Black body radiation

Black body – absorbs all energy, thermolizes, emits thermal radiation

<example spectrum>

y-axis how much enery is emitted per unit wavelength

Wien's law (1893, empirical law)

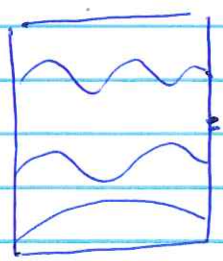
$$\text{spectral energy density} = \frac{\text{energy}}{\text{volume} \cdot \text{wavelength}} = \frac{f(\lambda T)}{\lambda^5}$$

Nobel prize, 1911

where $f(\lambda T)$ – universal function $\qquad \underline{b \approx 2.9 \cdot 10^{-3}\ \text{m.K}}$

$\lambda_{max} T = b$ – Wien's displacement constant

Model of a black body – closed conducting cavity



Because of the boundary conditions → standing waves

1D: $n \cdot \frac{\lambda_n}{2} = L \Rightarrow \lambda_n = \frac{2L}{n}$

~~wave vector~~ $\nu_n = \frac{c}{\lambda} = \frac{cn}{2L}$

How many modes?

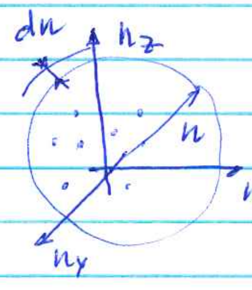
$\Delta k = \pi/L \, \Delta n$

3D: Standing waves in x, y, z

$$k_x = n_x \pi/L, \quad k_y = n_y \pi/L, \quad k_z = n_z \pi/L$$

Total wavevector

$$2\pi\nu/c = k = \sqrt{k_x^2 + k_y^2 + k_z^2} = \frac{\pi}{L} \underbrace{\sqrt{n_x^2 + n_y^2 + n_z^2}}_{n}$$



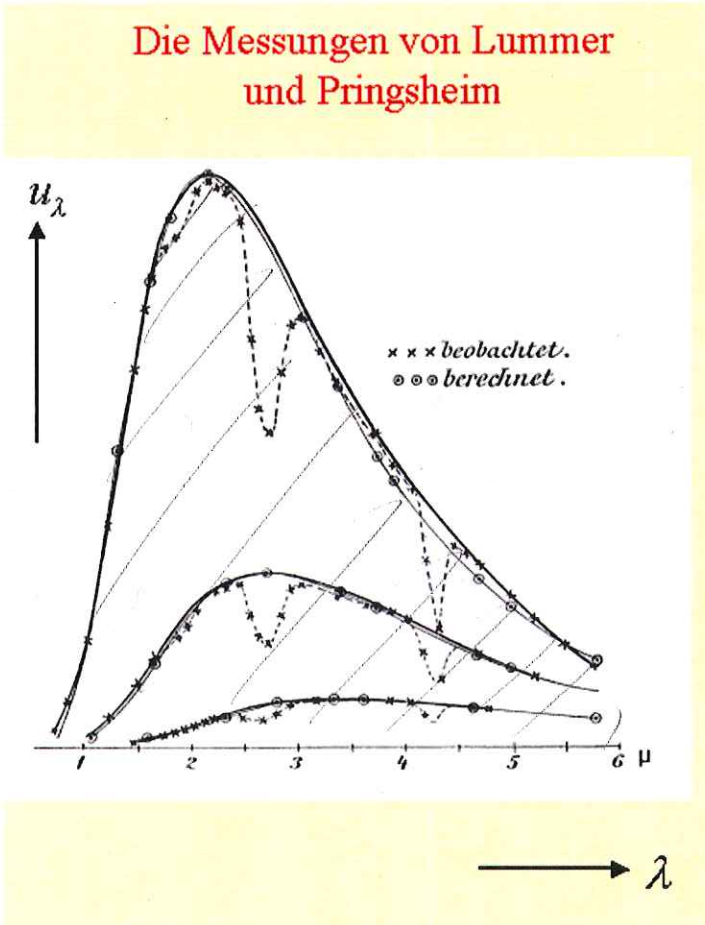
Number of modes for wavenumber values $k = |\vec{k}| \div k + dk$

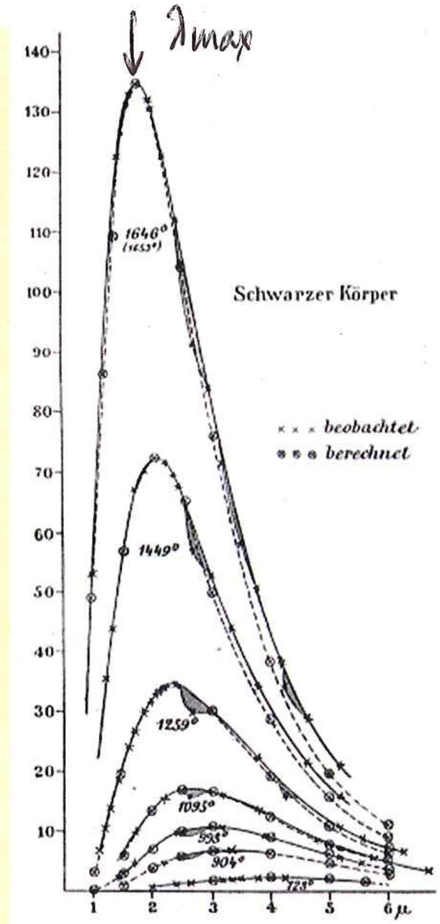
$\Downarrow$

area of the spherical layer

$$N(k)dk = 4\pi n^2 dn \cdot \frac{1}{8} \cdot 2$$

(to account for $k_{x,y,z}$ all > 0) → $\frac{1}{8}$; 2 ↑ polarization


Die Messungen von Lummer
und Pringsheim
u_λ
× × × beobachtet.
⊙ ⊙ ⊙ berechnet.
1
2
3
4
5
6 μ
λ


λmax
140
130
120
110
100
90
80
70
60
50
40
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10
1646°
1449°
1259°
1095°
904°
723°
Schwarzer Körper
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⊙ ⊙ ⊙ berechnet
1
2
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5
6 μ

~~Spectral~~ mode
density

$$\frac{\text{Number of modes}}{\text{volume} \cdot d\nu} = \frac{\frac{\pi}{\cancel{8}}\left(\frac{2L}{c}\right)^3 \nu^2 d\nu}{V} = \frac{8\pi}{c^3}\nu^2 d\nu$$

Energy density

$$u_\nu \, d\nu = \langle \text{energy per mode} \rangle \cdot \text{mode density}$$

Classical physics → Boltzman distribution

$$\langle E \rangle = \frac{\int_0^\infty E\, e^{-E/k_BT}\, dE}{\int_0^\infty e^{-E/k_BT}\, dE} = kT \frac{\int_0^\infty x e^{-x} dx}{\int_0^\infty e^{-x} dx} = k_BT$$

equipartition of energy

$$u_\nu \, d\nu = \frac{8\pi\nu^2}{c^3} k_BT \, d\nu$$

in terms of $\lambda = \frac{c}{\nu}$: $d\nu = -\frac{c}{\lambda^2} d\lambda$

$$u_\lambda \, d\lambda = -u_\nu \, d\nu = \frac{8\pi}{c^3} \frac{c^2}{\lambda^2} k_BT \left(\frac{c}{\lambda^2}\right) d\lambda$$

$$u_\lambda \, d\lambda = 8\pi k_B \cdot \frac{\lambda T}{\lambda^5} d\lambda$$ Rayleigh-Jeans expression

follows Wiens law

Works well for large λ, diverges at small λ

$$\int_0^\infty u_\lambda \, d\lambda = 8\pi k_BT \int_0^\infty \frac{d\lambda}{\lambda^4} = \frac{8\pi k_BT}{3} \left.\frac{1}{\lambda^3}\right|_0^\infty \rightarrow$$

total amount of radiating energy diverges

Vergleich der Strahlungsgesetze

Wien $u_\lambda = 8\pi hc\lambda^{-5}\exp\{-\frac{hc}{k\lambda T}\}$

Planck $u_\lambda = 8\pi hc\lambda^{-5}[\exp\{\frac{hc}{k\lambda T}-1\}]^{-1}$

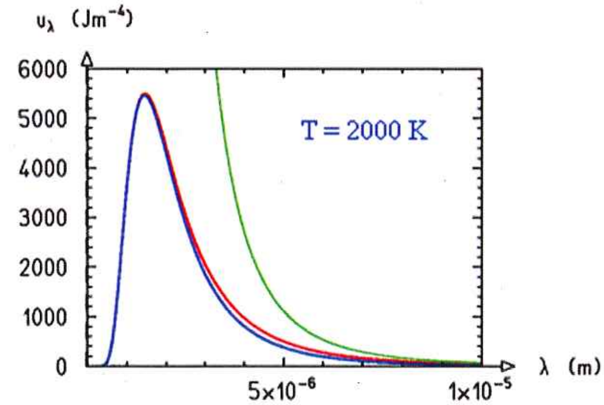
Reyleigh-Jeans $u_\lambda = \frac{8\pi}{\lambda^4}kT$

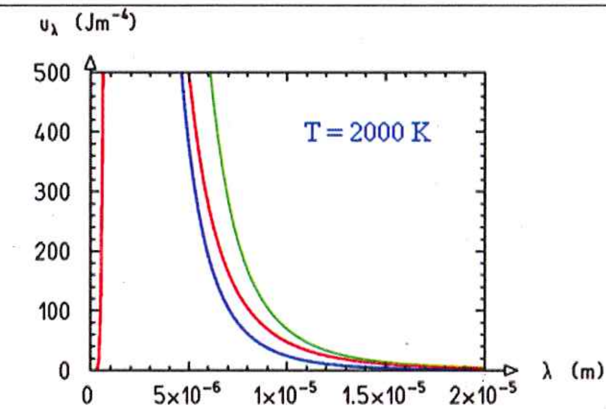
Vor Plancks Arbeiten waren die Konstanten nicht bekannt.

Das Plancksche Gesetz beschreibt den gesamten Wellenlängenbereich.

Das Wiensche Gesetz ist eine gute Näherung bei kurzen Wellenlängen.

Das Rayleigh-Jeans-Gesetz ist eine gute Näherung bei großen Wellenlängen. Es divergiert für kleine Wellenlängen.





Plank's quandization

Energy of each mode is quantized, namely it can only have values

$$\varepsilon_n = n h \nu$$

In this case the average energy per mode

$$\langle E \rangle = \frac{\sum_{n=0}^{\infty} \varepsilon_n e^{-\varepsilon_n / k_B T}}{\sum_{n=0}^{\infty} e^{-\varepsilon_n / k_B T}} = \frac{\hbar\nu \cancel{\frac{h\nu}{k_BT}} \sum_{n=0}^{\infty} n e^{-\hbar\nu \cdot n / k_B T}}{\cancel{k_BT} \sum_{n=0}^{\infty} e^{-(\hbar\nu / k_B T)\cdot n}}$$

$$\langle E \rangle = \frac{h\nu \, \cancel{k_B T \alpha} \sum_{n=0}^{\infty} n e^{-\alpha n}}{\sum_{n=0}^{\infty} e^{-\alpha n}} \qquad \alpha = \frac{h\nu}{k_B T}$$

$$\sum_{n=0}^{\infty} e^{-\alpha n} = \frac{1}{1 - e^{-\alpha}}$$

$$\sum_{n=0}^{\infty} n e^{-\alpha n} = -\frac{\partial}{\partial \alpha} \sum_{n=0}^{\infty} e^{-\alpha n} = -\frac{\partial}{\partial \alpha}\left(\frac{1}{1-e^{-\alpha}}\right) =$$

$$= \frac{e^{-\alpha}}{(1-e^{-\alpha})^2}$$

$$\langle E_\nu \rangle = h\nu \frac{e^{-\alpha}}{1 - e^{-\alpha}} = \hbar\nu \frac{1}{e^{\frac{h\nu}{k_B T}} - 1}$$

$$\langle E_\nu \rangle \to k_B T \quad \text{for } \nu \to 0$$

$$\to \hbar\nu e^{-\hbar\nu / k_B T} \qquad \nu \to \infty$$

$$u_\nu \, d\nu = \frac{8\pi}{c^3} \nu^2 \cdot h\nu \frac{1}{e^{h\nu/k_BT} - 1} \, d\nu =$$

$$= \frac{8\pi h}{c^3} \nu^3 \frac{1}{e^{h\nu/k_BT} - 1} \, d\nu$$

and rewriting it through λ s

$$\underline{u_\lambda \, d\lambda} = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda k_BT} - 1} \, d\lambda$$

energy density states "inside the box"

The radiation flux (intensity)

$$I = \frac{dE}{dt \cdot dA} = \frac{dE}{\frac{dx}{c} \cdot dA} = c \cdot \frac{dE}{dV} = c \cdot u$$

$$I_\lambda \, d\lambda = c \cdot u_\lambda \, d\lambda = \frac{8\pi h c^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_BT} - 1} \, d\lambda$$

Total emitted radiation

$$I(T) = \int_0^\infty I_\lambda \, d\lambda =$$

$$= \int_0^\infty 8\pi hc^2 \frac{1}{e^{hc/\lambda k_BT} - 1} \frac{d\lambda}{\lambda^5} = \left\{ \begin{array}{l} x = \frac{hc}{\lambda k_BT} \\ \lambda = \frac{hc}{x k_BT} \\ d\lambda = -\frac{hc}{x^2 k_BT} dx \end{array} \right.$$

$$= 8\pi hc^2 \int_0^\infty \frac{1}{e^x - 1} \frac{hc}{x^2 k_BT} dx \cdot x^5 \left(\frac{k_BT}{hc}\right)^5 =$$

$$= 8\pi hc^2 \left(\frac{k_BT}{hc}\right)^4 \underbrace{\int_0^\infty \frac{x^3 \, dx}{e^x - 1}}_{\pi^4/15}$$

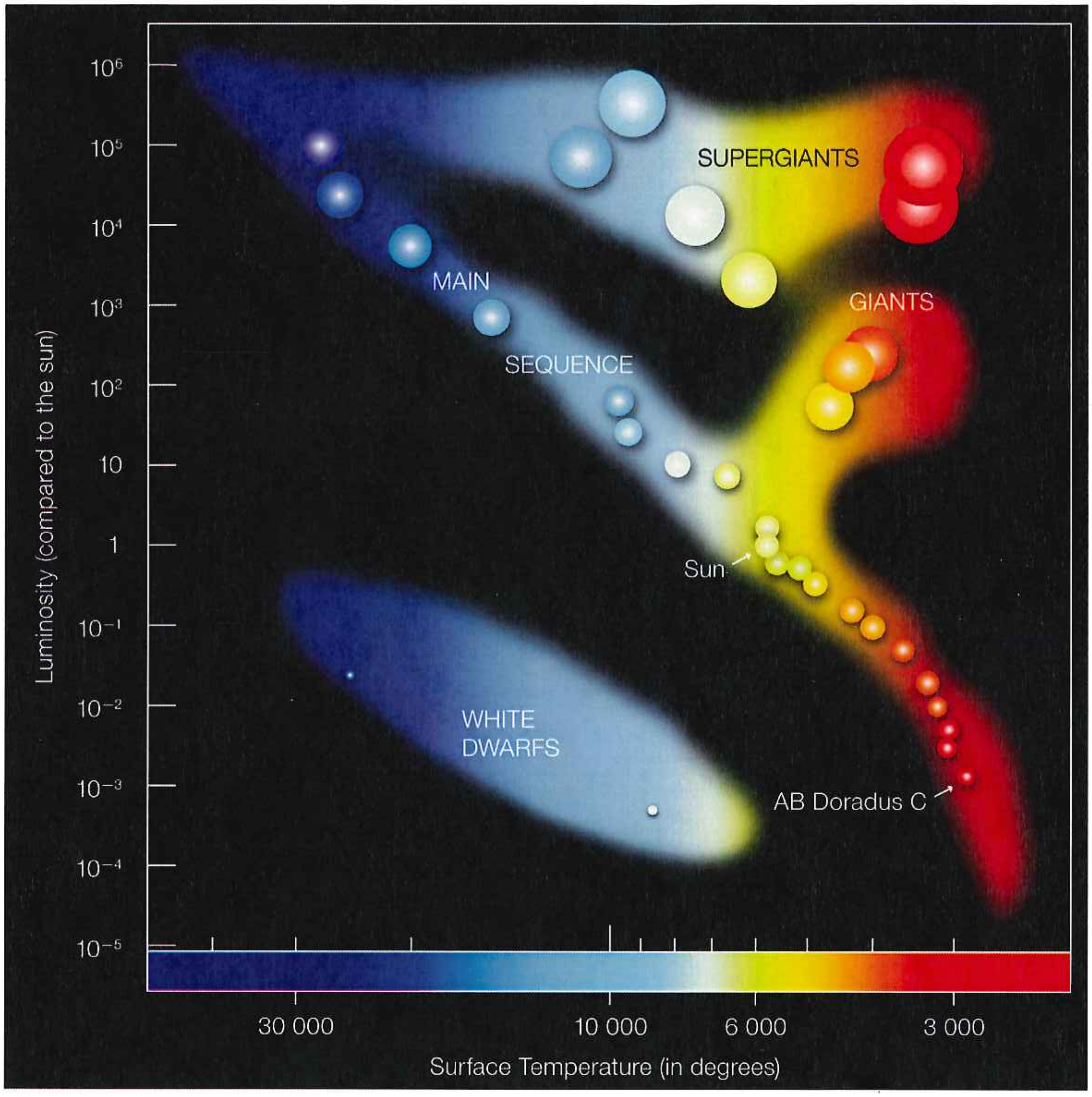
Total emitted intensity

$$I(T) = \underbrace{\left(\frac{8\pi^5 k_B^4}{15 h^3 c^2}\right)}_{} T^4$$

$\sigma = 5{,}67 \cdot 10^{-8} \frac{W}{m^2 K^4}$ Stephan - Boltzman constant

Luminocity of the star

$$L = 4\pi R^2 \cdot I = 4\pi R^2 \cdot \sigma T^4$$



10^6
10^5
10^4
10^3
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
Luminosity (compared to the sun)
SUPERGIANTS
MAIN
SEQUENCE
GIANTS
Sun
WHITE
DWARFS
AB Doradus C
30 000
10 000
6 000
3 000
Surface Temperature (in degrees)