Stars apparent and absolute magnitude

Hipparchus made a catalog of stars with apparent magnitude spanning from 1 to 6, brightest barely visible (dim).

To put some scientific merit it was agreed that if apparent brightness changes by 5 it correspond to change of flux $= 100$

\[
\frac{\text{energy}}{\text{time} \cdot \text{area}} = \left[ \frac{\text{W}}{m^2} \right]
\]

\[
\Delta \log_{10} \left( \frac{F_0 \cdot 100}{F_0} \right) = 0.5 = \Delta m
\]

\[
\Delta \log_{10} \left( \frac{F_1}{F_2} \right) = -5 \Rightarrow \Delta = -2.5
\]

\[
m_1 - m_2 \Delta m = -2.5 \log_{10} \left( \frac{F_1}{F_2} \right)
\]

\[
\Delta m = \sigma - 1 \Rightarrow \frac{F_1}{F_2} = 10^{1.05} = 10^{0.4} = 2.51
\]

\[
\text{ratio of fluxes}
\]
Absolute magnitude is given by the flux of a star as if it placed at the distance of 10pc

$$M = 4.74.$$  

$$m = -2.5 \log_{10} \frac{F_\star (d)}{10^2}$$  

$$M = -2.5 \log_{10} \frac{F_\star \cdot d^2}{10^2} =$$  

$$= (-2.5 \log_{10} F_\star) - 2.5 \log_{10} \frac{d^2}{10^2}$$  

$$= m - 5 \log \left( \frac{d}{10 \text{pc}} \right)$$  

So if we put sun at the position of the nearest star (Proxima Centauri),  
$$d = 1.3 \text{pc},$$ it would appear as  

$$m_0 = M + 5 \log \left( \frac{d}{10 \text{pc}} \right) =$$  

$$= 4.74 + 5 \log \left( \frac{1.3}{10} \right) \approx 4.74 - 0.88 =$$  

$$= 3.85.$$
Q: Are stars with the same temperature equally bright?

Going back to black body radiation.

Luminosity = Energy/time emitted by object

Stars are round so we can use Stefan-Boltzmann Equation, unit

\[ L = A \cdot \sigma T^4 \quad [\text{W}] \]

\[ \sigma = 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \cdot \text{K}} \]

So flux emitted by star at distance \( d \) All energy emitted per second over shell area

\[ F = \frac{L}{4\pi d^2} = \frac{4\pi R^2}{4\pi d^2} \sigma T^4 \]

\[ F = \frac{R^2}{d^2} \sigma T^4 \]

So large stars appear brighter.

So if we know star temperature and distance to it we can find its size, even if we cannot resolve it with telescope.
Example

Betelgeuse \Rightarrow \begin{align*}
\text{d} &= \frac{3.2\text{pc}}{200\text{pc}} \\
M_V &= -5.85 \\
T &= 3200\text{K} \\
\text{depends, since it is variable star (text book value 3600K)}
\end{align*}

\begin{align*}
-(M_{\text{Betelgeuse}} - M_\odot) &= \\
= 2.5 \log_{10} \left( \frac{R_\odot^2}{10^2\text{pc}} \cdot \frac{57^4}{R_B^2} \cdot \frac{R_B^4}{10^2\text{pc}} \right) &= 5800\text{K}
\end{align*}

R_\odot = 7.10^8\text{m}

-(M_B - M_\odot) \begin{align*}
-(-5.85-4.74) &= -2.5 \log_{10} \left( \frac{T_0^4 R_\odot^2}{T_B^4 R_B^2} \right) \\
10.59 &= -2.5 \log_{10} \\
&\begin{align*}
&=-10 \log_{10} \left( \frac{T_0}{T_B} \right) - 5 \log_{10} \left( \frac{R_\odot}{R_B} \right)
\end{align*}
\end{align*}

\begin{align*}
R_B &= \frac{R_\odot}{-(M_B - M_\odot) + 10 \log_{10}(T_0/T_B)} \\
&= \frac{R_\odot}{-5} \\
&= \frac{R_\odot}{-0.0023} \approx 431R_\odot
\end{align*}

Wiki says (950-1200)R_\odot

Note that B is a variable star.
All of it great but how do we know a star temperature?

\[ B_\lambda = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1} \]

Wein's displacement law

\[ \lambda_{\text{max}} T = 0.00289 \text{ m K} \]

Sun has \( \lambda_{\text{max}} = \frac{0.0029 \text{ m K}}{5800 \text{ K}} = 5 \times 10^{-7} \text{ m} \approx 500 \text{ nm} \]

\( \uparrow \) green!

Q: why there are white, yellow, red, blue stars but no green stars?

A: physiology we are sensitive to overall shape of \( B_\lambda \) so it perceived as yellow for sun and white
A bit about experimental difficulties now days we can do the whole spectrum of Bx.

But in the old days it was limited to a few detectors with fixed bands.

Now we measure absolute magnitudes within some envelope around these bands.

\[ M_U, M_B, M_V \]

for short \( U, B, V \)

\[ U - B \] and \( B - V \) is function of temperature only for B.B.

\[ \text{So stars are not ideal B.B. but rich closer too} \]
Sun luminosity $L = 3.8 \times 10^{26} \text{ W}$

Per $1 \text{ m}^3$ we have

$$\frac{L}{\frac{4\pi R_\odot^3}{3}} = \frac{3.8 \times 10^{26} \text{ W}}{\frac{4\pi}{3} \left(7 \times 10^8\right)^3} = 0.26 \frac{\text{ W}}{\text{ m}^3}$$

A typical cell phone generates

Power = $V_0 I = 4 V_0 0.5 \text{ A} = 2 \text{ W}$