From previous lecture we see that a particular element spectral line will have characteristic temperature dependence.

\[ \frac{N_{i+1}}{N_i} \propto (T)^{3/2} \propto \frac{e^{-\frac{\Delta E}{kT}}}{T} \]

smaller \( \chi \) earlier appearance at low \( T \)

This nice but what can we say about relative abundance? Above is a profile per particular element.

Overall line strength (observed)

\[ \sim \frac{N_{\text{particles}}}{\text{line strength per element}} \]

absorption cross section (depends on element dipole)
Hertzsprung–Russel Diagram (H–R) Diagram

B-V

Temperature

How do we know size?

Recall that position along B–V related to temperature,

\[ M \sim \log L \sim \log R^{2.5} \sim T^4 \]

so for the same temperature large star is more luminous 

\[ \Rightarrow \text{smaller } M. \]

Note also that temperature places a star in particular O, B, A, F, G, K, M

class right away.

But so far no way to say something about luminosity \( \Leftrightarrow \) or size
Fortunately there is one more parameter in spectra – line width, do not mix it with strength.

The same type let’s say ‘A’ will have a narrower lines as we go to a brighter star.

So now just looking at spectra we will now placement of a star at H–R diagram, i.e. its T and M notice absolute magnitude M now if one measure observed ‘m’ we will know the distance as well.

\[ d = 10 \left( m - M + 5 \right) / 5 \] [d in pc]
Let's draw the L-T diagram closely related to the H-R diagram. Larger stars as well as lines of same radius stars can be seen.

The equation $L \sim R^2 T^4$ is derived from the relation:

$$\log L / L_\odot = 4 \log (T / T_\odot) + 2 \log (R / R_\odot)$$

Recall experimental observation that $L / L_\odot$ is a function of $T / T_\odot$.

For stars around $M_\odot$:

- $M_\odot \sim R_\odot \sim L_\odot \sim T_\odot$

- $L / L_\odot \sim (M / M_\odot)^4 \sim R / R_\odot \sim T / T_\odot$

- $L / L_\odot \sim (M / M_\odot)^4 \sim R / R_\odot \sim T / T_\odot$

- $L / L_\odot \sim (M / M_\odot)^4 \sim R / R_\odot \sim T / T_\odot$
Finally, why lines a narrower for giant stars.

Line width related to pressure broadening (spectras show line width to large to be explained by Doppler broadening \( \propto \) thermal \( \propto \sqrt{T} \) average

Pressure broadening \( \propto \) between collision rate, \( \propto \) density \( \propto \) average velocity

\[ \Rightarrow \quad \Rightarrow \quad \text{density} \propto \frac{1}{\sqrt{T}} \]

So for the same \( \& \; T \), giants must have a less dense structure

\[ S_\odot = \frac{M_\odot}{4\pi R^2} = 1410 \frac{\text{kg}}{\text{m}^3} \] a bit more than water \( 1000 \frac{\text{kg}}{\text{m}^3} \)

Betelgeuse

\[ S_B = \frac{10 M_\odot}{4\pi (1000 R_\odot)^3} \approx \frac{10 S_\odot}{10^3} = \frac{S_\odot}{10^8} \]

\[ = \frac{1410}{10^8} \approx 1.4 \times 10^{-5} \]

\[ \approx 0.14 \times 10^{-6} \frac{\text{kg}}{\text{m}^3} \]

Compare to the air density \( \approx 1 \frac{\text{kg}}{\text{m}^3} \)