Light absorption coefficient:

\[
pI = -\alpha I \, ds
\]

\[
I = I_0 \, e^{-\alpha \, s}
\]

A more generally:

\[
l \rightarrow l(s, \lambda)
\]

\[
I_\lambda = \int_{s_{\text{start}}}^{s_{\text{end}}} I_0 \cdot e^{-\alpha \, l(s, \lambda)} \, ds
\]

\(\xi = \alpha \, s\) - referred as optical depth

\(\alpha\) - is absorption coefficient

\(s\) - is path traveled

\(l\) - is distance (position)

\(\lambda\) - wavelength
How to exclude the Earth atmosphere from consideration.

\[ I_{\text{eye}} = I_0 e^{-dh} \]

two unknowns one equation

\[ I_{\text{eye}}(\theta) = I_0 e^{-\frac{dh}{\cos\theta}} \]

\[ \log I(\theta) \approx \log I_0 - dh \cdot \sec\theta \]

So we can "pull out" \( I_0 \) and \((dh)\)

\[ \frac{1}{\cos\theta} = \sec\theta \]
So what is the mechanism for the absorption?

\[
\sigma = \pi a^2
\]

\[d \sim n \cdot V \sim n \cdot \sigma \cdot r \]

\[\text{concentration}
\]

\[\text{mean free path} \quad \ell_{\text{mfp}} = \frac{R \cdot r}{N} = \frac{1}{n \sigma}
\]

\(\sigma = A\) is just an approximation, not every "overlap" leads to scatter or particle removal.

So often \(\sigma\) is some experimentally determined parameter.
in optics

\[ \alpha = n \sigma = k \rho \]

\[ [ \text{m}^{-1} ] = [ \frac{1}{\text{m}^3} \cdot \text{m}^2 ] = \frac{\text{m}^2}{\text{kg} \cdot \text{m}^3} \]

in Solar atmosphere

\[ \rho = 2 \times 10^{-6} \text{ kg/m}^3 \] (lighter/more dilute than air)

\[ \ell_{\text{mfp}} = \frac{1}{k \rho} = 160 \text{ km} \]

\[ \text{500nm} \]
Random walk (scatter)

\[ S = (l_1 + l_2 + l_3 + \ldots + l_N) \]

\[ S^2 = \left( \sum_{i=1}^{N} l_i^2 \right) + \frac{1}{N} \sum_{i=1}^{N} \sum_{j=i+1}^{N} l_i \cdot l_j \]

Cross term

\[ l_i = l_f = e_{\text{mfp}} \]

\[ \sum \sum l^2_{\text{mfp}} \cdot \cos (\theta_{ij}) = 0 \]

\[ S^2 = N e_{\text{mfp}}^2 \]

Optical depth

Recall \[ z = \frac{S}{e_{\text{mfp}}} = \sqrt{N} \frac{e_{\text{mfp}}}{e_{\text{mfp}}} = \sqrt{N} \]

Interesting fact: travel time to the surface.

To travel \( S \) we need \( N \) bounces = \( \left( \frac{S}{e_{\text{mfp}}} \right)^2 \)

Time per bounce = \( \frac{e_{\text{mfp}}}{C} \)

So

Travel time

To travel \( R_{\odot} \):

\[ t = \left( \frac{S}{e_{\text{mfp}}} \right)^2 \cdot \frac{e_{\text{mfp}}}{C} = \left( \frac{7 \times 10^8 \text{ m}}{160 \times 10^3 \text{ m}} \right)^2 \cdot \left( \frac{160 \times 10^3 \text{ m}}{3 \times 10^8 \text{ m/s}} \right) \approx 10 \text{ s} \]

Effective speed \( \frac{R_{\odot}}{t} = 68 \text{ km/s} \); while \( R_{\odot}/c = 2 \text{ s} \)