lecture 16

light source function

Imagine

\[ \text{Block} \]

Naively we expect nothing here

\[ I_\lambda \sim I_0 e^{-k \lambda d} \to 0 \]

From other hand it energy delivered than it should go somewhere including forward,

Same with the stars they absorb but they are shiny,

So we introduce a correction a "source"

\[ dI_\lambda = \frac{1}{k \lambda} I_\lambda ds + j_{\lambda} \phi ds \]

absorption

\[ \frac{1}{k \lambda} \frac{dI_\lambda}{ds} = -I_\lambda + \frac{j_{\lambda}}{k \lambda} \phi s \]

\[ \frac{dI_\lambda}{d \lambda} = -I_\lambda + S_\lambda \]
Assuming \( K_\lambda = \text{const} \) and \( S_\lambda = \text{const} \)

\[
+ \frac{1}{k_\lambda s} \frac{dI_\lambda}{ds} = -I_\lambda + S_\lambda
\]

\[
I = I_0 e^{-k_\lambda s} + S_\lambda (1 - e^{-k_\lambda s})
\]

\( I_0 > S_\lambda \)

\( I_0 < S_\lambda \)

looks like feedback stabilization at \( S_\lambda \) level.

In equilibrium \( \frac{dI_\lambda}{ds} = 0 \) \( \Rightarrow \) \( I_\lambda = S_\lambda \)

So what is \( S_\lambda \)?

We know that in black body \( I_\lambda = B_\lambda \) and thus \( S_\lambda = B_\lambda \)

note this is "crazy" = \( \frac{W}{m^2 \cdot \text{s} \cdot \text{r}} \) = specific intensity
So we see if light propagates long enough it will "forget" its initial intensity distribution and will be replaced with "source" function of the underlying medium!

\[ I_x = I_0 e^{-\kappa_s s} + S_x(1 - e^{-\kappa_s s}) \]

This is due to the fact that \( \kappa_s > 0 \)

So when we look at the star we see properties of outer layer only, recall that for sun \( \frac{1}{\kappa_s} \approx 160 \text{ km} = 16 \times 10^5 \) when \( R_\odot = 7 \times 10^8 \text{ m} \)
In reality life is much harder

Since \( S_x \) depends on position, i.e. \( \neq \) const and also the \( k_x \neq \) const

Still simple const model

Here we see outer shell, \( S_x \)

So here we probe dipper

\[
S_x = \frac{1}{k_x S}
\]

Probing depth \( \Rightarrow e^{-k_x S_x} \approx e^{-1}

So when we look at the sun we have quite sharp edges though it is a gas object with no well defined edges.
Another amusing fact

$K_\lambda$ is a function of $\lambda$

$K_\lambda \uparrow$ so do we see deeper in the sun at $\lambda_0$ or not?

$S_\lambda \propto \frac{1}{K_\lambda \phi}$ so if $K_\lambda \to 0$, then $S_\lambda \to 0$

i.e. we can probe less deep

![Diagram](image)
What constitute $\Delta f$

It is $\sim$ to probability to catch/emit photon

$$\Delta f = \frac{1}{\pi} \left( \frac{1}{\Delta t_u} + \frac{1}{\Delta t_L} \right)$$

$$= \frac{1}{\pi} \frac{1}{\Delta t}$$

H$_d$ line

$$\lambda = 656.3 \text{ nm}$$

$$\Delta t = 10^{-8} \text{ s}$$
Doppler Broadening

D. shift \( \Delta f = f_0 \frac{v}{c} \)

\[ n(v) \sim e^{-\frac{mv^2}{2kT}} \]

\[ v_{1/2} \approx e^{-\frac{m v_{1/2}^2}{2kT}} = 1/2 \]

\[ \frac{m v_{1/2}^2}{2kT} = \ln 2 \]

\[ v_{1/2} = \sqrt{\frac{2kT}{m} \ln 2} \]

Typical doppler shift, i.e. broadening

\[ \Delta f_{1/2} = \left( \frac{f_0}{c} \right) \sqrt{\frac{2kT}{m} \ln 2} \approx \frac{12 \text{ GHz}}{c} \]

FWHM = 2 \cdot \Delta f_{1/2}

\[ FWHM = 2 \cdot \sqrt{\frac{kT}{m} \ln 2} \]

\[ m = 1.67 \times 10^{-24} \text{ Kg} \]

\[ k = 1.38 \times 10^{-23} \text{ J/K} \]

\[ T = 5800 \text{K} \]

For heavier elements, D. broadening is not that bad