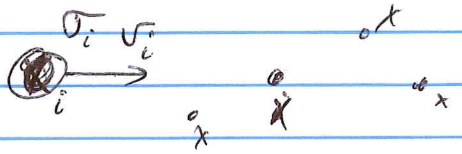
(P1)

lecture 21

Reaction rate



probability of fusion

\# of hits per ~~of~~ incident particle

$$dr \simeq p \cdot n_i \boxed{\sigma v \cdot n_x}\, n_v\, dv$$

\# collisions ~~hiting~~ per incoming particle

targets

has units of $\left[\frac{\#\text{ reactions}}{t \cdot \text{Volume}}\right]$

probability to have velocity v

$$\sigma \sim \lambda^2 \sim \left(\frac{h}{p}\right)^2 = \left(\frac{h}{mv}\right)^2 =$$

$$= \Big/ \frac{mv^2}{2} = E \Big/ = \sim \left(\frac{h}{\sqrt{E \cdot m}}\right)^2 = \frac{h^2}{mE} \sim \frac{1}{E}$$

$$\Rightarrow \sigma(E) \sim \frac{1}{E}$$

$n_v dv =$ Maxwell-Boltzmann distribution

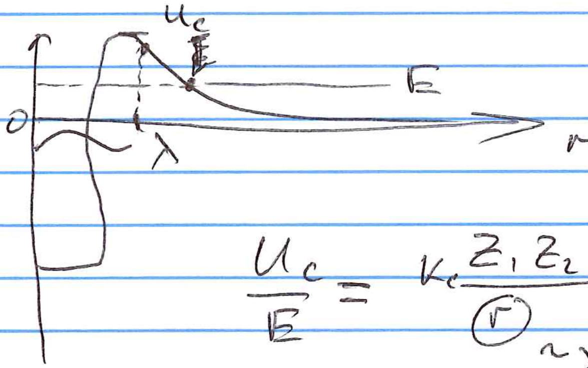
$$\sim \frac{1}{(kT)^{3/2}} v^2 e^{-\underbrace{\left(\frac{mv^2}{2kT}\right)}_{E/kT}} dv \sim \Big/ v = \sqrt{\frac{E}{m}2}$$

$$dv = \frac{1}{\sqrt{m}\sqrt{E}} dE$$

$$\sim \frac{1}{(kT)^{3/2}} e^{-\frac{E}{kT}} \sqrt{E}\, dE$$

~~Usually~~

Probability of fusion ~ probability of tunneling



$$\parallel$$

$$p(E) \sim e^{-2\pi^2 U_c/E}$$

Q.M.

$$\frac{U_c}{E} = k_c \frac{Z_1 Z_2 e^2}{\textcircled{r}} \cdot \frac{1}{E}$$

$$\sim \lambda \sim \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

$$\Rightarrow \quad -2\pi^2 \frac{U_c}{E} = -\frac{2\pi^2 k_c Z_1 Z_2 e^2}{h} \frac{\sqrt{2m}}{\sqrt{E}}$$

/To compare with the text recall that $k_c = \frac{1}{4\pi\epsilon_0}$/

$$-2\pi^2 \frac{U_c}{E} = -\frac{b}{\sqrt{E}}$$

Reduced mass $\frac{m_1 m_2}{m_1+m_2}$

$$\boxed{b = \frac{2\pi^2 k_c Z_1 Z_2 e^2 \sqrt{2m}}{h}}$$

$$dr = p_{(E)} n_i n_x \, \sigma(E) \cdot v \cdot \boxed{n_v \, dv}$$

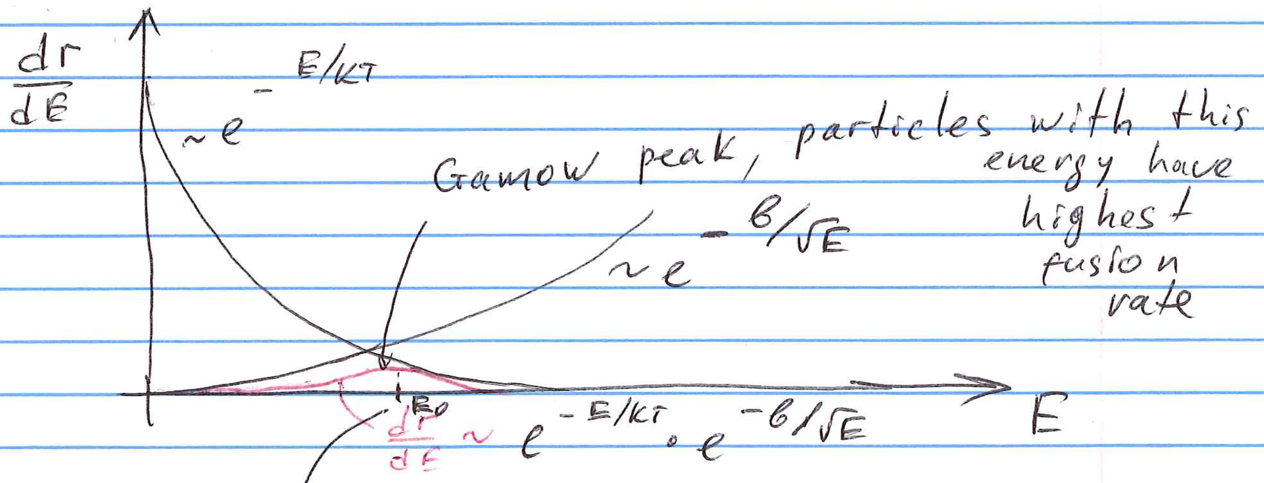
$$\sim p(E) \, n_i n_x \, \underset{\sim 1/E}{\sigma(E)} \, \frac{\sqrt{E}}{(kT)^{3/2}} \, e^{-\frac{E}{kT}} \, \sqrt{E} \, dE$$

$$\sim n_i n_x \, e^{-\frac{b}{\sqrt{E}}} \, \frac{1}{(kT)^{3/2}} \, e^{-E/kT} \, dE$$

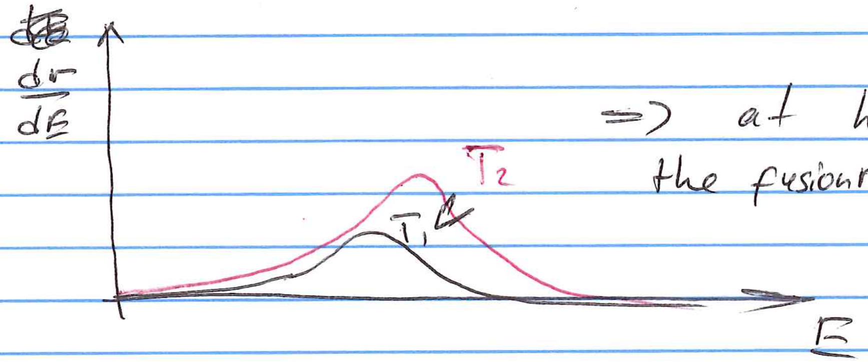
a more careful treatment

$$\boxed{dr = n_i n_x \left(\frac{2}{kT}\right)^{3/2} \frac{1}{(m\pi)^{1/2}} \, e^{-b/\sqrt{E}} \, e^{-\frac{E}{kT}} \, dE}$$

recall $b = f(Z_1, Z_2, m) = \frac{2\pi^2 k_c Z_1 Z_2 e^2}{h} \sqrt{2m}$



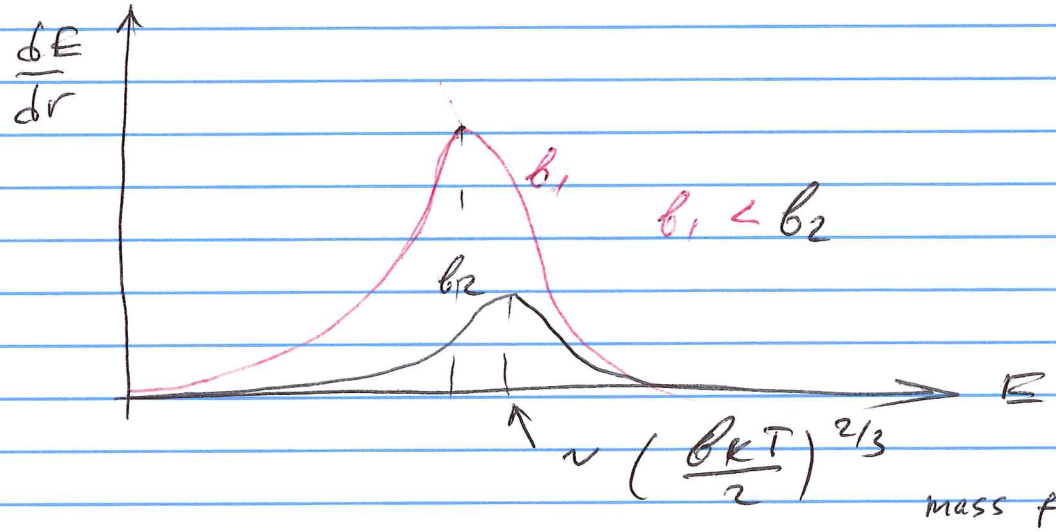
$$E_0 = \left(\frac{b kT}{2}\right)^{2/3}$$



⇒ at higher temperature the fusionreaction is faster

also note that high Z (charges) make b larger, and so does 'm' so for those overall reaction is slower

since $\frac{dr}{dE} \sim e^{-\frac{b}{\sqrt{E}}} e^{-E/kT}$



mass fraction

overall reaction rate is $r = \int dr = \int_0^\infty \dots dE \approx r_0 X_i X_x \rho^{\alpha'} T^{\beta}$

for two body $\alpha' = 2$

~~recall~~ recall that $r = \left[\frac{\#}{t\,V}\right]$

energy output per kg $\boxed{\epsilon \approx r/\rho} = \left[\frac{W}{kg}\right]$

So far we considered collision of 2 different elements.

But if we looking in fusion of H to He the reaction is

$$4\cdot{}^{1}_{1}H \rightarrow \underbrace{{}^{4}_{2}He} + \underbrace{2e^{+}}_{\text{positrons}} + \underbrace{2\nu_e}_{\text{electron neutrinos}} + \underbrace{2\gamma}_{\text{Photons}}$$

four particle collision required

this is highly unprobable

Most likely it realised in a chain when first 2 particles sticked, then one more, and yet one more

We will use notation

$${}^{\textcircled{A}}_{Z}X$$

protons + neutrons

element

number of protons

this unicly indentify chemical properties and thus this

Proton-Proton chain (PP I)

$^1_1H + ^1_1H \rightarrow ^2_1H + e^+ + \nu_e$ (requires $p^+ \rightarrow n + e^+ + \nu_e$)

$^2_1H + ^1_1H \rightarrow ^3_2He + \gamma$

$^3_2He + ^3_2He \rightarrow ^4_2He + 2\,^1_1H$ ~~69%~~

↖ High binding energy!! very stable

PP II

$^3_2He + ^4_2He \rightarrow ^7_4Be + \gamma$ ~~31%~~

$^7_4Be + e^- \rightarrow ^7_3Li + \nu_e$

$^7_3Li + ^1_1H \rightarrow 2\,^4_2He$

PP III

$^7_4Be + ^1_1H \rightarrow ^8_5B + \gamma$

$^8_5B \rightarrow ^8_4Be + e^+ + \nu_e$

$^8_4Be \rightarrow 2\,^4_2He$

Energy generation of all this chains

$$\underset{\frac{W}{kg}}{\varepsilon_0} \sim \underset{1.08\cdot 10^{-12}\,\frac{W\,m^3}{kg^2}}{\varepsilon_0'}\, T_6^4\, \underset{\text{density}}{\rho}\, \overset{\text{concentration of H}}{X^2} \quad \text{where} \quad T_6 = \frac{T}{10^6\,K}$$

CNO Cycle

1st Branch

$$\underbrace{{}^{12}_{6}C} + {}^{1}_{1}H \rightarrow {}^{13}_{7}N + \gamma$$

$${}^{13}_{7}N \rightarrow {}^{13}_{6}C + e^{+} + \nu_e$$

$${}^{13}_{6}C + {}^{1}_{1}H \rightarrow {}^{14}_{7}N + \gamma$$

$${}^{14}_{7}N + {}^{1}_{1}H \rightarrow {}^{15}_{8}O + \gamma$$

$${}^{15}_{8}O \rightarrow {}^{15}_{7}N + e^{+} + \nu_e$$

$${}^{15}_{7}N + {}^{1}_{1}H \rightarrow \underbrace{{}^{12}_{6}C} + {}^{4}_{2}He$$

Carbon is catalyzer

2nd Branch (0.04% of time)

$${}^{15}_{7}N + {}^{1}_{1}H \rightarrow {}^{16}_{8}O + \gamma$$

$${}^{16}_{8}O + {}^{1}_{1}H \rightarrow {}^{17}_{9}F + \gamma$$

$${}^{17}_{9}F \rightarrow {}^{17}_{8}O + e^{+} + \nu_e$$

$${}^{17}_{8}O + {}^{1}_{1}H \rightarrow {}^{14}_{7}N + {}^{4}_{2}He$$

$$\varepsilon_{CNO} \simeq \varepsilon'_{0_{CNO}} \rho X X_{CNO} T_6^{19.9}$$

– sharp dependence on Temperature

$$\varepsilon'_0 = 8.24 \cdot 10^{-31} \frac{W\,m^3}{kg^2}$$

The triple Alpha Process —
— Burning of He

$${}^4_2He + {}^4_2He \rightleftharpoons {}^8_4Be$$

$${}^8_4Be + {}^4_2He \rightarrow {}^{12}_6C + \gamma$$

(recall that α-particle is ${}^4_2He^+$)

Above looks like

$$3 \cdot {}^4_2He \rightarrow {}^{12}_6C + \gamma$$

$$\varepsilon_{3\alpha} \simeq \varepsilon'_{0,3\alpha}\, \rho^2 Y^3 \ldots T_8^{\boxed{41.0}}$$

← super sharp dependence on T

notice

$$T_8 = T/10^8$$

3α - process kicks in at high temperatures