P1

Lecture 08

Q: How to measure mass of Mercury and Venus which have no satelites.

A: you cannot (at least reliably) until you make such satelite your self. So mass of this planet were measured only a few decades ago. Before the were estimates which were wildly different

perihelion and aphelion speed

$\dot{r} = 0$

$$E = \frac{\mu v_p^2}{2} - \frac{\alpha}{\underbrace{a(1-e)}_{r_p}} = \frac{\mu v_a^2}{2} - \frac{\alpha}{\underbrace{a(1+e)}_{r_a}}$$

note $\cancel{r_a = a(1+e)}$
$\cancel{r_p = a(1-e)}$

$$L = \mu r_a v_a = \mu r_p v_p$$

$$\Rightarrow \frac{r_a}{r_p} = \frac{v_p}{v_a} = \frac{a(1+e)}{a(1-e)}$$

$$\frac{\mu v_p^2}{2} - \frac{\alpha}{a(1-e)} = \frac{\mu v_p^2}{2} \cdot \left[\frac{(1-e)}{(1+e)}\right]^2 - \frac{\alpha}{a(1+e)}$$

$$v_p^2 = \frac{\frac{\alpha}{a}\left[\frac{1}{1-e} - \frac{1}{1+e}\right]}{\frac{\mu}{2}\left[1 - \left(\frac{1-e}{1+e}\right)^2\right]} =$$

$$= \frac{\frac{\alpha}{a}\,\frac{2e}{1-e^2}}{\frac{\mu}{2}\left[\frac{4e}{(1+e)^2}\right]} = \frac{\alpha}{a\mu}\,\frac{(1+e)^2}{1-e^2} =$$

$$= \frac{\alpha}{a\mu}\,\frac{1+e}{1-e} =$$

$$= \boxed{\frac{G M_T}{a}\,\frac{1+e}{1-e} = v_p^2}$$

Note $\boxed{v_a^2 = v_p\left[\frac{1-e}{1+e}\right]^2 = \frac{G M_T}{a}\,\frac{1-e}{1+e}}$

Halley comet revisited

See previous lecture

$e = 0.9673, \quad a = 18\,\text{au}, \quad b = 4.55\,\text{au}$

$$v_p = \sqrt{\frac{GM_\odot}{a}\,\frac{1+e}{1-e}} =$$

$$= \sqrt{\frac{6.67\cdot 10^{-11}\cdot 2\cdot 10^{30}}{18\cdot 1.5\cdot 10^{11}}}\sqrt{\frac{1+e}{1-e}} =$$

$$= \cancel{4.943} \simeq \sqrt{4.9\cdot 10^{7}}\sqrt{\frac{1+0.9673}{1-0.9673}} =$$

$$= 7031 \cdot 7.75$$

$$\simeq 54.5\ \text{km/s}$$

$$v_a = 906\ \text{m/s} = \sqrt{\frac{GM_\odot}{a}\,\frac{1-e}{1+e}}$$

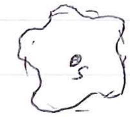
What about Sun - Earth - Moon system.

Q: How orbit of moon looks like

$M_{\odot} = 2 \cdot 10^{30}$ kg

$M_{\oplus} = 6 \cdot 10^{24}$ kg

$M_M = 7.3 \cdot 10^{22}$

 or 

$r_S = 1\,au = 1.5 \cdot 10^{11}$ m

$r_{EM} = 4 \cdot 10^{8}$ m

not too far apart!

$$F_{Sun\text{-}Moon} = G \frac{M_{\odot} M_m}{r_S^2} =$$

$$= 4.33 \cdot 10^{20} \text{ N}$$

$$\cancel{F_{Sun}}\ F_{Earth\text{-}moon} = \frac{G M_{\oplus} M_M}{(r_{EM})^2} =$$

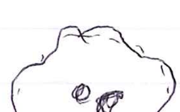
$$= 1.8 \cdot 10^{20}$$

Earth speed on orbit $\approx$
$e \simeq 0$

$$v_E \approx v_{Moon} = \sqrt{\frac{G M_{\odot}}{a}} = \underline{\underline{30 \text{ km/s}}}$$

$$v_{Moon \text{ around Earth}} = \sqrt{\frac{G M_{\oplus}}{r_{em}}} = 10^3 \text{ m/s} = 1 \text{ km/s}$$

So Moon orbit with respect to stars looks like



In C.M system of Earth - Moon what is the speed of Earth?

recall $r_1 = -\frac{\mu}{m_1} r$

$r_2 = \frac{\mu}{m_2} r$

$\Rightarrow$ $v_1 = \dot{r}_1$ $\quad \mu = \frac{m_1 \cdot m_2}{m_1 + m_2} =$

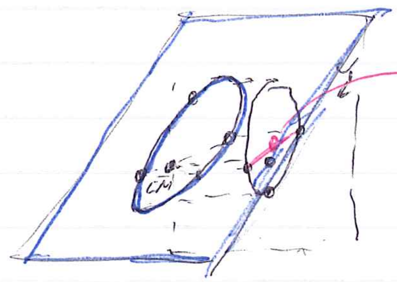
$v_2 = \dot{r}_2$ $\Rightarrow$ $= \frac{M_M \cdot M_\oplus}{M_M + M_\oplus} \simeq$

$\frac{v_1}{v_2} = \frac{m_2}{m_1}$ $\quad \approx M_M$

So if Moon speed v_M then ~~$v_E = v_M \cdot M$~~

$$\boxed{v_E = \frac{M_M}{M_E} \cdot v_M = \frac{7.3 \cdot 10^{22}}{6 \cdot 10^{24}} \cdot 1000 \simeq \approx 12\ m/s}$$

Almost always plane of the orbit is inclined:



C.M' – appeared C.M. but for observed ellipce it should be in other point

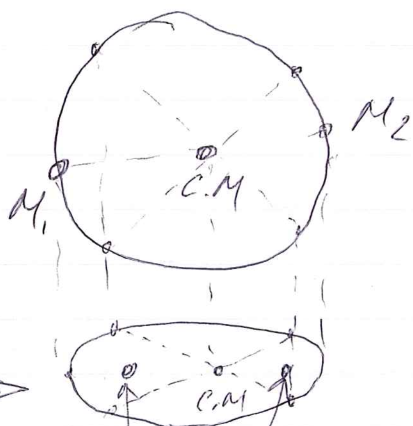
so there is a way to figure out inclanation angle 'i'

Note: move forward present as puzzle / Q

Simple example:

$M_1 = M_2$ ~~...~~ $e = 0$ i.e. circular orbits

in the inclined projection it will look like



is this possible?

But for ellipse C.M. must be in the focus!

So observations contradict each other and need to be corrected

Star mass measurements

We need 2 star bounded by gravitation, ideally we should be able to ~~to~~ see (resolve) both of them.

This put some constrains, on top of above orbital period should be ~ 1 year

Let's asume 2 stars with $T = 1$ year, what should be the distance between them.

$$M_1 = M_\odot = M_2 \Rightarrow M_T = 2M_\odot$$

Recall deriv

$$1\text{ year} = P^2 = \frac{a^3}{\alpha} = \frac{a^3}{2} \sim \quad \alpha = \frac{M_T}{M_s}$$

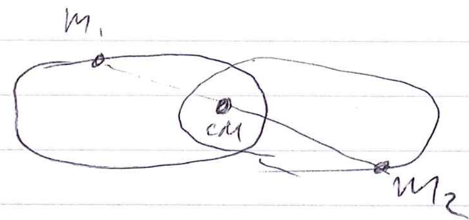
$$a = \sqrt[3]{2} \approx \underline{1.26\ au}$$

This quite close $\simeq$ distance of Sun-Earth

$$a_1 = a\frac{\mu}{m_1}$$

$$a_2 = a\frac{\mu}{m_2}$$

$$\Rightarrow a_1 + a_2 = a\left(\frac{\mu}{m_1} + \frac{\mu}{m_2}\right) = a$$



$$\frac{a_1}{a_2} = \frac{m_2}{m_1}$$

observable

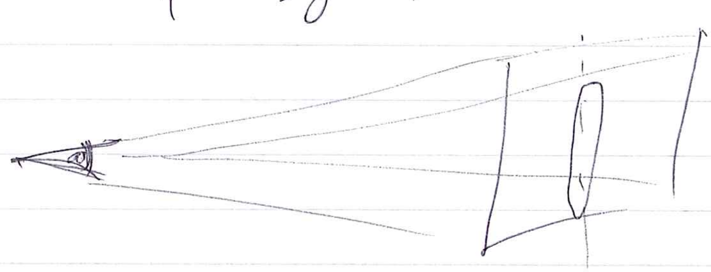
Once we know $a = a_1 + a_2$ we can find total mass

$$M_T = \frac{4\pi^2 a^3}{G P^2}$$

$$M_T = m_1 + m_2 = m_1 + m_1\frac{a_1}{a_2} =$$

$$\Rightarrow \boxed{m_1 = \frac{M_T}{1 + a_1/a_2} = \frac{a_2}{a_2 + a_1} M_T \qquad m_2 = \frac{a_1}{a_1 + a_2} M_T}$$

So life sounds easy. But this only in the case of orbits in plane orthogonal to lign of sight.



Once we know star masses,

we can observe striking correlation between star mass and its luminocity (how much energy is emited by star)