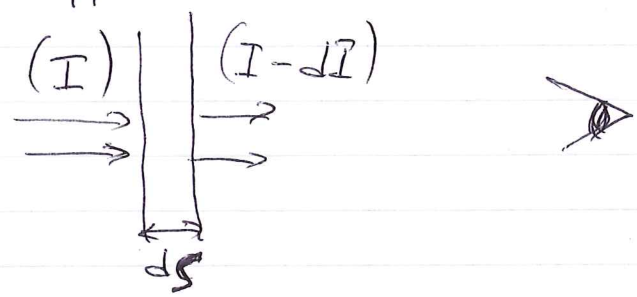
lecture 15

(P1)

Light absorption coefficient

$$dI = -\alpha I \, d\xi$$



$$I = I_0 e^{-\alpha \xi}$$

A more generally
$\alpha \rightarrow \alpha(\xi, \lambda)$

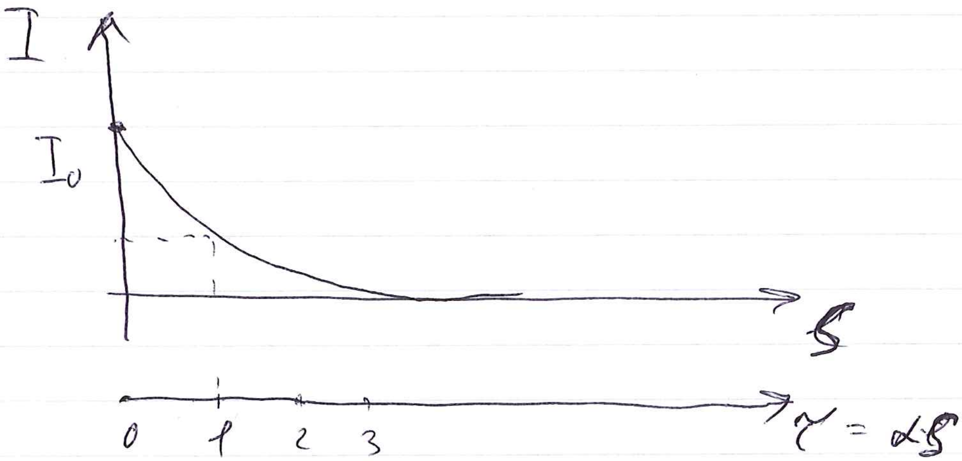
$$I_\lambda = I_{0\lambda} e^{\int_{\xi_{start}}^{\xi_{end}} \alpha(\lambda \xi) d\xi}$$

ξ - distance (position)

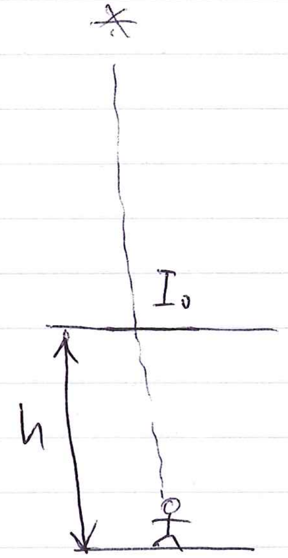
λ - wavelength

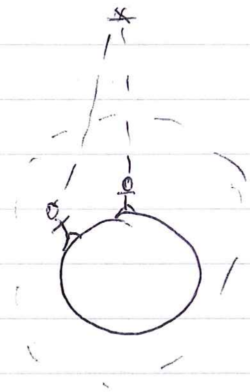
α - is absorption coefficient
ξ - is path traveled

$\boxed{\tau = (\alpha \cdot \xi)}$ - reperred as optical depth



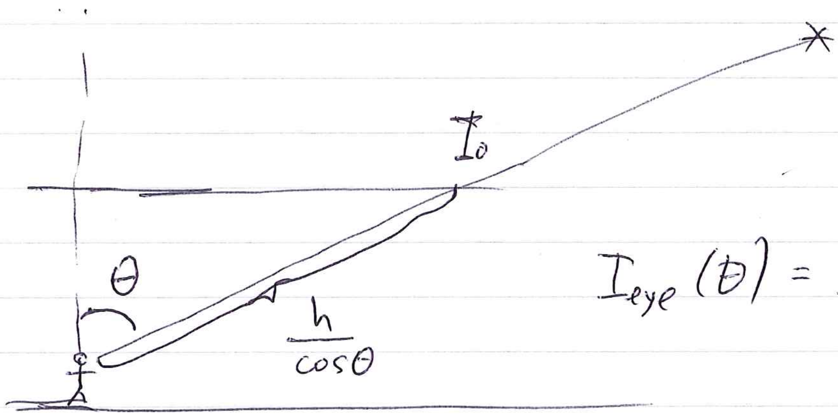
How to exclude the Earth atmosphere from consideration.



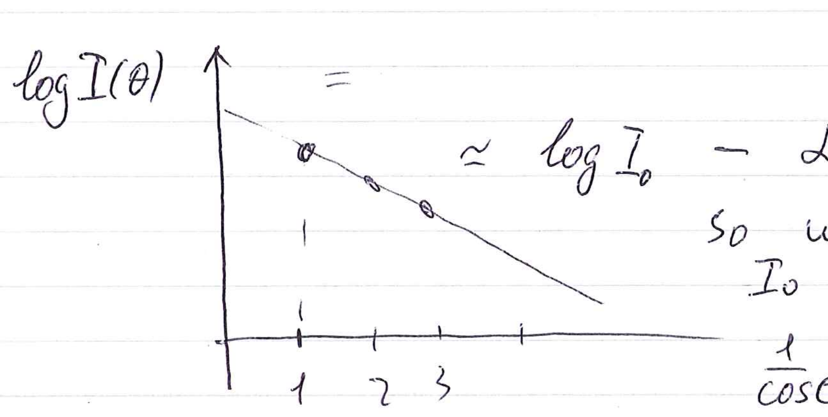


$$I_{eye} = I_0 \, e^{-\alpha h}$$

two unknowns one equation



$$I_{eye}(\theta) = I_0 \, e^{-\alpha \frac{h}{\cos\theta}}$$



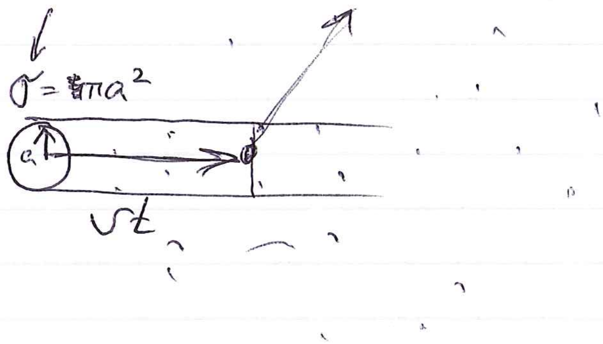
$$\log I(\theta) = \simeq \log I_0 - \alpha h \cdot \sec\theta$$

So we can "pull out" I_0 and $(\alpha \cdot h)$

So what is the mechanism for the absorption?

cross section

$\sigma = \pi a^2$



$\alpha \sim$ to # of particle with which interaction might happen

$$N = n \cdot V = n \cdot \sigma \cdot \underbrace{v t}_{\xi}$$

concentration

mean free path — path to the next collision

$$\ell_{m.f.p.} = \frac{v \cdot t}{N} = \frac{1}{n \sigma}$$

$\sigma = A$ is just an approximation
not every "overlap" leads to scatter. or particle removal.

so often σ is come experimentally determined parameter in optics — absorption coef

in astro — absorption coefficient, opacity

$$\alpha = \cancel{n\sigma} \; n\sigma = \kappa_\lambda \rho \leftarrow \text{density}$$

$$[m^{-1}] = \left[\frac{1}{m^3} \cdot m^2\right] = \frac{m^2}{kg} \cdot \frac{kg}{m^3}$$

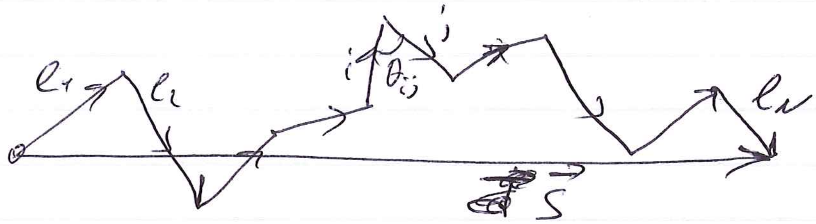
in Solar atmosphere

$\rho = 2.1 \cdot 10^{-4}$ kg/m³ (lighter / more diluted then air)

$$\ell_{mfp} = \frac{1}{\kappa_\lambda \rho} = 160 \text{ km}$$

500 nm

Random walk (elastic scatter)



$$\vec{S} = (\vec{l}_1 + \vec{l}_2 + \vec{l}_3 + \dots \vec{l}_N)$$

$$S^2 = \left(\sum_i^N \vec{l}_i^{\,2}\right) + \underbrace{\sum_{i=1}^{N} \sum_{j \neq i}^{N} l_i \cdot l_j}_{\text{cross term}}$$

$$l_i = l_j = l_{m.f.p.} \qquad \sum\sum l_{m.f.p}^2 \cdot \cos(\theta_{ij}) = 0$$

$$S^2 = N l^2$$

recall τ (optical depth) $= \dfrac{S}{l_{m.f.p}} = \dfrac{\sqrt{N}\, l_{m.f.p}}{l_{m.f.p}} = \sqrt{N}$

interesting fact : travel time ~~to the surface~~.

to travel S we need N bounces $= \left(\dfrac{S}{l_{m.f.p}}\right)^2$

time per bounce $= \dfrac{l_{m.f.p}}{c}$ so

travel time

to travel $R_\odot$: $t = \left(\dfrac{S}{l_{m.f.p}}\right)^2 \cdot \dfrac{l_{m.f.p}}{c} = \underbrace{\left(\dfrac{7 \cdot 10^8\, m}{160 \cdot 10^3\, m}\right)^2 \cdot \left(\dfrac{160 \cdot 10^3\, m}{3 \cdot 10^8\, m/s}\right)}_{\approx 10\, kS}$

effective speed $\dfrac{R_\odot}{t} = 68\, km/s$; while $R_\odot / c \approx 2\, s$