lecture 24

gravitational mechanism of stellar pulsation

$g$-mode - bubble squashing

Buoyant force

\[ \text{net } = \left( -\rho^{(6)} g + \sigma^{(6)} \right) V = \left( \rho^{(5)} - \rho^{(6)} \right) g V \]

g - gravity pull down

once bubble displaced $\delta r$ dr

we write Taylor expansion

\[ \rho^{(5)} = \rho_{\text{initial}} + \frac{d \rho^{(5)}}{dr} \delta r \]

\[ \rho^{(6)} = \rho^{(6)}_{\text{initial}} + \frac{d \rho^{(6)}}{dr} \delta r \]

assuming that $\rho^{(5)}_{\text{initial}} = \rho^{(6)}_{\text{initial}}$ i.e. they have the same starting point from which bubble emerge

\[ \text{net } = g \left( \frac{d \rho^{(5)}}{dr} \delta r - \frac{d \rho^{(6)}}{dr} \delta r \right) V = \]

\[ = \left( \frac{d \rho^{(5)}}{dr} \delta r - \frac{d \rho^{(6)}}{dr} \delta r - \frac{d \rho^{(6)}}{dr} \delta r \right) g V \]
we require adiabatic process for bubble

\[ PV^\gamma = \text{const} \]

since \( m \) of the bubble is a const, i.e. we have mass conservation

\[ P \left( \frac{V}{m} \right)^\gamma = \text{const} \]

\[ d \left( PS^{-\gamma} \right) = dP S^{-\gamma} + (-\gamma) P S^{-\gamma-1} dP = 0 \]

\[ \frac{dP}{dP} = \frac{S^{-\gamma}}{1 + \gamma S^{-\gamma-1} P} = \frac{S'(\theta)}{dP'(\theta)} \]

recall that this is about bubble

now we can plug it to the net eq.

\[ f_{\text{net}} = V g \left( \frac{dP'(\theta)}{d\theta} - \frac{S'(\theta)}{dP'(\theta)} \right) S'r = V g \left( \frac{dP}{d\theta} - \frac{1}{dP} \right) S'r \]

since \( P'(\theta) = P'(\theta) \) to maintain the bubble

and \( S'(\theta) = S'(\theta) \) we can drop \' \theta \' and \' s \' subscript, keeping in mind that it is actually \' s \' everywhere now

\[ f_{\text{net}} = V g A \rho S' r \]

\[ A = \frac{1}{S} \frac{dP}{d\rho} - \frac{1}{dP} \frac{dP}{dr} \]
if \( A < 0 \) we have a restoring force

think about \( < \), so our bubble will go back to equilibrium and thus oscillate around this point.

Since \( \mathcal{P}V = m \) of the bubble

\[
\text{fnet} = ma = mg(\delta r)
\]

\[
a = (\delta r)
\]

get again equation of harmonic oscillator

\[
\omega = \sqrt{-gA} = \sqrt{\left( \frac{1}{\delta P} \frac{dP}{dr} - \frac{1}{\delta r} \right) g}
\]

\[
\Omega = \frac{2\pi}{\omega}
\]

Note that if \( A > 0 \) then there is no restoring force. Once the bubble starts to move up, it will keep going to do it \( \Rightarrow \) convection condition
Oscillation Driving mechanism

It is not enough to have conditions for oscillations; some mechanism must drive this oscillation since otherwise oscillation energy will dissipate and oscillation will seize.

On top of it whatever the source of energy (fusion) it must be applied right to do a positive work on a slice of a star.

\[
\begin{align*}
\text{work} & \geq 0 & \text{if we go } \Theta \\
\text{and negative if we do } \Theta \text{ direction} & \\
W &= \int PdV
\end{align*}
\]
Opacity effects, $K$ and $J$ - mechanism

Generally $K \approx \frac{3}{4} \text{cool}$

We need $K$-to be increased with $T$ can be achieved in partially ionization zone if injection of heat goes not to kinetic energy of atoms (and thus increase of $T$) but to increased ionization and thus $\rightarrow K$

This will introduce delay:
Max pressure will happen after maximum compression

Additionally if some region is heated less during compression (i.e. $T$ is smaller than surrounding)
then heat additionally will go to such region (J-mechanism)

Partial ionization zones thus acts like a piston, and their location in star defines what mode will oscillate if any

This in turn depends on star temperatures
See figs. 14.14
p-modes, going back to pressure governed node, we discussed motion of the spherical layer as a whole. But we can also observe ripples on the surface or in deeper parts of the star, this described by spherical harmonic function $\ell^m(\theta, \phi)$.

\[ l = 0, 1, 2, 3 \ldots \]
\[ m = -l, \ldots, 0, \ldots, l \quad \text{overall} \quad 2l+1 \text{ values} \]

\[ Y^0_l(\theta, \phi) = K^0_l \text{(const)} \]
\[ Y^1_l(\theta, \phi) = K^1_l \cos \theta \]
\[ Y^\pm_l(\theta, \phi) = K^\pm_l \sin \theta e^{\pm i\phi} \]

Wavelength of oscillation $\lambda = \frac{2\pi R}{\sqrt{2(2l+1)}}$

\[ f_p = \frac{f_{\text{sound}}}{\lambda} = \sqrt{\frac{8P}{\pi}} \sqrt{\frac{2(2l+1)}{2\pi R}} \]

Solar spectra

Distance from center not the star radius!

3 mHz $= 300$ sec $= 5$ min oscillation