxed{M_{BE} = C_{BE} \cdot \frac{v_T^4}{G^{3/2} P_0^{1/2}}}, \quad \text{where } C_{BE} \simeq 1.18$$