Lecture 19

~~How~~ What is the sun output per kg?

$$\frac{L_\odot}{M_\odot} = \frac{3.8 \cdot 10^{26}\ W}{2 \cdot 10^{30}} \approx 2 \cdot 10^{-4}\ \frac{W}{kg}$$

So to power 100W bulb (or make heat output comparable to a human which is ~ the same 100W)

$$\text{we } \cancel{m}\text{eed } m = \frac{100W}{2 \cdot 10^{-4}\ W/kg} \approx 5 \cdot 10^5 kg = 500\ \text{Tonn}$$

Good news this bulb will shine for 10¹¹ years as we saw in prev lecture.

Recall that 1kg of H when converted to He is capable of generating energy/heat

$$E = 1 \cdot kg \cdot \underbrace{0.007}_{\text{mass conversion coef.}} \cdot c^2 = 7 \cdot 10^{-3} \cdot (3 \cdot 10^8)^2$$
$$= 63 \cdot 10^{-3} \cdot 10^{16} = 6.3 \cdot 10^{14}\ J$$

But the Sun "drinks" this ocean with a tea spoon. :(Which might be a good thing.

So what governs the rate of fusion?

Probability of collision, since we need to fuse (nuclei), i.e. bring them in contact

nuclei size $\sim 10^{-15}\,m = 1\,fm \Rightarrow$ ①

Probability $\sim n \sigma v =$ collision rate

$\sigma = \pi \cdot (1\,fm)^2$

$n = \frac{\rho}{m_H} \sim \frac{1\,kg}{1\,m^3} \cdot \frac{1}{1.7 \cdot 10^{-27}\,kg} = 6 \cdot 10^{26}\,\frac{1}{m^3}$

$v = c$

collision rate $\sim 6 \cdot 10^{26} \cdot \pi (10^{-15})^2 \cdot 3 \cdot 10^{8}\,\frac{m}{s}$

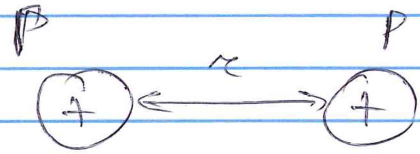
$= 60 \cdot 10^{4} \approx 6 \cdot 10^{5}\,\frac{hits}{sec}$

So it seems that ~~energy~~ all hydrogen should fuse within a fraction of a second

But

this is for each nuclei which does not have repulsion

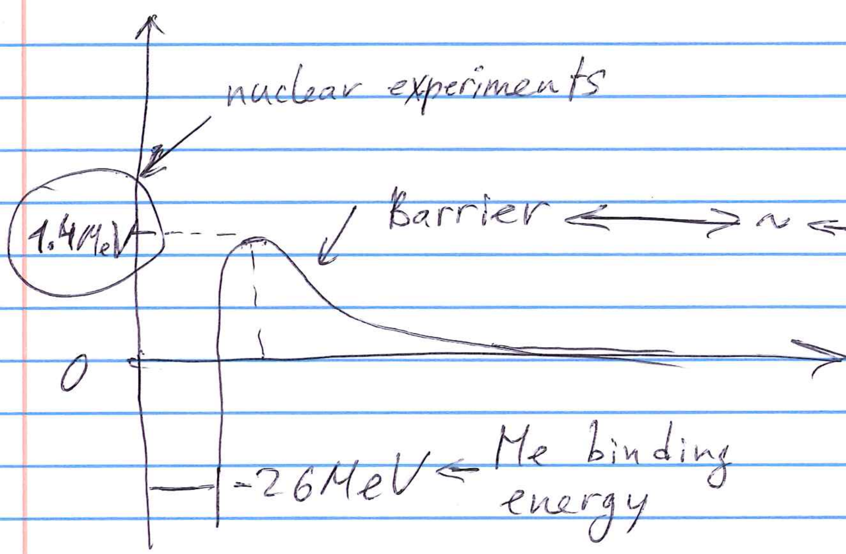
Repulsion due to Coulomb force



$$F = K_e \frac{Z_1 Z_2}{r^2} \qquad Z_1 = Z_2 = e$$

$$F(r = 1\,fm) = 9 \cdot 10^{9} \frac{(1.6 \cdot 10^{-19})^2}{(10^{-15})^2}$$

$$\simeq 230\, N$$



$$\underline{1\,MeV = 1.6 \cdot 10^{-13}\,J}$$

Estimate

$\sim \leftarrow U_c = K_e \frac{Z_1 Z_2}{r}$

$$\simeq 9 \cdot 10^{9} \cdot \frac{(1.6 \cdot 10^{-19})^2}{10^{-15}}$$

$$\approx 230 \cdot 10^{-15}$$

$$\approx 2.3 \cdot 10^{-13}\,J$$

So to overcome the Barrier we need ~~v = 1.4 MeV~~

$$\frac{mv^2}{2} = 1.4\ MeV = E_{Barier}$$

$$v = \sqrt{\frac{2 E_{Barier}}{m}} = \sqrt{\frac{2 \cdot 1.4 \cdot 10^6 \cdot 1.6 \cdot 10^{-19}}{1.67 \cdot 10^{-27}}}$$

$$\simeq \sqrt{\frac{2.8 \cdot 1.6}{1.67} \cdot 10^{14}} \simeq 1.6 \cdot 10^7\ \frac{m}{s} \approx \frac{c}{20}$$

not that far from c

From thermal point of view

$$E_{Bar} = \frac{3}{2} k T$$

$$T = \frac{2}{3} \frac{E_{Bar}}{k} = \frac{2}{3} \cdot \frac{1.4 \cdot 10^6 \cdot 1.6 \cdot 10^{-19}}{1.38 \cdot 10^{-23}} =$$

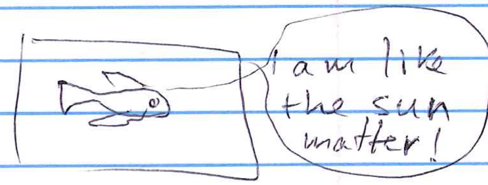
$$= \frac{4.48}{4.14} \frac{10^{-13}}{10^{-23}} \simeq 10^{10}\ K$$

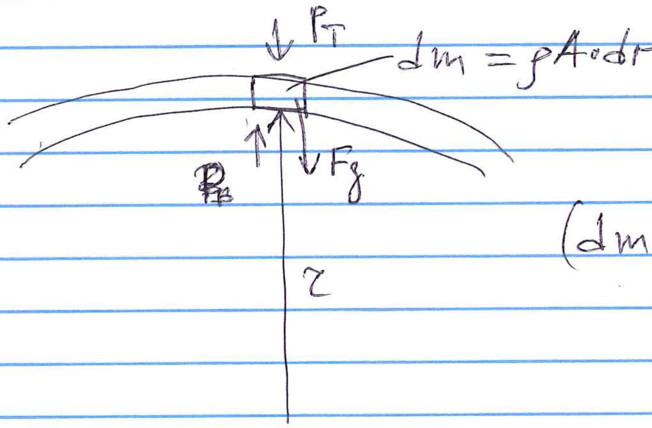
This seems to be hot when we compare $T_{\odot surface} = 6000K$

To see if we can have such ~~pressure~~ temperature we will relate it to pressure.

$$\boxed{\text{Recall that for ideal gas} \quad PV = NkT \Rightarrow P = \frac{\rho}{m_{part}} kT}$$

Pressure equation





$$(dm)a = -F_g - P_T A + P_B A$$

$$(dm)a = -(dm)\underbrace{g}_{\text{local acceleration due to gravity}} - (P_T - P_B)A$$

$$(\rho \cdot \cancel{A}\, dr)\underbrace{a}_{\frac{d^2r}{dt^2}} = -(\rho \cancel{A}\, dr)g - dP \cdot \cancel{A}$$

$$\boxed{\rho \frac{d^2 r}{dt^2} = -\rho \underbrace{g}_{G\frac{M(r)}{r^2}} - \frac{dP}{dr}}$$

In hydrostatic equilibrium $\frac{d^2r}{dt^2} = 0$

$$\boxed{\frac{dP}{dr} = -\rho G \frac{M(r)}{r^2}}$$

Q: For 2 stars of the same mass but different radius, which one has higher pressure?

Assuming $\rho = \text{const}$

$$M(r) = \frac{4\pi}{3} r^3 \rho$$

$$\frac{dP}{dr} = -\rho\, G \frac{\frac{4\pi}{3} r^3 \cdot \rho}{r^2} = -\rho^2 G \frac{4\pi}{3} r$$

$$P(r) = \underbrace{P(\text{at surface})}_{=0} + \int_{R_\odot}^{r} dP =$$

$$= 0 - \rho^2 G \frac{4\pi}{3} \int_{R_\odot}^{r} r\,dr =$$

$$= -\rho^2 G \frac{4\pi}{3} \cdot \frac{1}{2} \left[r^2 - R_\odot^2 \right]$$

$$P(r=0) = P_{center} = \frac{G}{2} \underbrace{\left(\frac{4\pi}{3} \rho R_\odot^3\right)}_{=M_\odot} \cdot \underbrace{\left(\frac{4\pi}{3} R_\odot^3 \rho\right)}_{=M_\odot} \frac{3}{4\pi} \frac{1}{R_\odot^4}$$

$$\boxed{P_c = \frac{3}{8\pi} \frac{M_\odot^2}{R_\odot^4}}$$

A: So star with large R has smaller central pressure

$$P_{\odot c} = \frac{3}{8\pi} G \frac{(2 \cdot 10^{30})^2}{(7 \cdot 10^8)^4} \approx G \frac{3 \cdot 4}{8 \cdot \pi \cdot 2500} \frac{10^{60}}{10^{32}}$$

$$\approx \frac{G}{5} \cdot \frac{10^{28}}{10^3} = \frac{6.67 \cdot 10^{-11} \cdot 10^{28}}{5 \cdot 10^3} \approx 1.3 \cdot 10^{14}\ \text{Pa}$$

if we lift $\rho = \text{const}$ assumption, a better model gives $P_c = 3.34 \cdot 10^{16}$ Pa