lecture 18

Sun / star / solar energy sources

Sun emits $L_\odot = 3.8 \cdot 10^{26}$ W

So what can fuel it?

Oil burn? Diesel gives $\sim \rho_E \approx 43$ MJ/kg ← energy density

So we need to burn

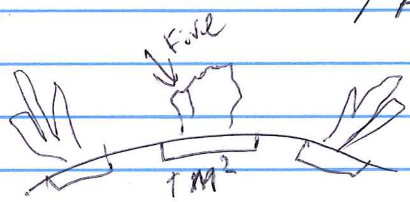
$$\frac{dM}{dt} = \frac{L_\odot}{\rho_E} = \frac{3.8 \cdot 10^{26}\ \mathrm{J/s}}{43 \cdot 10^{6}\ \mathrm{J/kg}} = \approx 10^{+19}\ \frac{\mathrm{kg}}{\mathrm{s}}$$

Mass of the Sun $M_\odot = 2 \cdot 10^{30}$ kg
so there is enough mass for it to be plausible.

~~Mass~~ $R_\odot = 7 \cdot 10^{8}$ m $\Rightarrow A_\odot = 4\pi R_\odot^2 = 6.2 \cdot 10^{18}\ \mathrm{m}^2$

So we need to burn about

$$10^{19}\ \mathrm{kg/s} / A_\odot \approx 1.6\ \frac{\mathrm{kg}}{\mathrm{s \cdot m^2}}$$



it is somewhat hard since we need spread oil in some volume but possible

One can say we do not see oil in the Sun spectra, but so what?, oil burns underneath the Sun surface / in the area of high opacity.

How long our sun will last?

$$T = \frac{M_\odot}{dm/dt} = \frac{2\cdot 10^{30}\,kg}{10^{19}\,kg/s} = 2\cdot 10^{11}\,s$$

$$1\,\text{year} \approx \pi\cdot 10^{7}\,s$$

$$T \approx \frac{2\cdot 10^{11}}{\pi\cdot 10^{7}} \approx 6400 \text{ years.}$$

Wow looks like Biblical time!

Except it is time to the "end of the word" from ~~current~~ ~~time~~ the start of the burn.

Well Egypt ~~is~~ civilization is at least 6000 old

$\Rightarrow$ Oil is no good.

May be gravitational energy?

A rock with mass m falling from ∞ to the sun would convert its potential energy to kinetic and once it hit the "surface" it would generate othe form of energy like heat.

$$E = \Delta U = -\frac{GM_\odot m}{\underset{\infty}{r_i}} - \left(-\frac{GM_\odot m}{r_f}\right) =$$

$r_f = R_\odot$

$$= G\frac{M_\odot}{R_\odot} m$$

Energy per kg $\Rightarrow \frac{E}{m} = G\frac{M_\odot}{R_\odot} = \frac{6.67\cdot 10^{-11}\cdot 2\cdot 10^{30}}{7\cdot 10^{8}} \approx$

$\approx 2\cdot 10^{11}$ J/kg

This better than oil's 43 MJ/kg

So we should see a matter fall on the sun with mass

$$\frac{dm}{dt} = \frac{L_\odot}{E/m} = \frac{3.8\cdot 10^{26}}{2\cdot 10^{11}} \approx 2\cdot 10^{15}\ kg/s$$

So per year required mass

$$M = 2\cdot 10^{15}\frac{kg}{s}\cdot\left(\pi\cdot 10^{7}\frac{s}{year}\right) = \cancel{6\cdot 10} \; 6\cdot 10^{22}\frac{kg}{year}$$

for comparison $\cancel{M_\odot}$ $M_E = 6\cdot 10^{24}$ kg

we would notice something that big

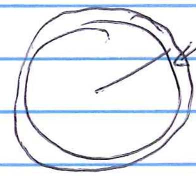
Another problem with it
that we would have optimistic
sun acreation / asembly time

of $\Pi = \frac{M_\odot}{dm/dt} = \frac{2 \cdot 10^{30}\ kg}{6 \cdot 10^{22}\ kg/years} = 3 \cdot 10^{7}$

$\hat{=}$ 30 M years

we have fossils which that long
(oldest living creatures like lobsters
have been around for 110 M years)

Ok. falling rocks are no good.
lets consider gravitational collapse
of sun on itself.



$$dm_r = 4\pi r^2 dr \cdot \rho_\odot$$

$$dU = -G \frac{dm_r M_r}{r}$$

$$U_{Total} = -\int_0^{R_\odot} G \frac{\overset{\frac{4\pi}{3} r^3 \cdot \rho_\odot}{M_r}\, dm_r}{r} =$$

$$= -\int_0^{R_\odot} G \frac{\frac{4\pi}{3} r^3 \cdot \rho_\odot \cdot 4\pi r^2 dr \cdot \rho_\odot}{r} =$$

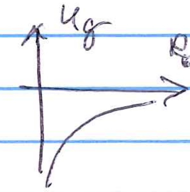
$$= / \rho_\odot = const / =$$

$$= -G \rho_\odot^2 \frac{(4\pi)^2}{3} \int_0^{R_\odot} r^4 dr =$$

$$= -G \rho_\odot^2 \cdot \frac{(4\pi)^2}{3} \frac{R_\odot^5}{5} =$$

$$= / \frac{4\pi}{3} R_\odot^3 \cdot \rho_\odot = M_s / =$$

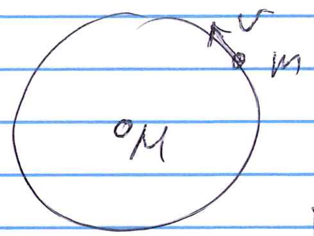
$$= \boxed{-G \frac{3}{5} \frac{M_\odot^2}{R_\odot} = U_g}$$



But we need to be concerned with <u>total</u> energy

$E = K + U$, how to figure out K?

let's consider simple exam



to maintain circular orbit we need

$$F_g = \frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\frac{GMm}{r} = \left(\frac{mv^2}{2}\right) \cdot 2$$

$$\boxed{-U_g = 2K}$$

Well, there is Virial theorem which states a more general statement.

In equilibrium

$$\boxed{-\bar{U}_g = 2\bar{K}}$$

$\bar{K}$, $\bar{u}$ are averaged over time and it ~~needs~~ needs to be counted in C.M. ref. frame.

Bonus: when Virial was born or died?

P8

Kelvin-Helmholtz time scale

$$\frac{E_{in} - E_{final}}{L_\odot} = \frac{1.2 \cdot 10^{41}\ J}{3.8 \cdot 10^{26}\ W} \simeq$$

$$\simeq 3 \cdot 10^{14}\ s \simeq 10^{7}\ \text{years}$$

$$= 10\ M\ \text{years}$$

Yet, again we have problem with fossils.

Let's recall $E = mc^2$

$$\frac{dm}{dt} = \frac{L}{c^2} = \frac{3.8 \cdot 10^{26}}{(3 \cdot 10^8)^2} = 4.2 \cdot 10^9 \ kg/s$$

this is quite modest

$$\text{So} \quad T_{mc^2} = \frac{M_\odot}{dm/dt} = \frac{2 \cdot 10^{30}}{4.2 \cdot 10^9} \simeq 0.5 \cdot 10^{21}$$

$$\simeq 5 \cdot 10^{20} \ sec \approx 1.7 \cdot 10^{13} \ years$$

yey, plenty of time to "the end of the world"

We assume total mass to energy conversion but this is quite unrealistic. We probably convert only fraction of mass.

What fraction? Stars have plenty of H let's convert it to He

$$m_H = 1.0078 \ u$$
$$m_{He} = 4.002603 \ u$$

$1u = 1.66 \cdot 10^{-27} \ kg$
$mc^2 = 931.49 \ MeV$
$1 \ MeV = 1.6 \cdot 10^{-13} \ J$
$\Rightarrow \boxed{1u \Rightarrow 1.5 \cdot 10^{-10} \ J}$

$$4H \rightarrow He + \Delta E$$

$$\Delta E = 4 \cdot 1.0078_u - 4.002603_u =$$

$$= 0.028 \ u = 4.3 \cdot 10^{-12} \ J$$

How much mass is converted

$$\frac{4M_H - M_{He}}{4M_H} \simeq \frac{0.028}{4} \approx$$
$$\simeq 0.7\%$$

So instead of T_{mc^2} we have about 1000 less
$\simeq 10^{10}$ years $= 10$ billion we are fine!